

Address-of and dereference operators

Copyright (c) 2025 - 2010 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.
This document was produced by using LibreOffice.

& address-of operator

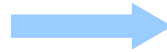
***** dereference operator

Address-of operator and dereferencing operator

address-of operator **&** : the address of a variable

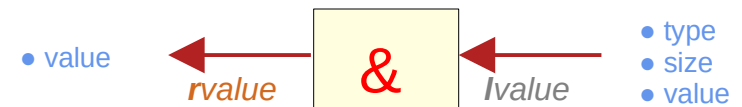
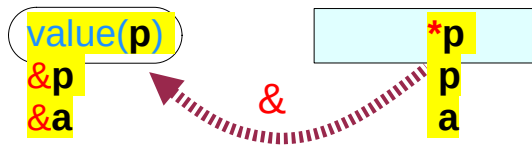
a **variable** has a memory location
whose value can be changed
by an assignment

a **variable** must be an **lvalue**



&variable returns
the address of a variable

&variable returns an **rvalue**



Address-of operator and dereferencing operator

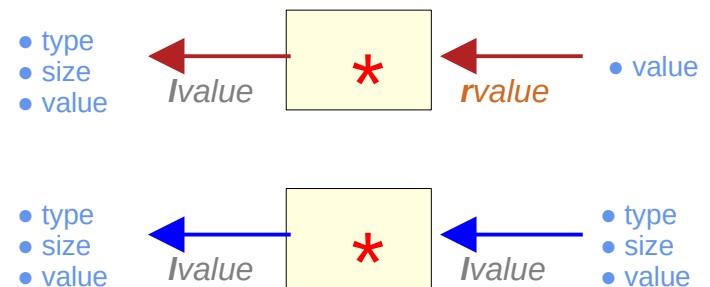
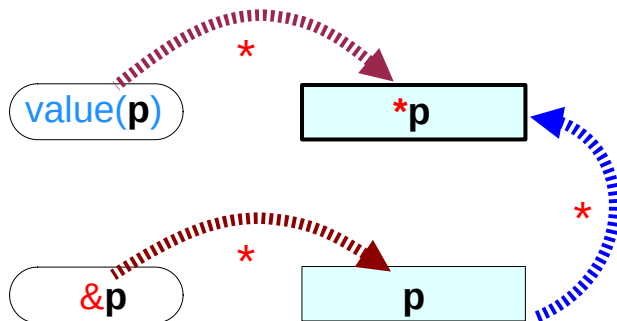
*dereferencing operator ***** : the content at an address*

*an **address** can be an address value or a pointer variable that holds an address value*

*an **address** must be an **rvalue**, or evaluated to be an **rvalue***

****** **address** returns the content value at the address*

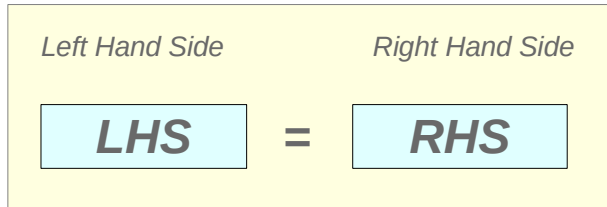
****** **address** returns an **lvalue***



***p** $\equiv$ ***value(p)**

Ivalue and rvalue in assignments

an assignment statement



- in the **LHS**, only **Ivalue** can exist
- **rvalue** can exist only in the **RHS**

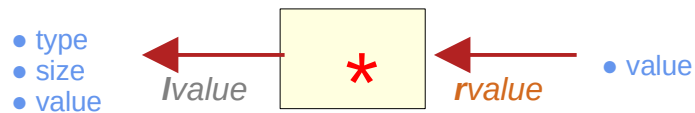
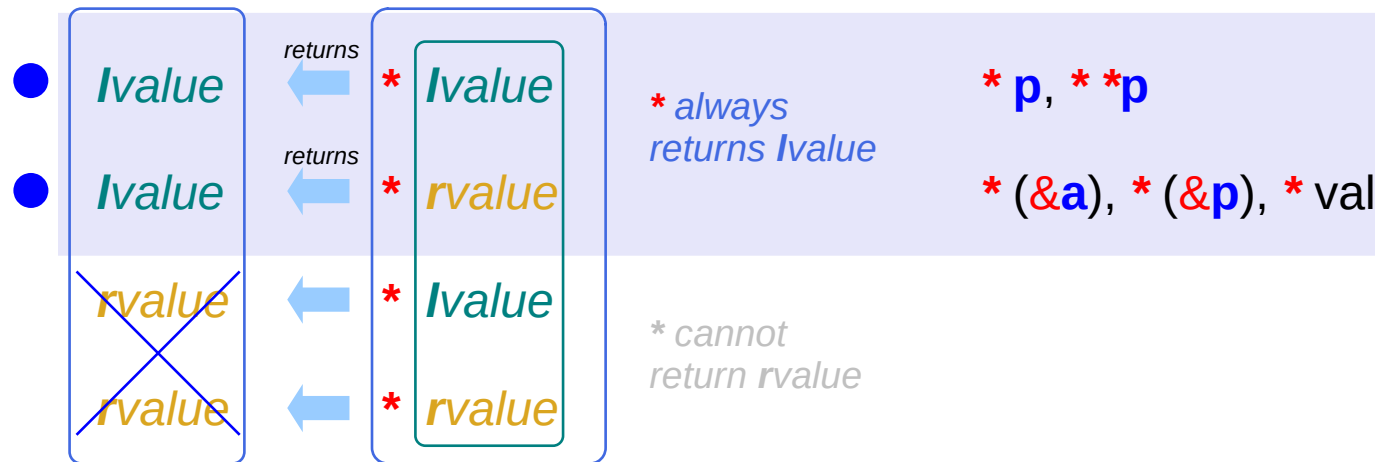
```
int  a, b = 10 ;  
int  *p, *q = &a ;
```

a, b, p, q	: Ivalues	... variables	... RW
*p, *q	: Ivalues	... variables	... RW
&a, &b	: rvalues	... constants	... RO

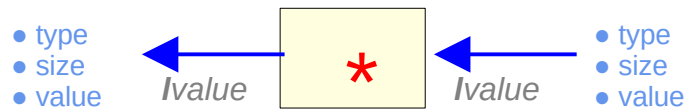
Ivalue	=	Ivalue
Ivalue	=	rvalue
rvalue	=	Ivalue
rvalue	=	rvalue

p	=	q ;
p	=	&a ;
&a	=	p ;
&a	=	&b ;

Ivalue and rvalue with * and & operators



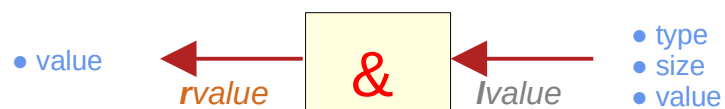
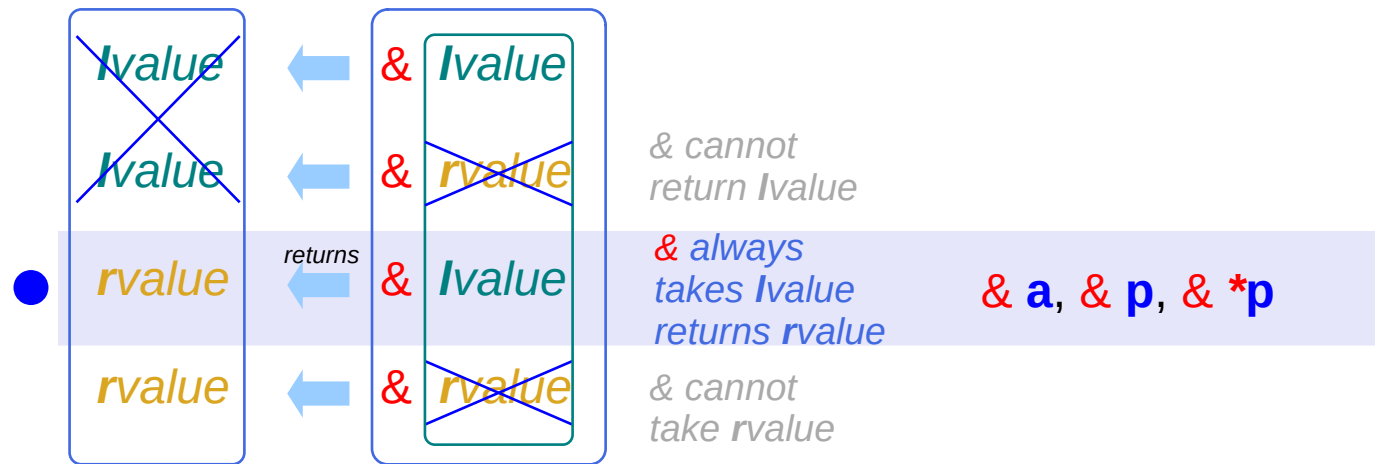
`*` can be applied to either an **lvalue** variable or a **rvalue** address



`*` **operand** becomes an **lvalue** variable thus it can be applied successively.

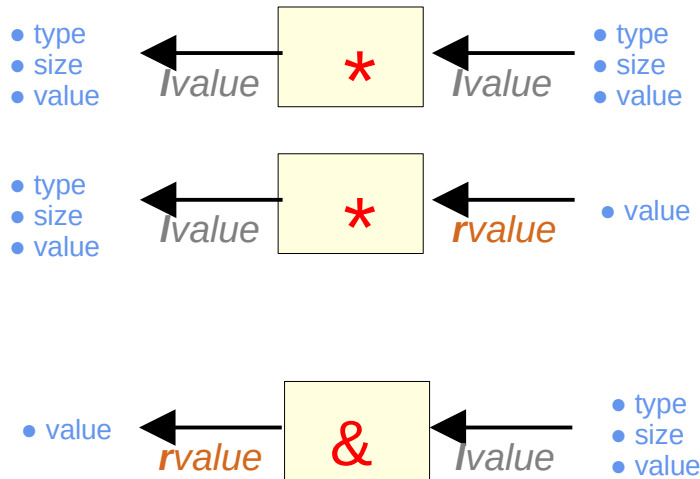
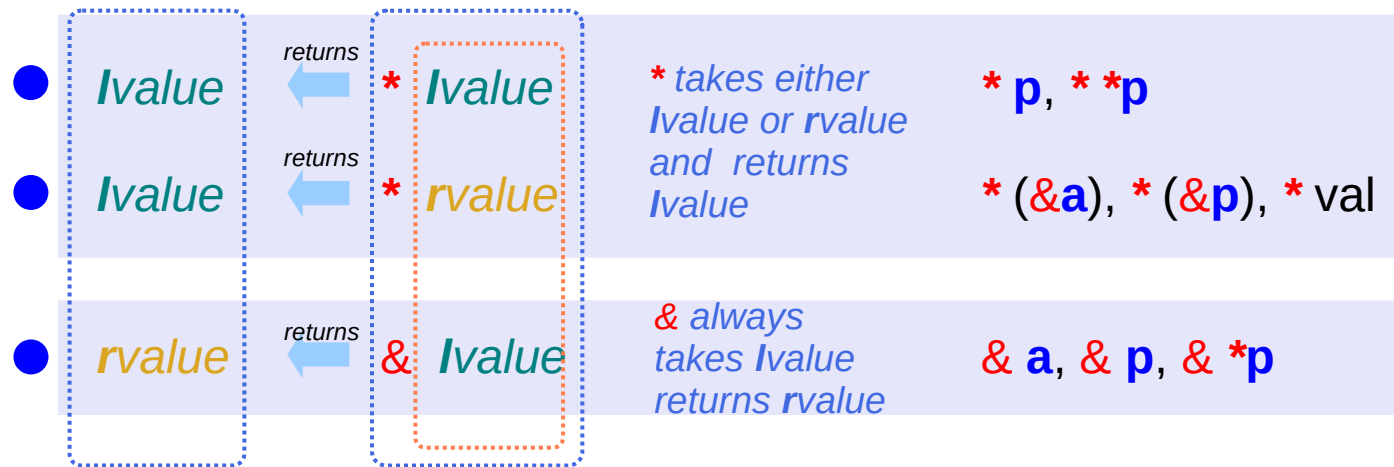
$$*p \equiv *value(p)$$

lvalue and rvalue with * and & operators



& can be applied to only an **lvalue** variable and returns only an **rvalue** address

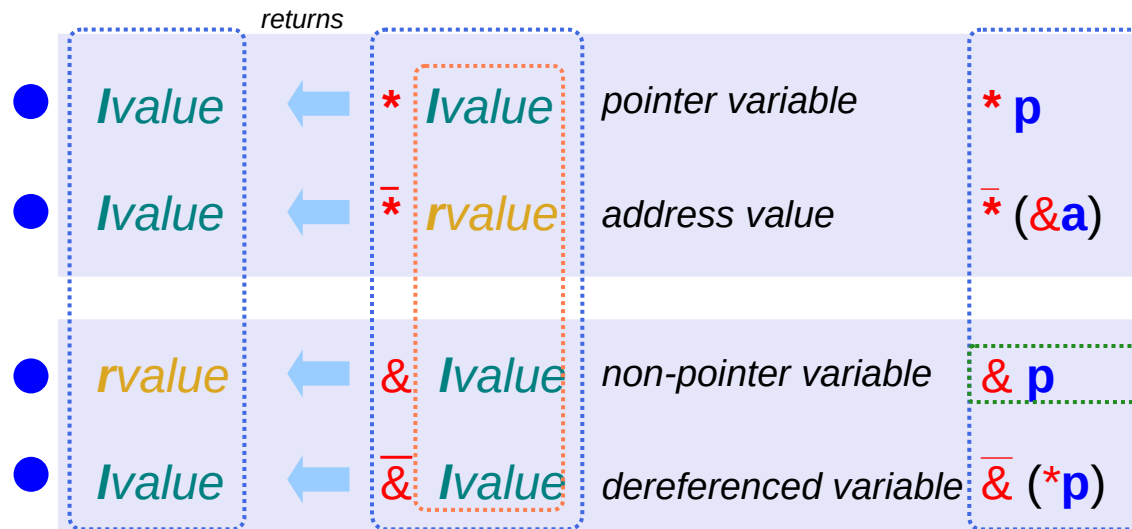
Ivalue and rvalue with * and & operators



* (an **Ivalue** variable)
 * (an **rvalue** address)
 * **operand** : an **Ivalue** variable
 * can be applied successively.

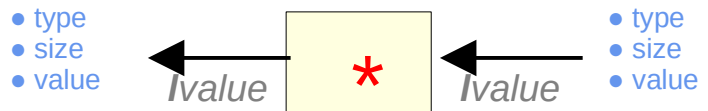
& (an **Ivalue** variable)
 & **operand** : an **rvalue** address,
 & cannot be applied successively.

Ivalue and rvalue with * and & operators

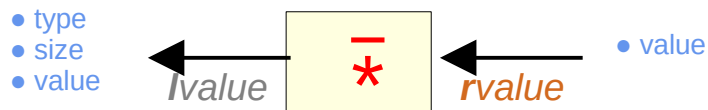


* can be applied successively.
 $\overline{\&}$ can be applied successively.

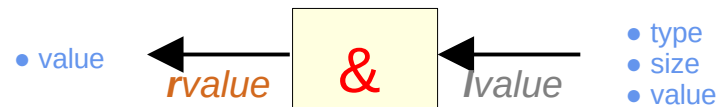
**p
 $\overline{\&}^{**}p = *p$
 $\overline{\&\&}^{**}p = \overline{\&}^{*}p = p$



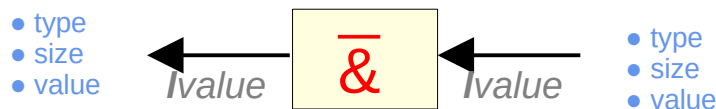
* (an Ivalue variable)
 * operand: an Ivalue variable



$\overline{*}$ (an rvalue address)
 $\overline{*}$ operand: an Ivalue variable



& (an Ivalue variable)
 & operand: an rvalue address



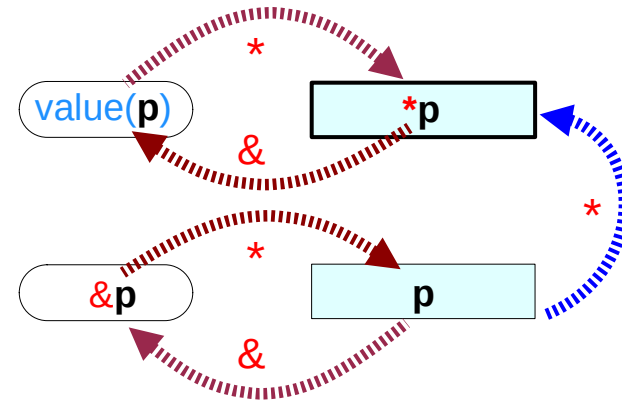
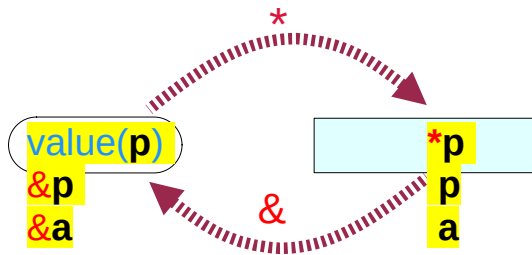
$\overline{\&}$ (an dereferenced Ivalue variable)
 $\overline{\&}$ operand: an Ivalue variable

Variable types

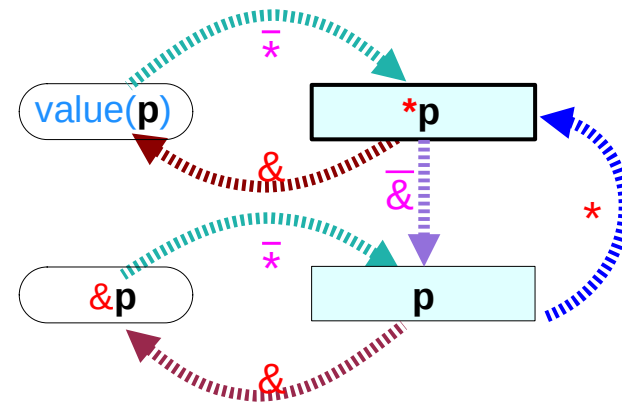
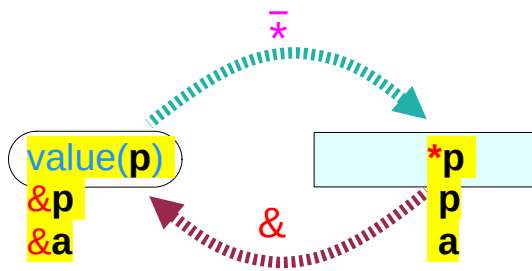
a	non-pointer variables	<i>lvalue</i>
p	pointer variables	<i>lvalue</i>
*p	dereferenced variables	<i>lvalue</i>
val	address values	<i>rvalue</i>

Comparisons (1)

C operators : $*$, $\&$



augmented operators : $\bar{*}$, $\bar{\&}$



Comparisons (2)

C operators : *, &

	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	val	<i>rvalue</i>

<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>

augmented operators : $\bar{*}$, $\bar{\&}$

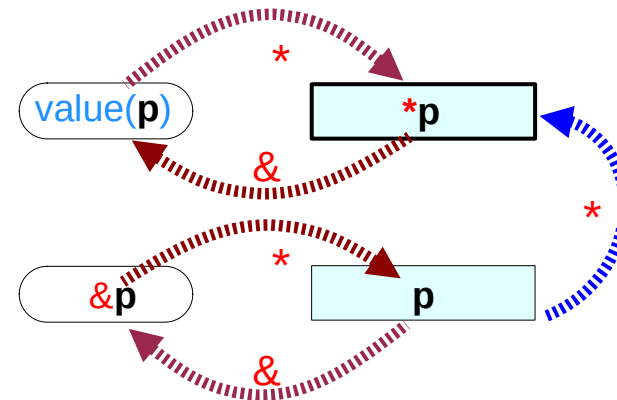
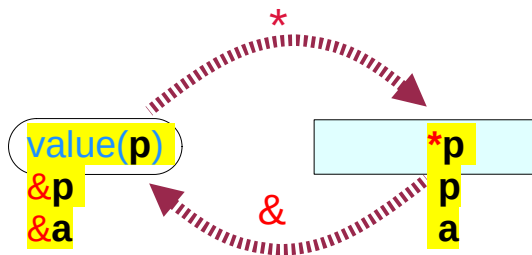
	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{*}$	val	<i>rvalue</i>

<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{\&}$	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>

C operators : *, &

	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	val	<i>rvalue</i>

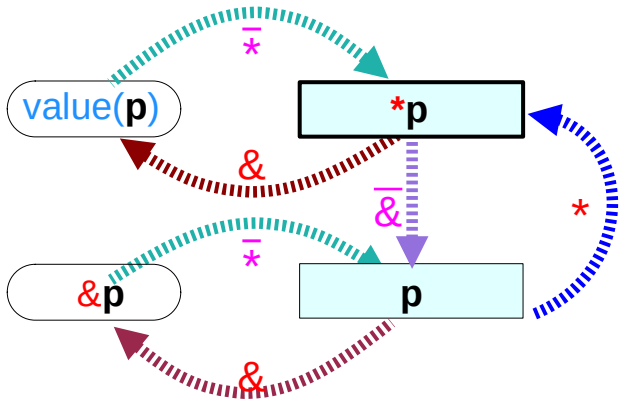
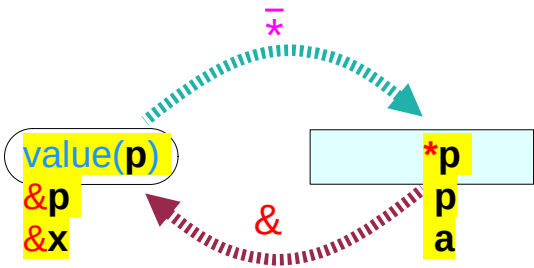
<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>



Augmented operators : $\bar{*}$, $\bar{\&}$

	$\times$	a	<i>Ivalue</i>
<i>Ivalue</i> ←	$*$	p	<i>Ivalue</i>
<i>Ivalue</i> ←	$*$	* p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{*}$	val	<i>rvalue</i>

<i>rvalue</i> ←	$\&$	a	<i>Ivalue</i>
<i>rvalue</i> ←	$\&$	p	<i>Ivalue</i>
<i>rvalue</i> ←	$\&$	* p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{\&}$	* p	<i>Ivalue</i>
	$\times$	val	<i>rvalue</i>



Recursive application of the address-of operator

~~$\&(\&(\&(c[i])[j])[k])$~~

$\&$ C operator

takes only **lvalue** variable

returns **rvalue** address value

thus, the above expression is not possible

successive application of $\&$ is not possible

but, $*p$ becomes a **lvalue** variable

$*$ operator can be applied successively.

$\overline{\&}(\overline{\&}(\overline{\&}(c[i])[j])[k])$

$\overline{\&}$ mathematical operator

takes a dereferenced **lvalue** variable

returns **lvalue** variable

successive application of $\overline{\&}$ is not possible

but, $*p$ becomes a **lvalue** variable

$*$ operator can be applied successively.

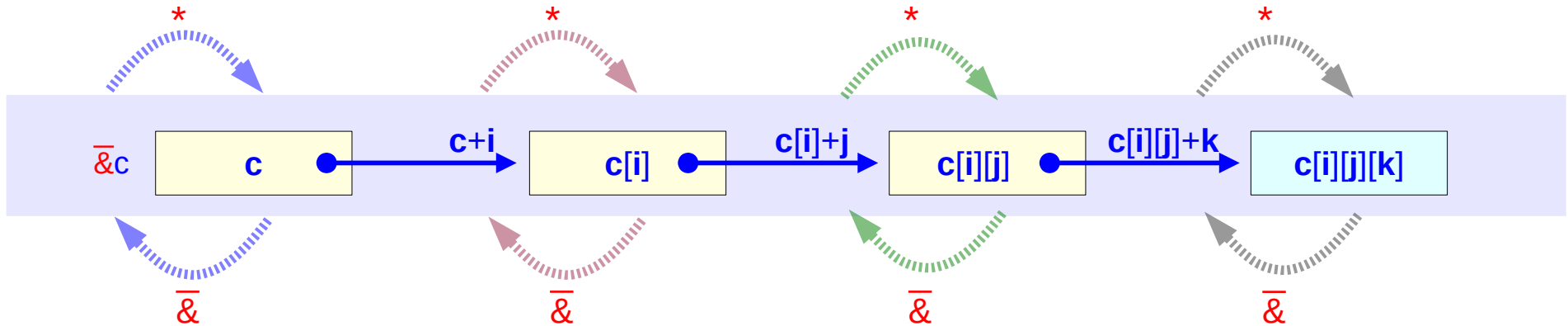
Recursive application of the address-of operator

$$*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$$

$$\begin{aligned} &*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k}) = \\ &*(*(\mathbf{c}[\mathbf{i}]+\mathbf{j})+\mathbf{k}) = \\ &*(\mathbf{c}[\mathbf{i}][\mathbf{j}]+\mathbf{k}) = \\ &\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}] \end{aligned}$$

$$\overline{\&}(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}])$$

$$\begin{aligned} &\overline{\&}(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}]) = \\ &(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])+\mathbf{k}) = \\ &((\overline{\&}(\mathbf{c}[\mathbf{i}])+\mathbf{j})+\mathbf{k}) = \\ &(((\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k}) \end{aligned}$$



Recursive application of the address-of operator

$\bar{*}v(\bar{*}v(\bar{*}v(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$ $*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$\text{value}(\mathbf{c}[\mathbf{i}][\mathbf{j}]+\mathbf{k})$	$= \mathbf{c}[\mathbf{i}][\mathbf{j}]$	$+ \mathbf{k} * \text{sizeof}(\mathbf{c}[0][0][0])$
$\text{value}(\mathbf{c}[\mathbf{i}] + \mathbf{j})$	$= \mathbf{c}[\mathbf{i}]$	$+ \mathbf{j} * \text{sizeof}(\mathbf{c}[0][0])$
$\text{value}(\mathbf{c}+\mathbf{i})$	$= \mathbf{c}$	$+ \mathbf{i} * \text{sizeof}(\mathbf{c}[0])$

Recursive application of the address-of operator

$*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$\overline{*}\mathbf{v}(\overline{*}\mathbf{v}(\overline{*}\mathbf{v}(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$*(*(\mathbf{c}[\mathbf{i}]+\mathbf{j})+\mathbf{k})$

$*(\mathbf{c}[\mathbf{i}][\mathbf{j}]+\mathbf{k})$

$\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]$

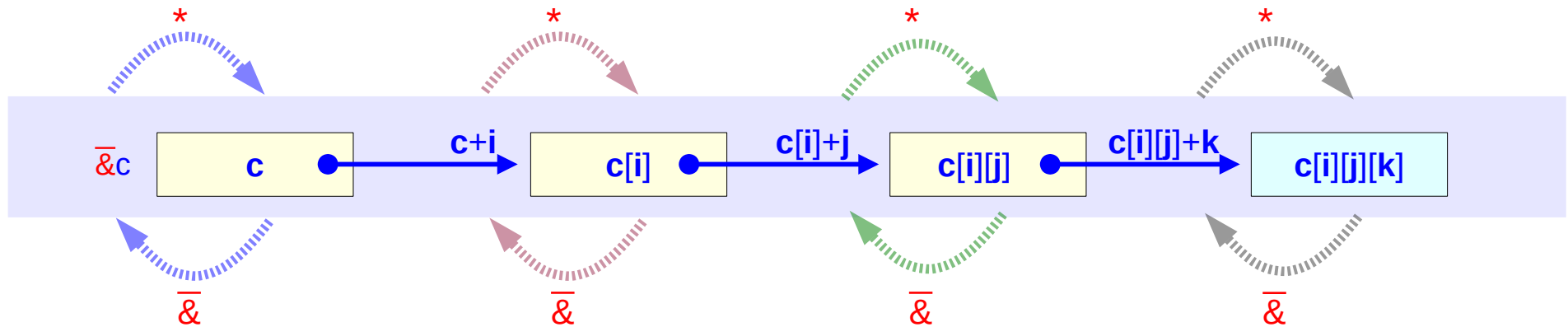
$\overline{\&}(\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}])$	$= \mathbf{c}[\mathbf{i}][\mathbf{j}]$	$+ \mathbf{k}$
$\overline{\&}(\mathbf{c}[\mathbf{i}][\mathbf{j}])$	$= \mathbf{c}[\mathbf{i}]$	$+ \mathbf{j}$
$\overline{\&}(\mathbf{c}[\mathbf{i}])$	$= \mathbf{c}$	$+ \mathbf{i}$

$\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]$	$= *(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})$
$\mathbf{c}[\mathbf{i}][\mathbf{j}]$	$= *(\mathbf{c}[\mathbf{i}] + \mathbf{j})$
$\mathbf{c}[\mathbf{i}]$	$= *(\mathbf{c} + \mathbf{i})$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}
 \overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\
 \overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\
 \overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i
 \end{aligned}$$

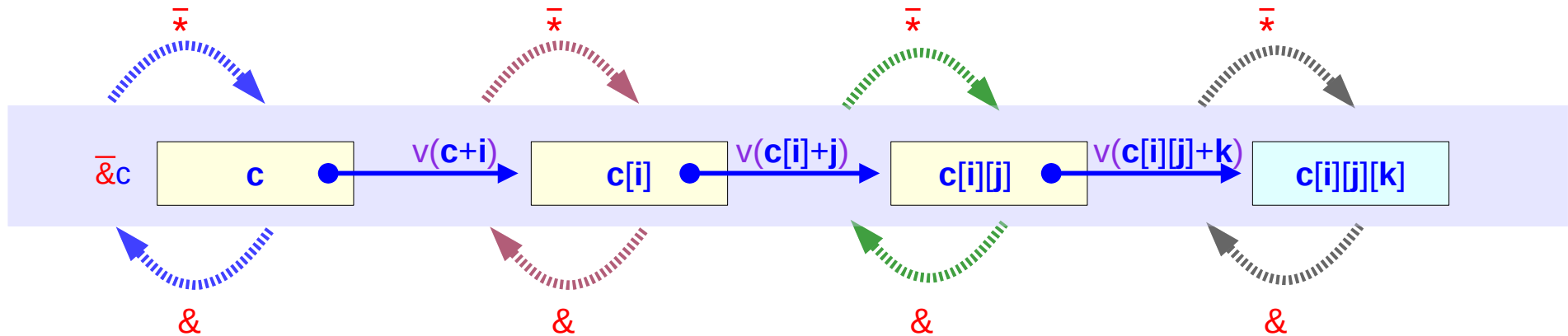
$$\begin{aligned}
 * \overline{\&}(c[i][j][k]) &= * \overline{\&}(* (c[i][j] + k)) &= *(c[i][j] + k) \\
 * \overline{\&}(c[i][j]) &= * \overline{\&}(* (c[i] + j)) &= *(c[i] + j) \\
 * \overline{\&}(c[i]) &= * \overline{\&}(* (c + i)) &= *(c + i)
 \end{aligned}$$



Two step deferencing in type II (1) – without skipping

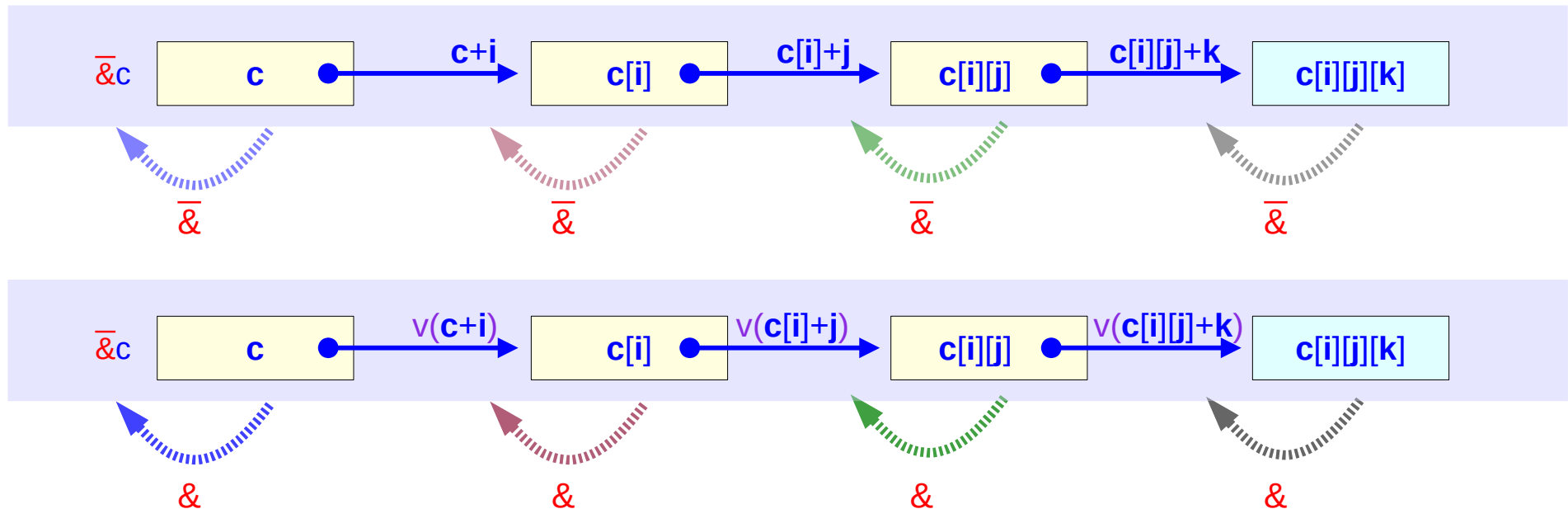
$$\begin{aligned}
 \&(\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]) &= \&(\ast(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})) &= \text{value}(\mathbf{c}[\mathbf{i}][\mathbf{j}]) + \mathbf{k} &= \mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k} \ast \text{sizeof}(\mathbf{c}[0][0][0]) \\
 \&(\mathbf{c}[\mathbf{i}][\mathbf{j}]) &= \&(\ast(\mathbf{c}[\mathbf{i}] + \mathbf{j})) &= \text{value}(\mathbf{c}[\mathbf{i}]) + \mathbf{j} &= \mathbf{c}[\mathbf{i}] + \mathbf{j} \ast \text{sizeof}(\mathbf{c}[0][0]) \\
 \&(\mathbf{c}[\mathbf{i}]) &= \&(\ast(\mathbf{c} + \mathbf{i})) &= \text{value}(\mathbf{c}) + \mathbf{i} &= \mathbf{c} + \mathbf{i} \ast \text{sizeof}(\mathbf{c}[0])
 \end{aligned}$$

$$\begin{aligned}
 \bar{\ast}\&(\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]) &= \bar{\ast}\&(\ast(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})) &= \bar{\ast}\text{value}(\mathbf{c}[\mathbf{i}][\mathbf{j}]) + \mathbf{k} &= \bar{\ast}(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k} \ast \text{sizeof}(\mathbf{c}[0][0][0])) \\
 \bar{\ast}\&(\mathbf{c}[\mathbf{i}][\mathbf{j}]) &= \bar{\ast}\&(\ast(\mathbf{c}[\mathbf{i}] + \mathbf{j})) &= \bar{\ast}\text{value}(\mathbf{c}[\mathbf{i}]) + \mathbf{j} &= \bar{\ast}(\mathbf{c}[\mathbf{i}] + \mathbf{j} \ast \text{sizeof}(\mathbf{c}[0][0])) \\
 \bar{\ast}\&(\mathbf{c}[\mathbf{i}]) &= \bar{\ast}\&(\ast(\mathbf{c} + \mathbf{i})) &= \bar{\ast}\text{value}(\mathbf{c}) + \mathbf{i} &= \bar{\ast}(\mathbf{c} + \mathbf{i} \ast \text{sizeof}(\mathbf{c}[0]))
 \end{aligned}$$



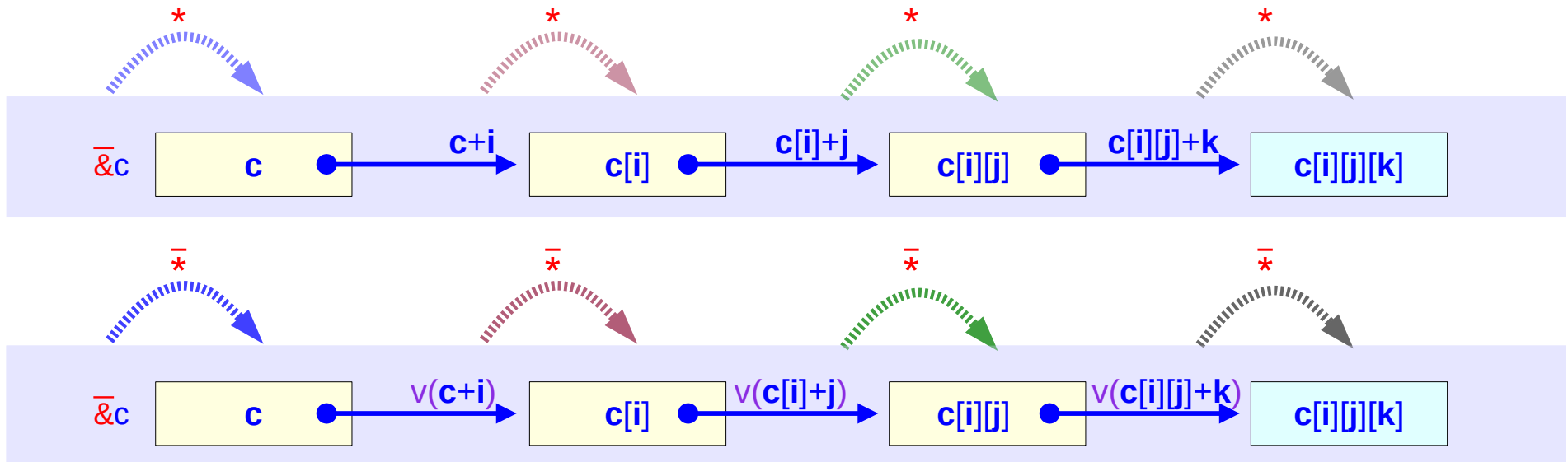
$$\begin{aligned}
\overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\
\overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\
\overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i
\end{aligned}$$

$$\begin{aligned}
\&(c[i][j][k]) &= \&(* (c[i][j] + k)) &= c[i][j] &+ k * \text{sizeof}(c[0][0][0]) \\
\&(c[i][j]) &= \&(* (c[i] + j)) &= c[i] &+ j * \text{sizeof}(c[0][0]) \\
\&(c[i]) &= \&(* (c + i)) &= c &+ i * \text{sizeof}(c[0])
\end{aligned}$$



$$\begin{aligned}
*\bar{\&}(c[i][j][k]) &= *\bar{\&}(* (c[i][j] + k)) &= *(c[i][j] + k) \\
*\bar{\&}(c[i][j]) &= *\bar{\&}(* (c[i] + j)) &= *(c[i] + j) \\
*\bar{\&}(c[i]) &= *\bar{\&}(* (c + i)) &= *(c + i)
\end{aligned}$$

$$\begin{aligned}
*\bar{\&}(c[i][j][k]) &= *\bar{\&}(* (c[i][j] + k)) &= *\bar{\&}(c[i][j] + k * \text{sizeof}(c[0][0][0])) \\
*\bar{\&}(c[i][j]) &= *\bar{\&}(* (c[i] + j)) &= *\bar{\&}(c[i] + j * \text{sizeof}(c[0][0])) \\
*\bar{\&}(c[i]) &= *\bar{\&}(* (c + i)) &= *\bar{\&}(c + i * \text{sizeof}(c[0]))
\end{aligned}$$



Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\ \overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\ \overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i\end{aligned}$$

$$\begin{aligned}* \overline{\&}(c[i][j][k]) &= * \overline{\&}(* (c[i][j] + k)) &= * (c[i][j] &+ k) \\ * \overline{\&}(c[i][j]) &= * \overline{\&}(* (c[i] + j)) &= * (c[i] &+ j) \\ * \overline{\&}(c[i]) &= * \overline{\&}(* (c + i)) &= * (c &+ i)\end{aligned}$$

$$\begin{aligned}\&(c[i][j][k]) &= \&(* (c[i][j] + k)) &= c[i][j] &+ k * \text{sizeof}(c[0][0][0]) \\ \&(c[i][j]) &= \&(* (c[i] + j)) &= c[i] &+ j * \text{sizeof}(c[0][0]) \\ \&(c[i]) &= \&(* (c + i)) &= c &+ i * \text{sizeof}(c[0])\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= * \overline{\&}(* (c[i][j] + k)) &= * (c[i][j] &+ k * \text{sizeof}(c[0][0][0]) \\ c[i][j] &= * \overline{\&}(* (c[i] + j)) &= * (c[i] &+ j * \text{sizeof}(c[0][0])) \\ c[i] &= * \overline{\&}(* (c + i)) &= * (c &+ i * \text{sizeof}(c[0]))\end{aligned}$$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= c[i][j] + k \\ \overline{\&}(c[i][j]) &= c[i] + j \\ \overline{\&}(c[i]) &= c + i\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= *(c[i][j] + k) \\ c[i][j] &= *(c[i] + j) \\ c[i] &= *(c + i)\end{aligned}$$

$$\begin{aligned}\&(c[i][j][k]) &= c[i][j] + k * \text{sizeof}(c[0][0][0]) \\ \&(c[i][j]) &= c[i] + j * \text{sizeof}(c[0][0]) \\ \&(c[i]) &= c + i * \text{sizeof}(c[0])\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= \overline{*}(c[i][j] + k * \text{sizeof}(c[0][0][0])) \\ c[i][j] &= \overline{*}(c[i] + j * \text{sizeof}(c[0][0])) \\ c[i] &= \overline{*}(c + i * \text{sizeof}(c[0]))\end{aligned}$$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= c[i][j] + k \\ \overline{\&}(c[i][j]) &= c[i] + j \\ \overline{\&}(c[i]) &= c + i\end{aligned}$$

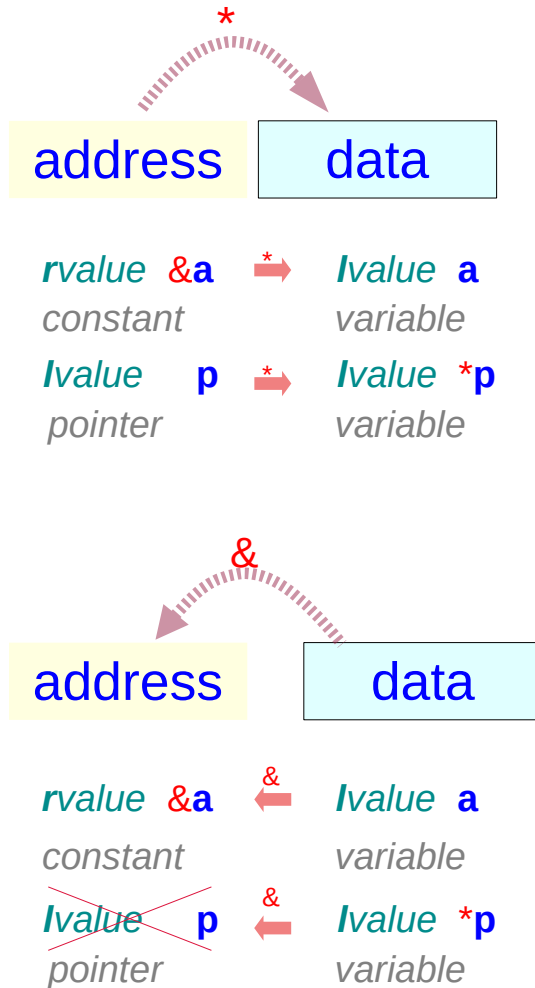
$$\begin{aligned}\&(c[i][j][k]) &= \text{value}(c[i][j] + k) \\ \&(c[i][j]) &= \text{value}(c[i] + j) \\ \&(c[i]) &= \text{value}(c + i)\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= *(c[i][j] + k) \\ c[i][j] &= *(c[i] + j) \\ c[i] &= *(c + i)\end{aligned}$$

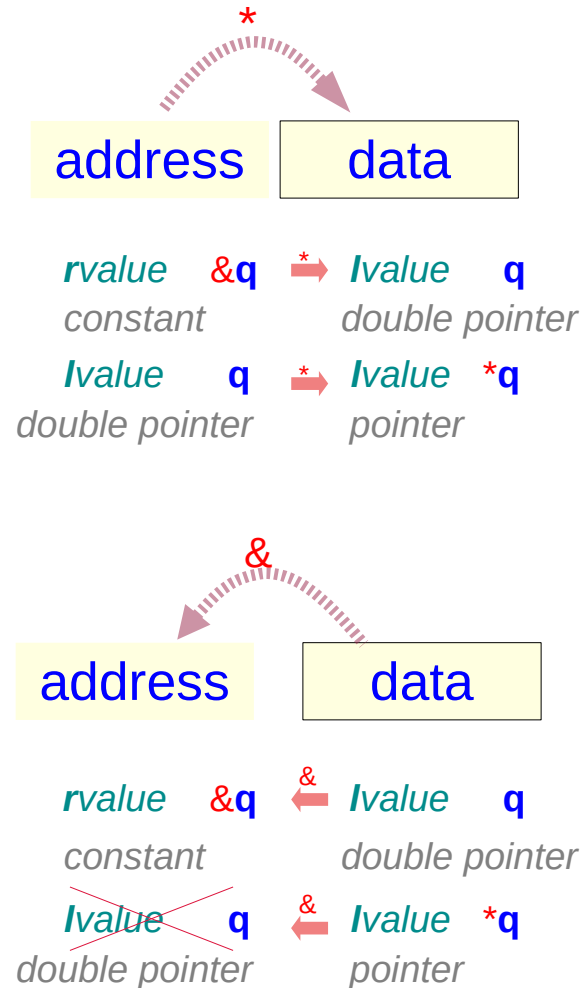
$$\begin{aligned}c[i][j][k] &= \overline{*} \text{value}(c[i][j] + k) \\ c[i][j] &= \overline{*} \text{value}(c[i] + j) \\ c[i] &= \overline{*} \text{value}(c + i)\end{aligned}$$

Address-of & and dereference * C operators (1)

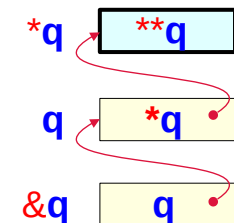
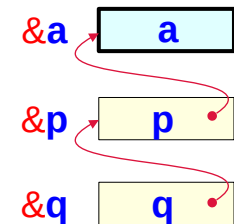
Primitive Data Type



Pointer Data Type

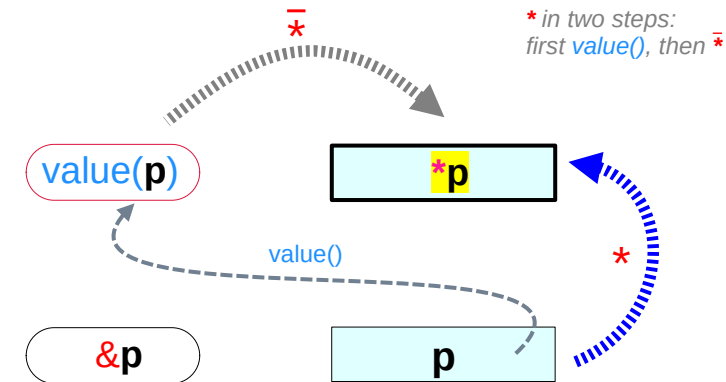
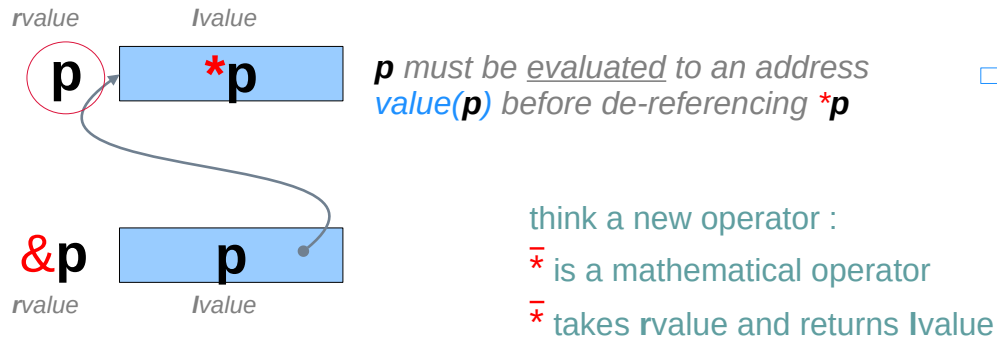


```
int a ;  
int * p ;  
int ** q ;
```

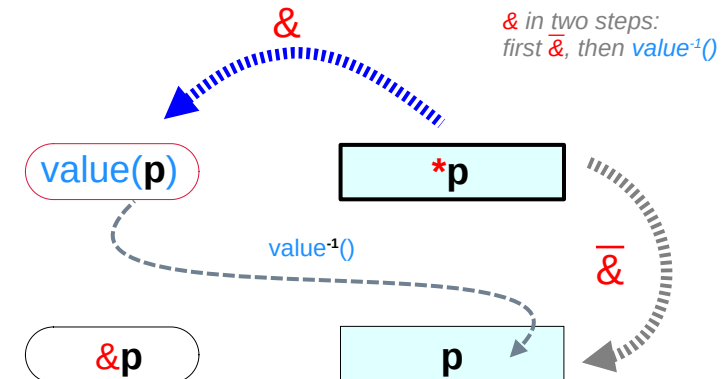
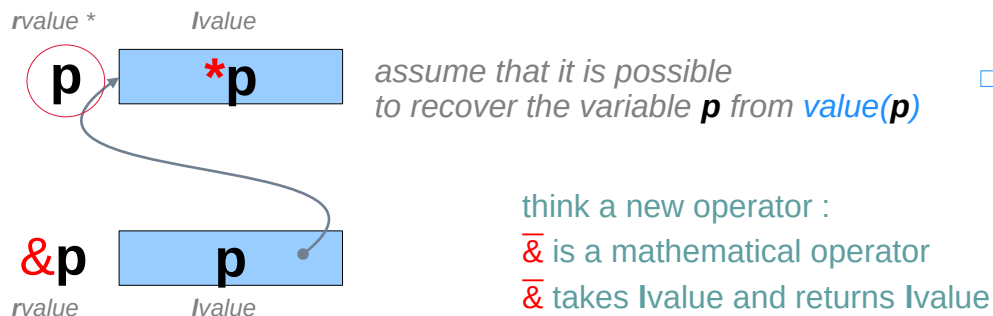


& and * operators in pointer de-referencing

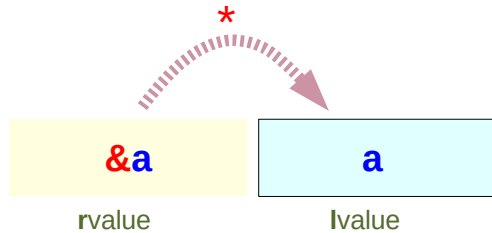
Two step De-reference * operation



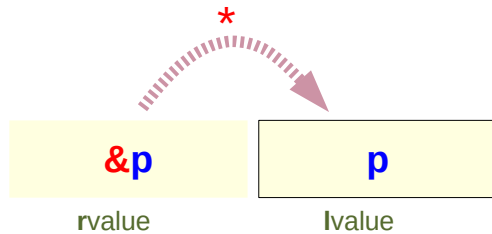
Two step Address-of & operation



Type, Size, and Value attributes of an **lvalue**



lvalue ← **rvalue*
a **&a*



lvalue ← **rvalue*
p **&p*

lvalue is associated with a memory location

lvalue has the following attributes

- Type
- Size
- Value

rvalue has the only attribute

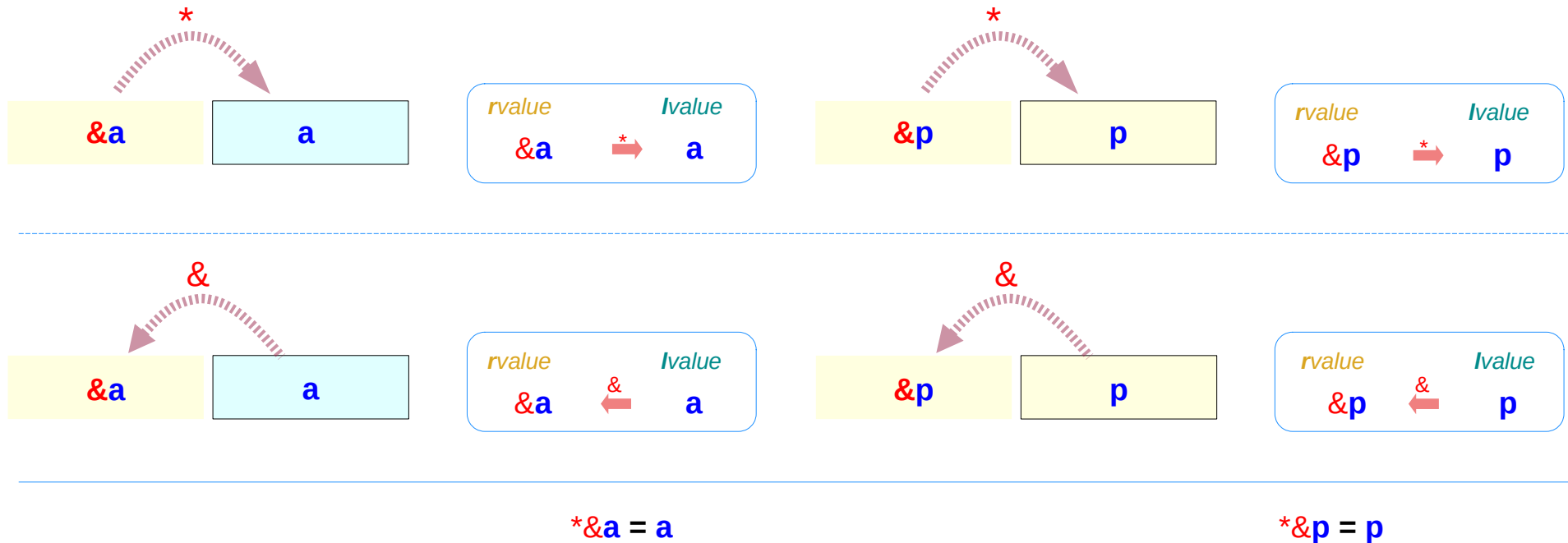
- Value

assume the function `value()`

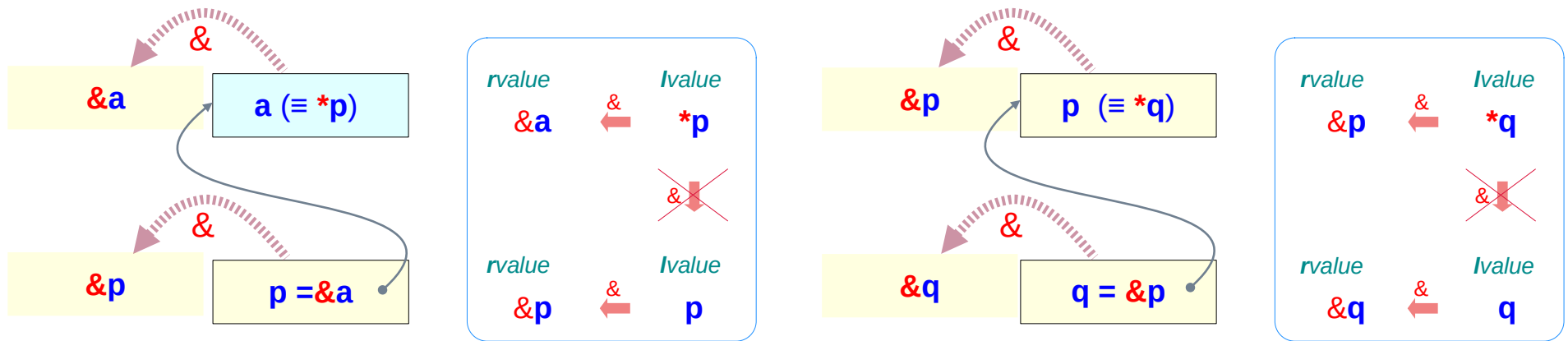
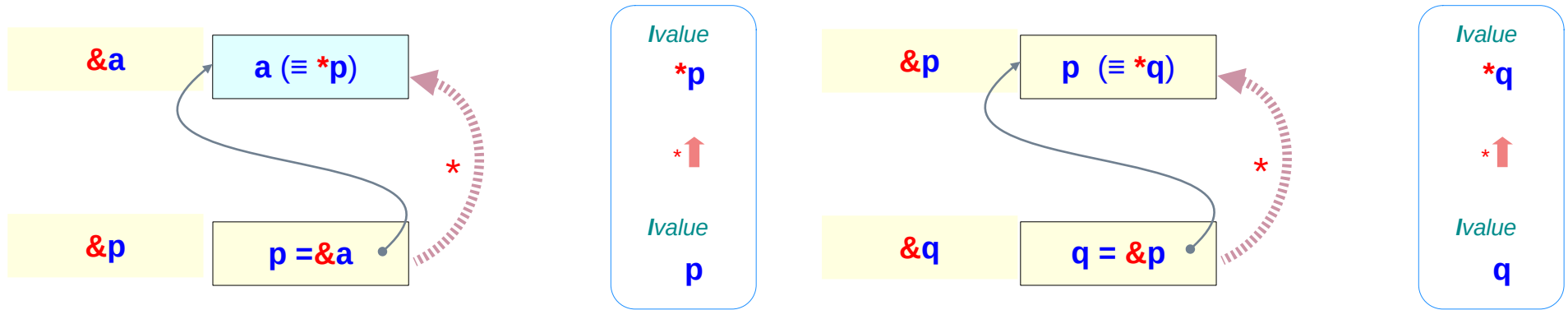
`value(lvalue)` returns
the **Value** attribute of **lvalue**

`value(rvalue)` returns
the **Value** attribute of **rvalue**

Address-of & and dereference * C operators (1)



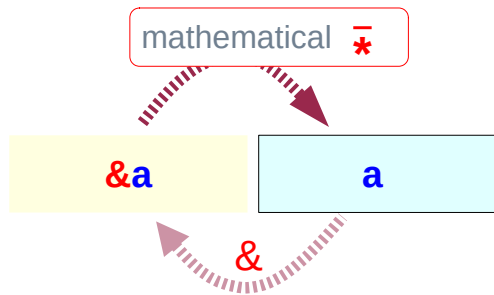
Address-of & and dereference * C operators (2)



***&p = p**
~~**&*p = p**~~ *value(p)*

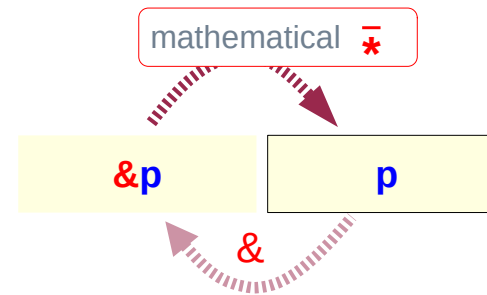
***&q = q**
~~**&*q = q**~~ *value(q)*

Introducing mathematical operators : $\bar{\&}$ and $\bar{*}$



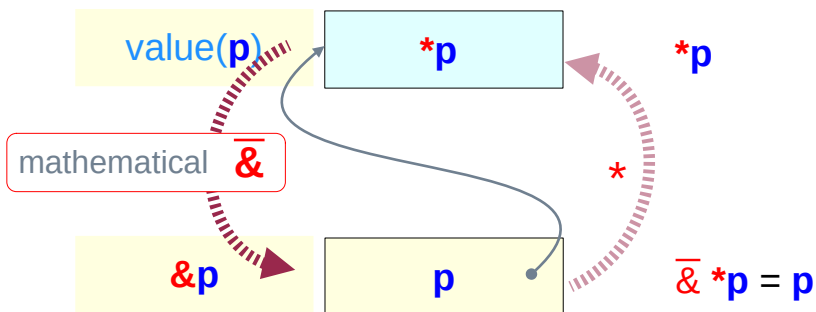
$$\bar{*} \&a = a$$

$\&a$

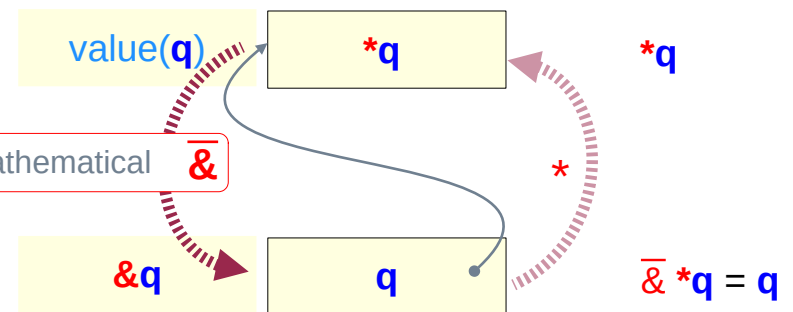


$$\bar{*} \&p = p$$

$\&p$

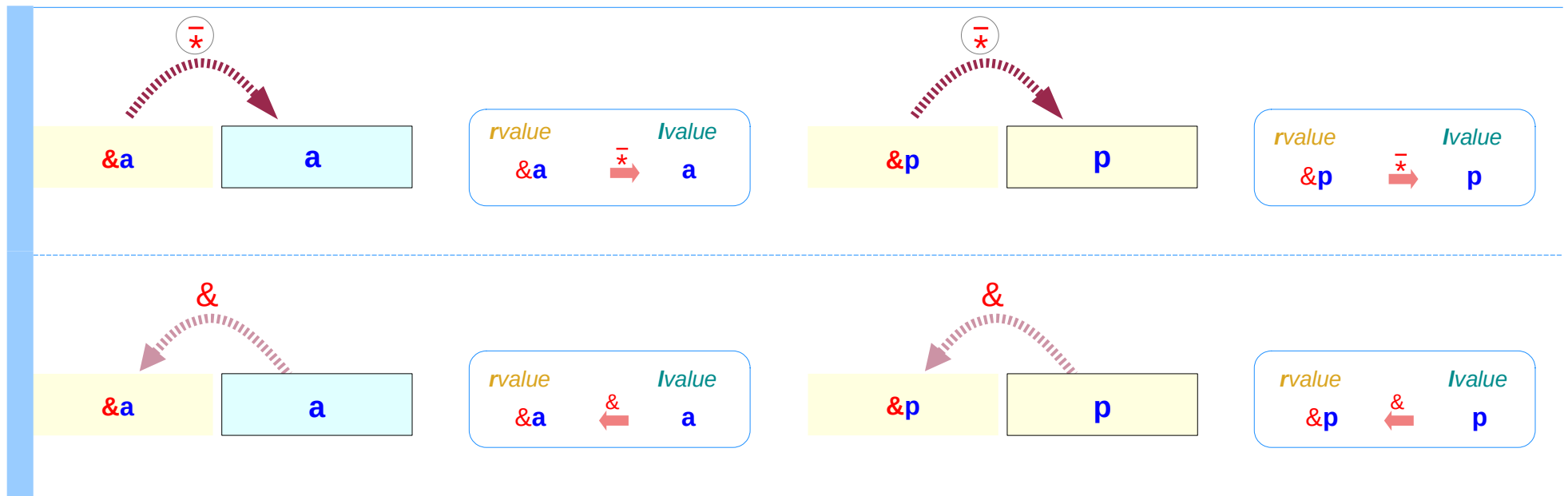


$$\bar{\&} * p = p$$



$$\bar{\&} * q = q$$

Inverse operators $\bar{*}$ and $\&$



$$\bar{*}\&a = a$$

$$\&\bar{*}\&a = \&a$$

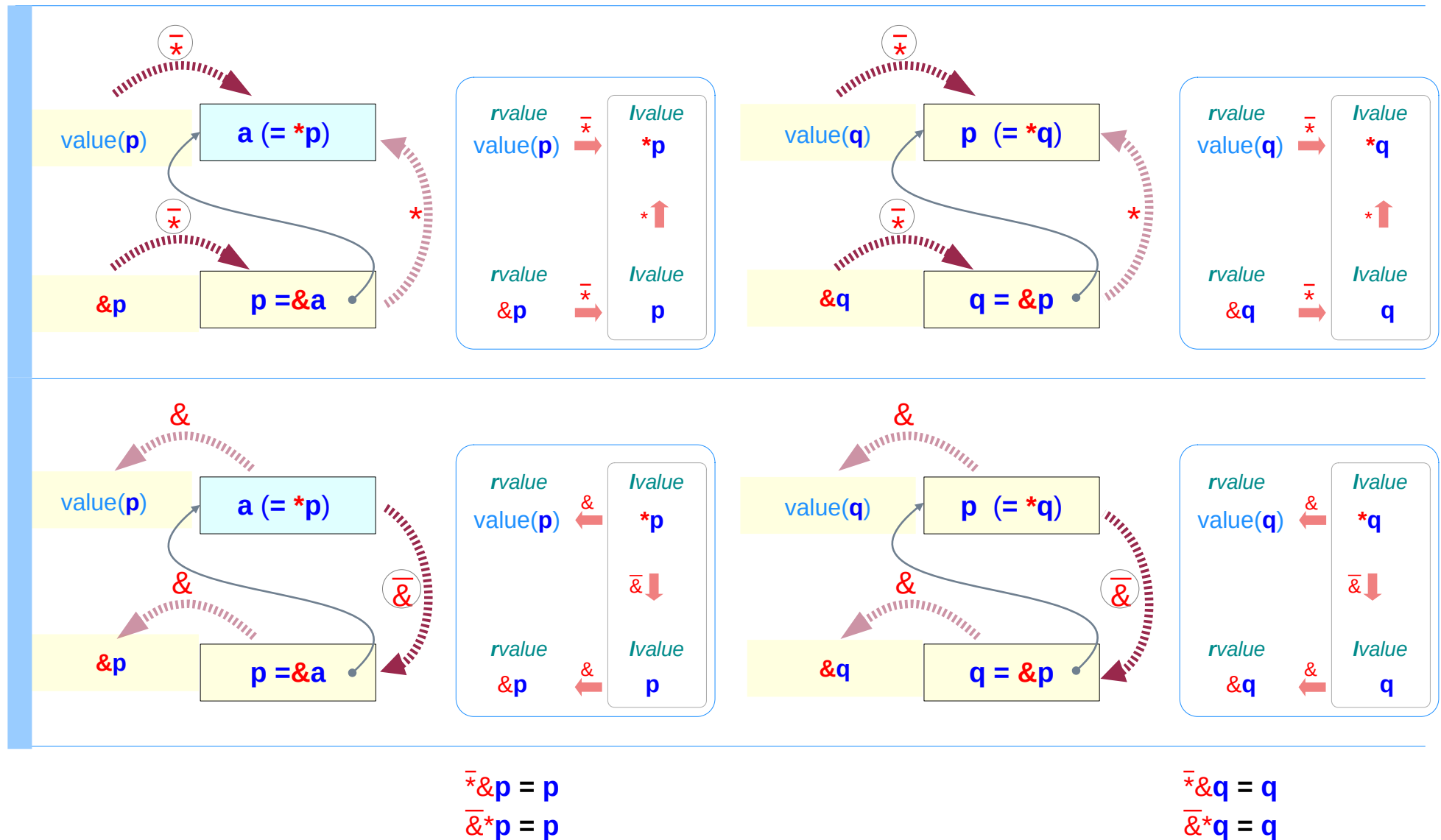
$\bar{*}$ and $\&$ are
inverse operators
to each other

$$\bar{*}\&p = p$$

$$\&\bar{*}\&p = \&p$$

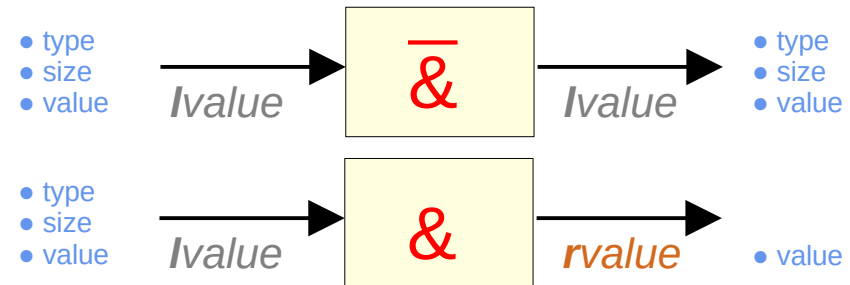
$\bar{*}$ and $\&$ are
inverse operators
to each other

Inverse operators $\bar{*}$ and $*$

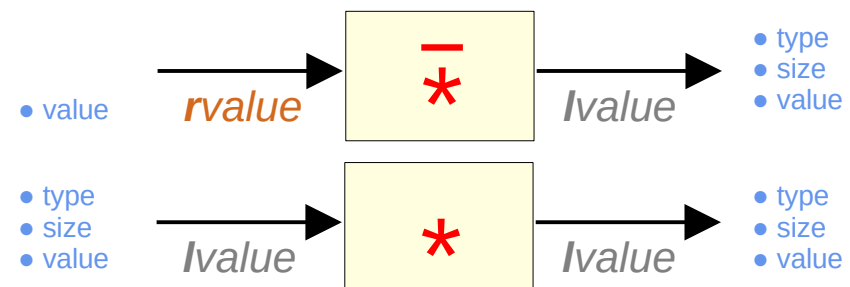
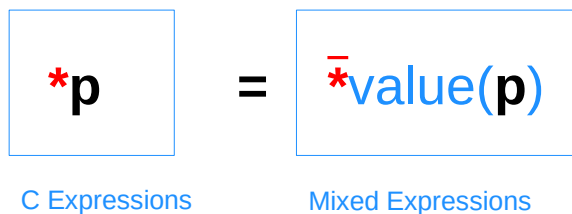


C operators and mathematical operators

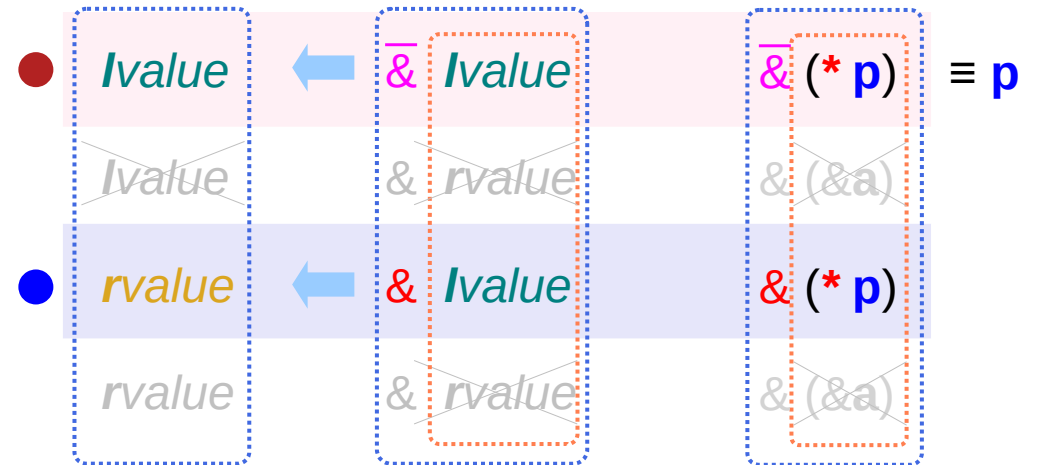
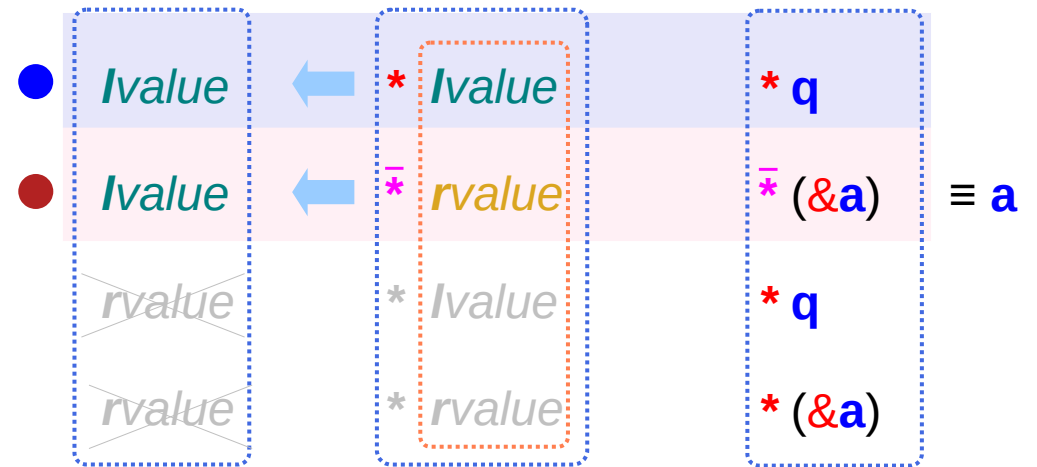
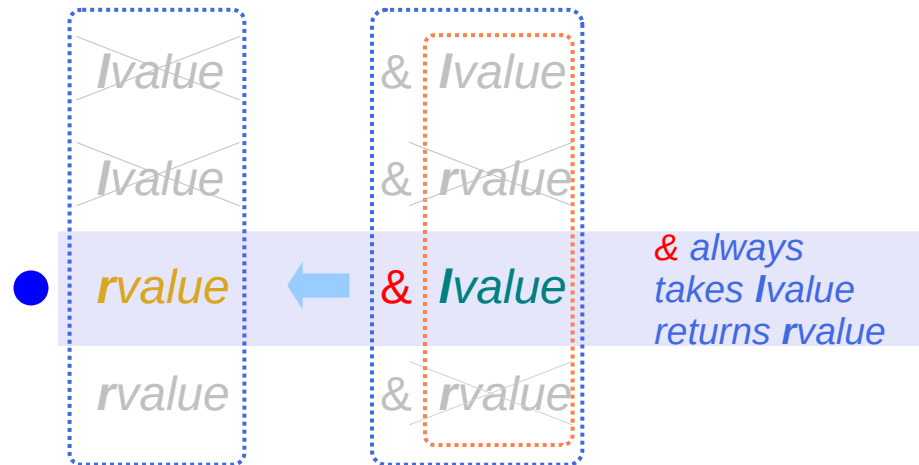
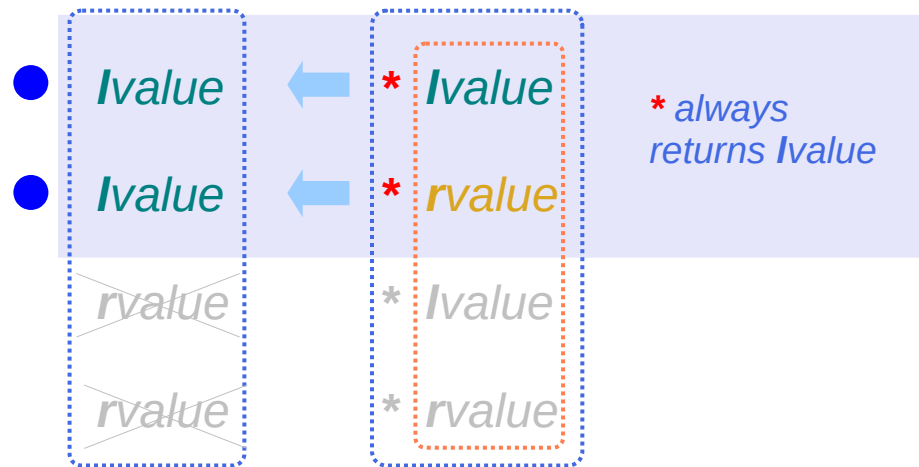
Address-of operation



De-reference operation



Ivalue and rvalue with * and & operators



a, p, q : Ivalues ... variables ... RW
 *p, *q, **q : Ivalues ... variables ... RW
 &a, &p, &q : rvalues ... constants ... RO

Examples of inverse operators

$\overline{*}\&a \rightarrow a$

$\overline{*}\&p \rightarrow p$

$\overline{*}\&q \rightarrow q$

$\overline{*}\text{value}(p) \rightarrow *p \rightarrow a$

$\overline{*}\text{value}(q) \rightarrow *q \rightarrow p$

$\overline{*}\text{value}(*q) \rightarrow **q \rightarrow *p \rightarrow a$

$\overline{\&}*p \rightarrow p$

$\overline{\&}*q \rightarrow q$

$\overline{\&}**q \rightarrow *q$

Extended Operators

$*\&a \rightarrow a$

$*\&p \rightarrow p$

$*\&q \rightarrow q$

$*p \rightarrow a$

$*q \rightarrow p$

$**q \rightarrow *p \rightarrow a$

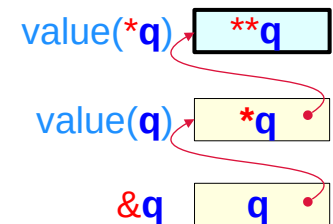
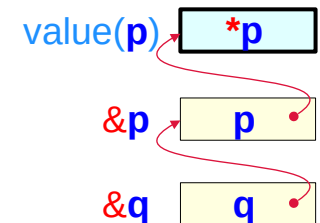
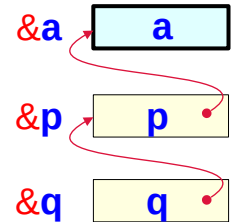
$\&*p \rightarrow \&a$

$\&*q \rightarrow \&p$

$\&**q \rightarrow \&a$

C Operators

```
int a;  
int * p = &a;  
int ** q = &p;
```



Operands of mathematical operators : $\bar{\&}$ and $\bar{*}$

$\bar{*}$ address value

$\bar{\&} \bar{*} \text{value(P)}$ $\longrightarrow$ value(P)

$\bar{*}$ $\bar{\&}$ variable

$\bar{*} \bar{\&} X$ $\longrightarrow$ X

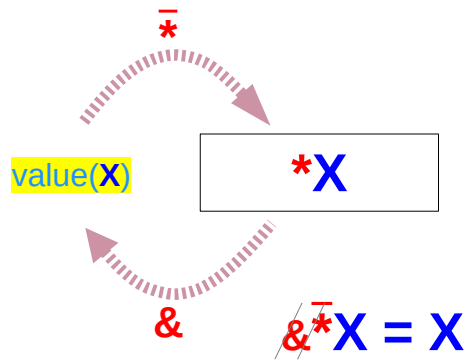
$\bar{\&}$ $* \text{pointer}$
dereferenced pointer

$\bar{\&} * P$ $\longrightarrow$ P

$\bar{\&}$ ~~pointer~~
must be a dereferenced pointer
not considering
simultaneous pointing

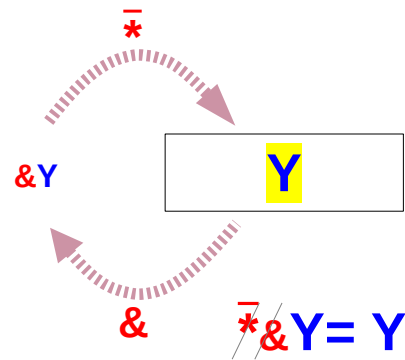
$* \bar{\&} Q$ $\longrightarrow$ Q
 Q must be a dereferenced pointer
i.e, $Q = *p \longrightarrow \bar{\&} Q = p$

& and mathematical $\bar{*}$



$\&\bar{*}\text{value}(a) = \text{value}(a)$
 $\&\bar{*}\text{value}(p) = \text{value}(p)$
 $\&\bar{*}\text{value}(q) = \text{value}(q)$

$\&*a = \text{value}(a)$
 $\&*p = \text{value}(p)$
 $\&*q = \text{value}(q)$



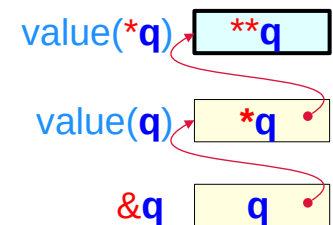
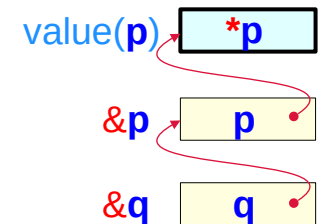
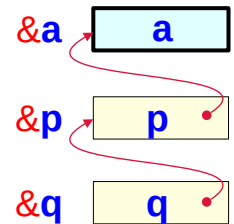
$\bar{*}\&a = a$
 $\bar{*}\&p = p$
 $\bar{*}\&q = q$

$\bar{*}\&a = a$
 $\bar{*}\&p = p$
 $\bar{*}\&q = q$

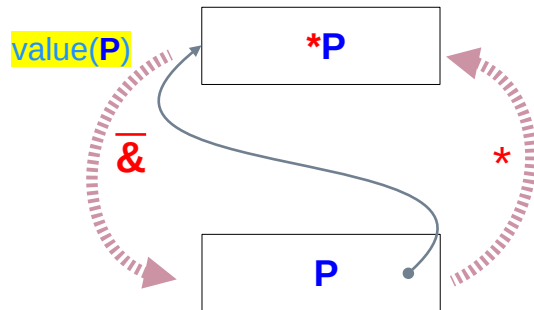
$*\&a = a$
 $*\&p = p$
 $*\&q = q$

C expressions

```
int a;  
int * p = &a;  
int ** q = &p;
```



* and mathematical $\bar{\&}$



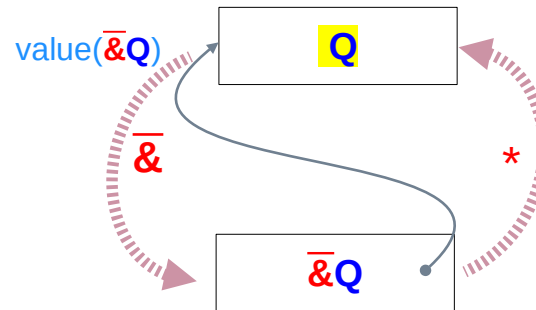
$$\bar{\&*P} = P$$

$$\bar{\&}*p = p$$

$$\bar{\&}*q = q$$

$$\&*p = \text{value}(p)$$

$$\&*q = \text{value}(q)$$



$$*\bar{\&Q} = Q$$

$$*\bar{\&}*p = *p$$

$$*\bar{\&}*q = *q$$

$$\bar{*}\bar{\&}*p = *p$$

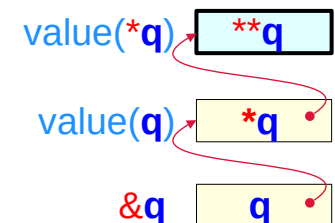
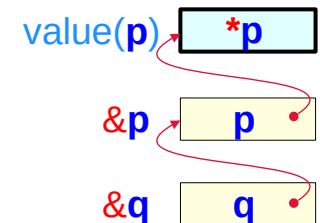
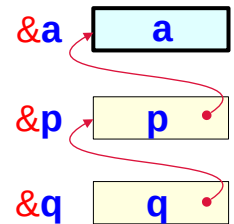
$$\bar{*}\bar{\&}*q = *q$$

$$\begin{aligned} * \bar{\&}*p &= *p \\ * \bar{\&}*q &= *q \end{aligned}$$

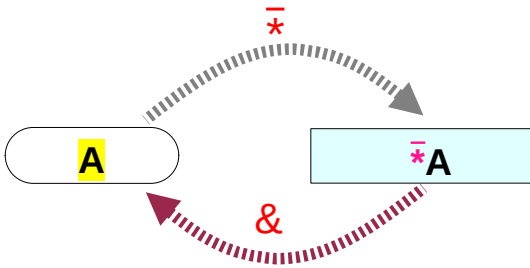
C operators

if not considering
simultaneous pointing

```
int a;
int * p = &a;
int ** q = &p;
```

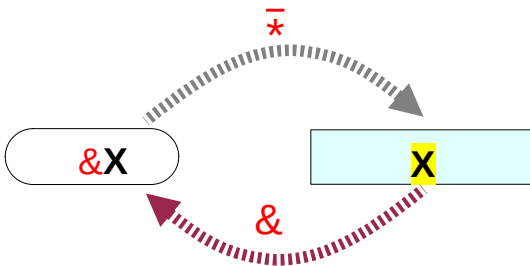


Requirements of address and data

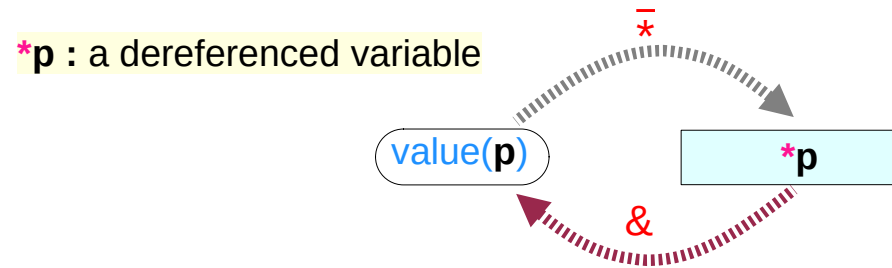


A : an address value - a number

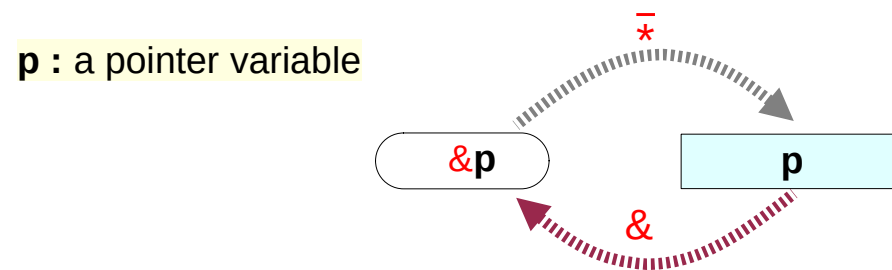
$\text{value}(p)$, $\&p$, $\&x$



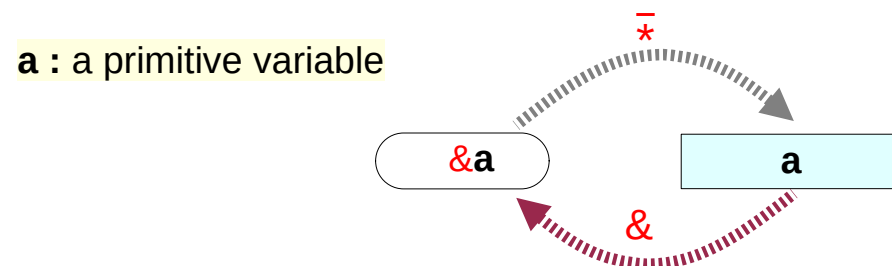
X : a variable $*p$, p , a



Pointer Dereferencing



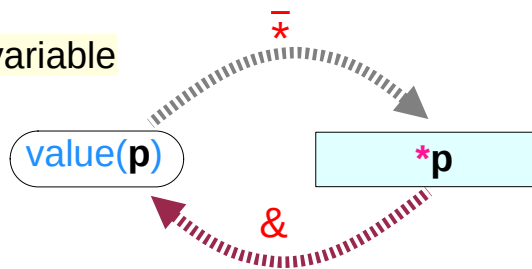
Pointer Data



Non-pointer Data

Inverse relations of $\&$ and $\bar{*}$ operators

$*p$: a dereferenced variable

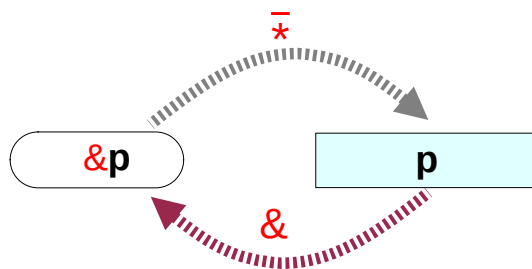


$$\bar{*} \text{value}(p) = *p$$

$$\&\bar{*} \text{value}(p) = \&*p = \text{value}(p)$$

$$\bar{*}\&\text{value}(p) = \bar{*}\&p = \bar{*} \text{value}(p) = *p$$

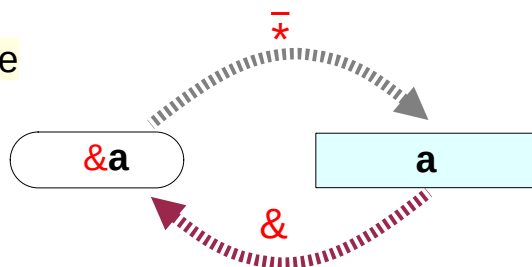
p : a pointer variable



$$\bar{*}\&p = p$$

$$\&\bar{*}\&p = \&p$$

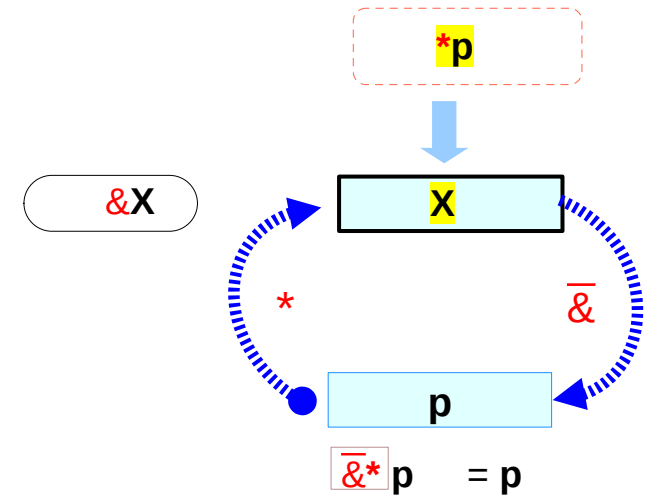
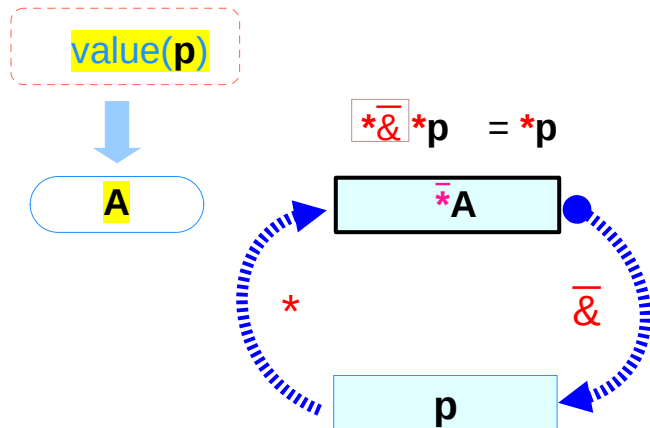
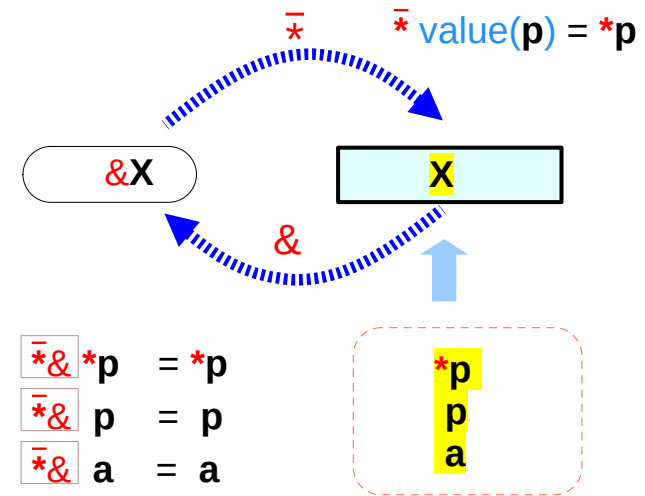
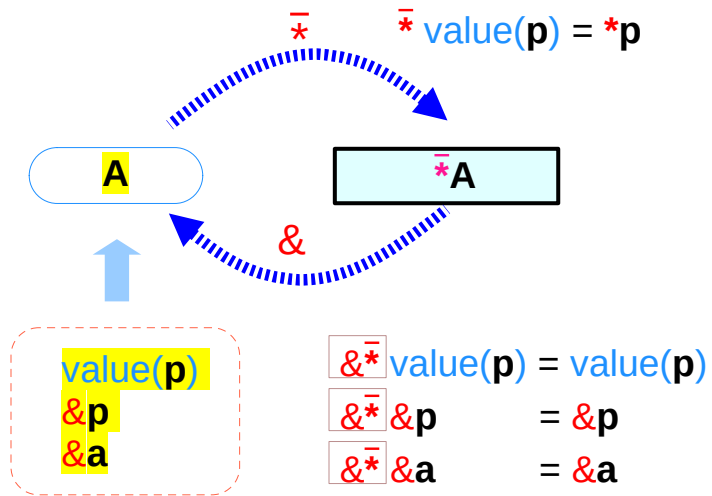
a : a primitive variable



$$\bar{*}\&a = a$$

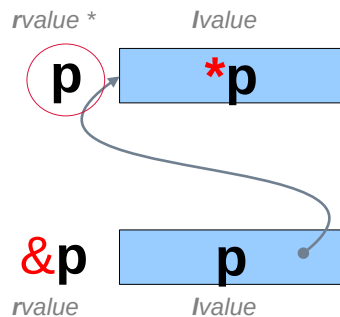
$$\&\bar{*}\&a = \&a$$

Requirements of pointed address and data

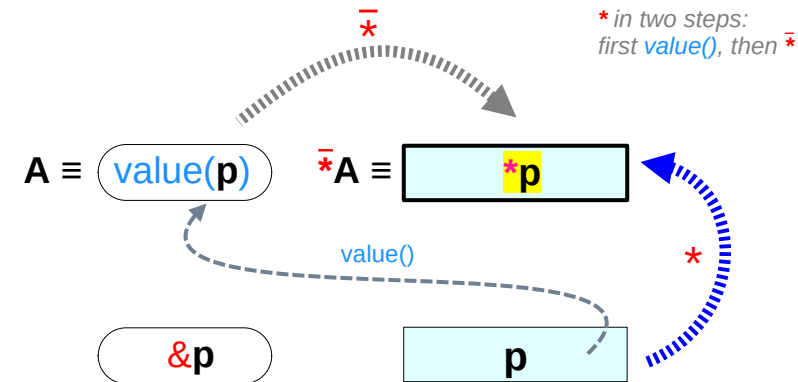


& and * operators in pointer de-referencing (1)

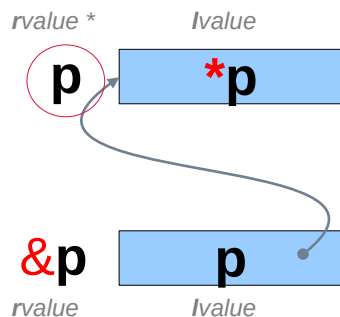
Two step De-reference * operation



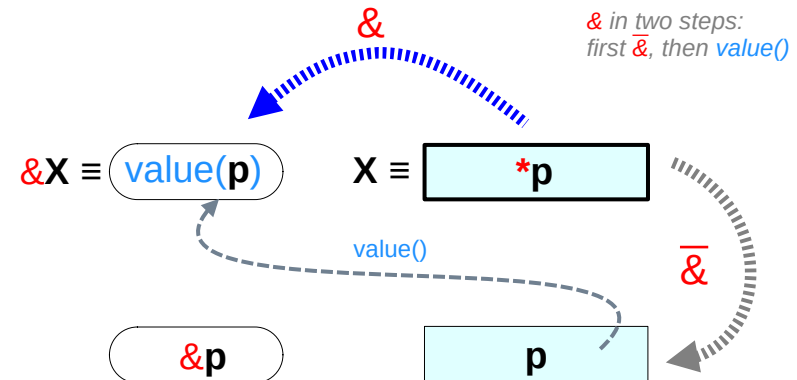
$$\begin{array}{lcl} \boxed{*p} & = & \boxed{\bar{*}\text{value}(p)} \\ \text{C Expressions} & & \text{Mixed Expressions} \\ \boxed{\bar{*}A} & = & \boxed{* \text{value}^{-1}(A)} \end{array}$$



Two step Address-of & operation

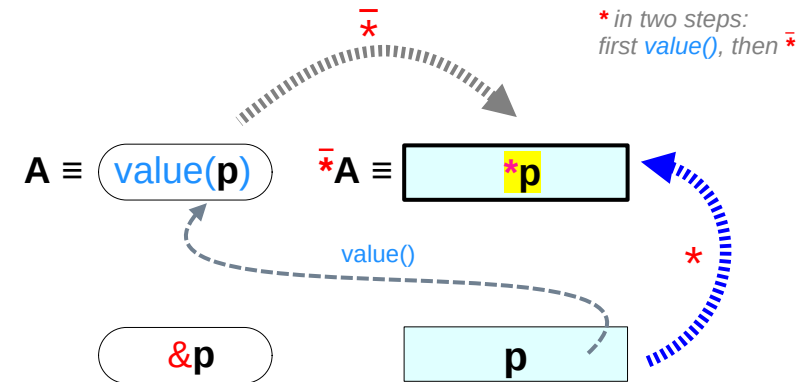
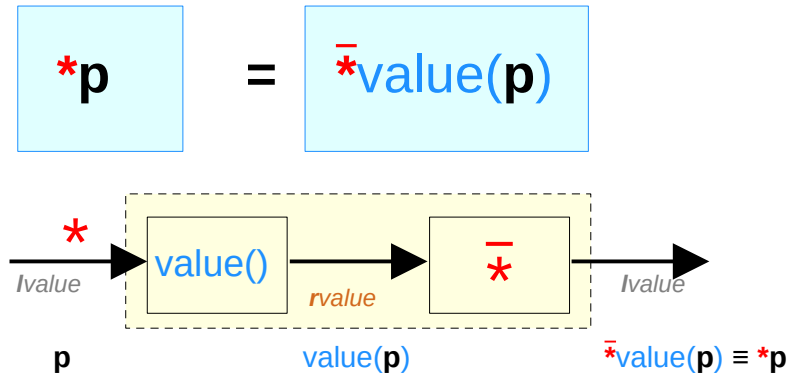


$$\begin{array}{lcl} \boxed{\&X} & = & \boxed{\text{value}(\bar{\&}X)} \\ \text{C Expressions} & & \text{Mixed Expressions} \\ \boxed{\bar{\&}X} & = & \boxed{\text{value}^{-1}(\&X)} \end{array}$$

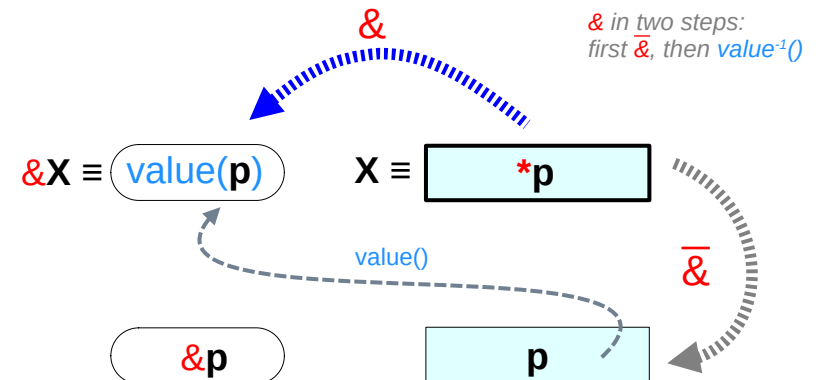
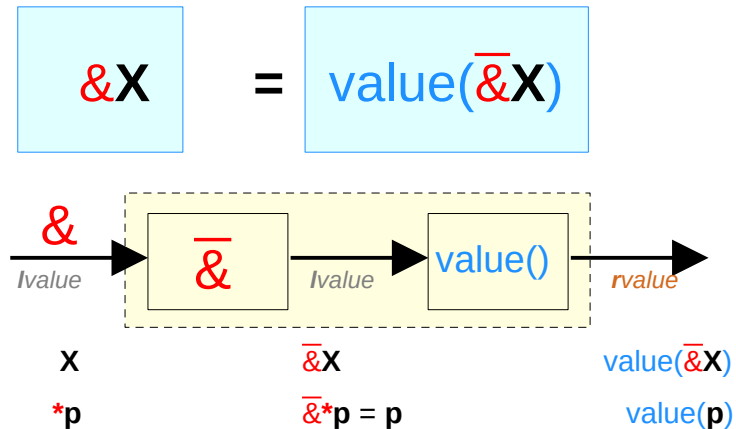


& and * operators in pointer de-referencing (2)

Two step De-reference * operation

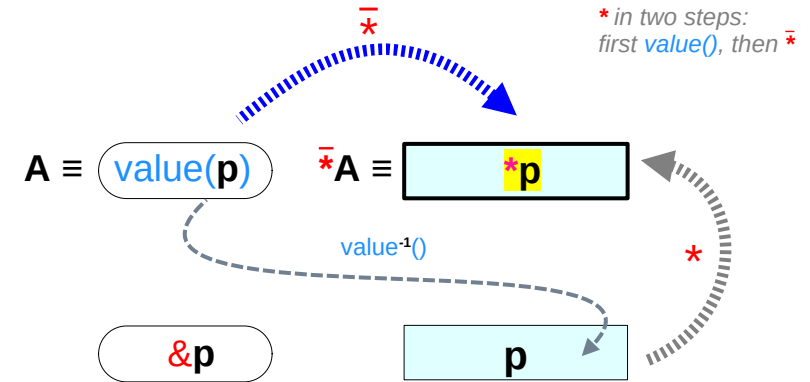
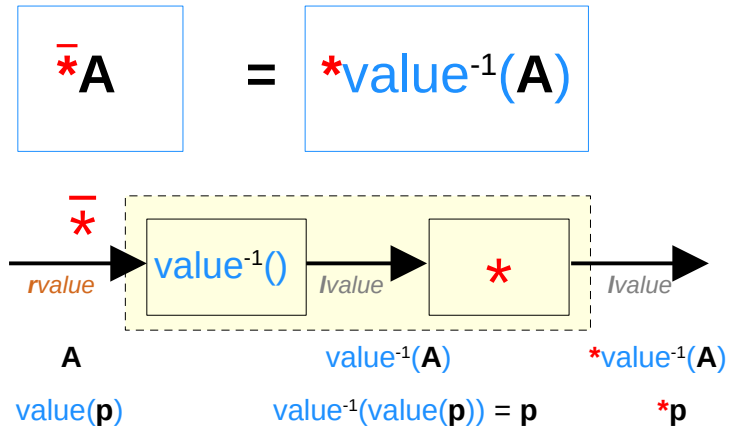


Two step Address-of & operation

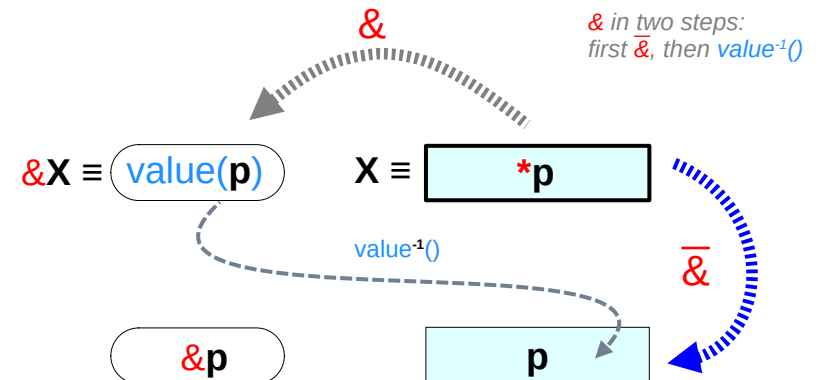
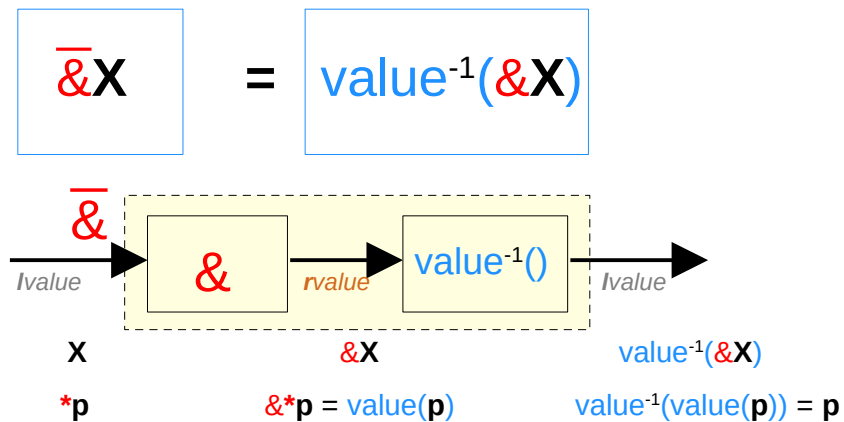


$\bar{*}$ and $\bar{*}$ operators in pointer de-referencing (3)

Two step De-reference $\bar{*}$ operation



Two step Address-of $\bar{\&}$ operation



Two step address-of & and $\bar{\&}$ operators

for $\&X$, X can be $*p$, p , a

$$\&X = \text{value}(\bar{\&X})$$

C Expressions

for $\bar{\&X} = p$, X must be $*p$

$$\bar{\&X} = \text{value}^{-1}(\&X)$$

$*p$ a dereferenced variable
 p a pointer variable
 a a primitive variable

$\&X$

$$\&*p = \&* \text{value}(p) = \text{value}(p)$$

$\&p$

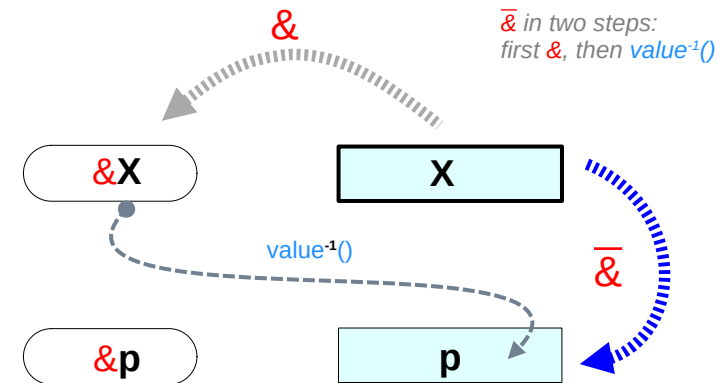
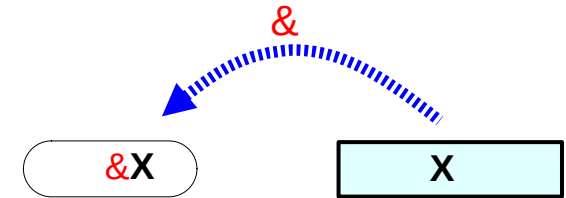
$\&a$

$*p$ a dereferenced variable
 ~~p a pointer variable~~
 ~~a a primitive variable~~

$$\bar{\&X} = p$$

$$\bar{\&*p} \equiv p$$

~~$\bar{\&p}$~~
 ~~$\bar{\&a}$~~



Two step de-reference $\bar{*}$ and $*$ operators

for $\bar{*}A$, A can be $\text{value}(p)$, $\&p$, $\&a$

$$\bar{*}A = * \text{value}^{-1}(A)$$

$\text{value}(p)$ value of a pointer variable
 $\&p$ address of a pointer variable
 $\&a$ address of a primitive variable

$$\begin{aligned} \bar{*}A \\ \bar{*} \text{value}(p) &= *p \\ \bar{*} \&p &= p \\ \bar{*} \&a &= a \end{aligned}$$

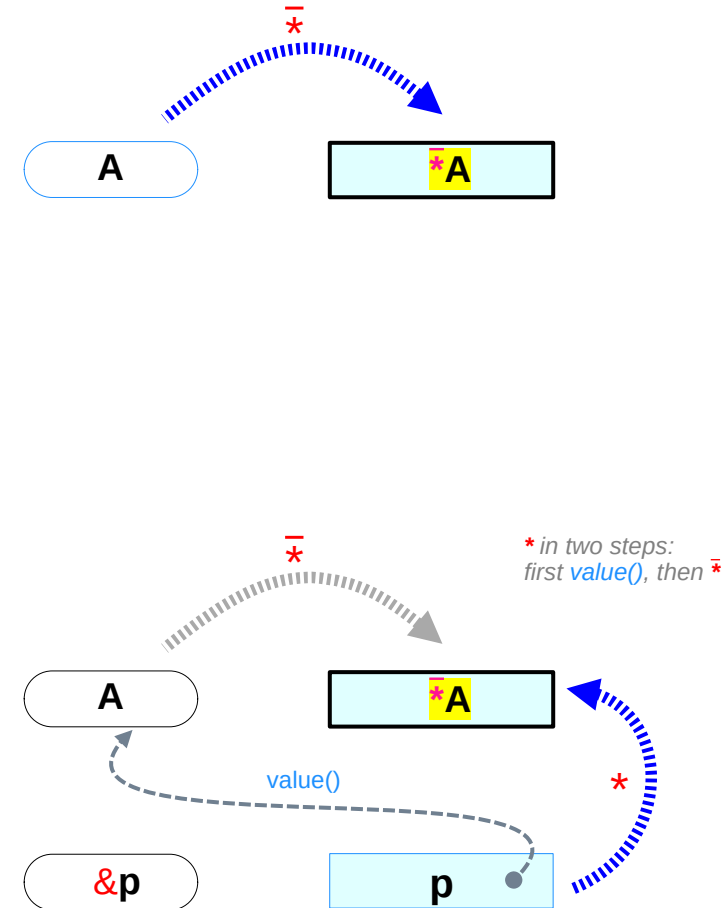
for $\bar{*}A = *p$, A must be $\text{value}(p)$

$$*p = \bar{*} \text{value}(p)$$

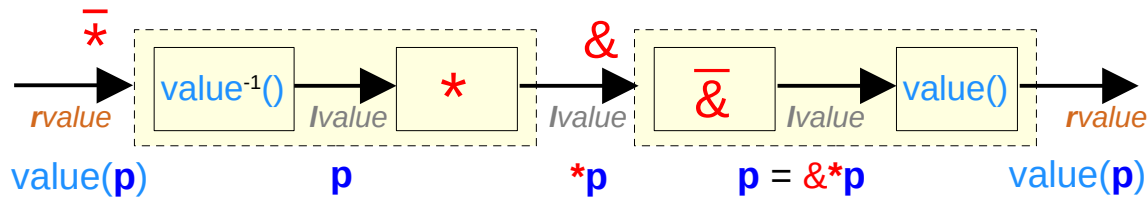
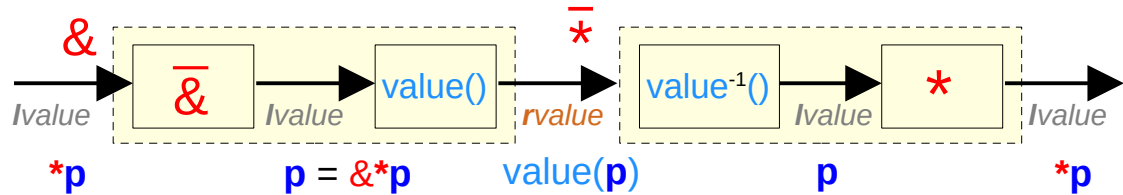
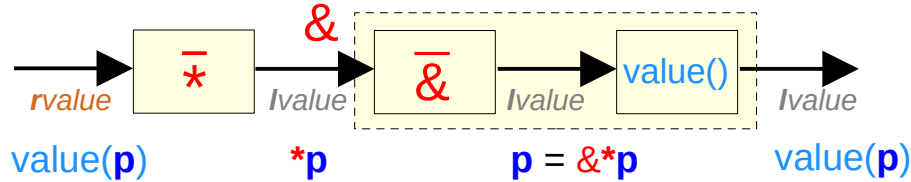
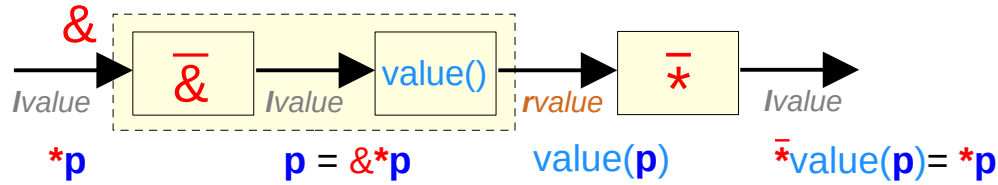
C Expressions

$\text{value}(p)$ value of a pointer variable
 ~~$\&p$ address of a pointer variable~~
 ~~$\&a$ address of a primitive variable~~

$$\begin{aligned} *p \\ *p &\equiv \bar{*} \text{value}(p) \\ \cancel{* \&p} &= p \\ \cancel{* \&a} &= a \end{aligned}$$



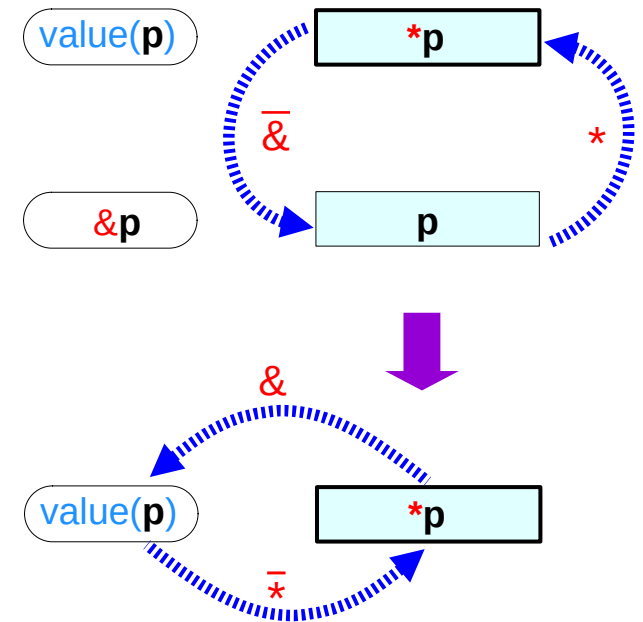
Inverse operators : $\&$ and $\bar{*}$ in pointer de-referencing



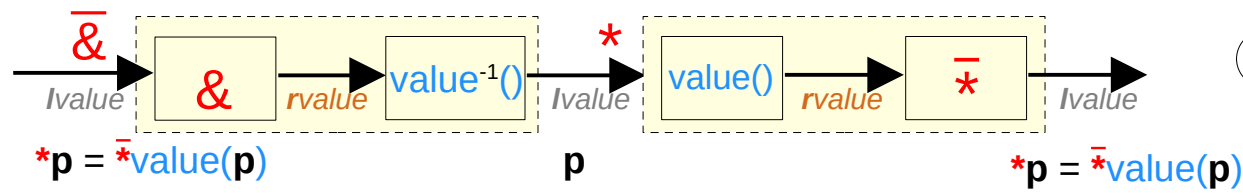
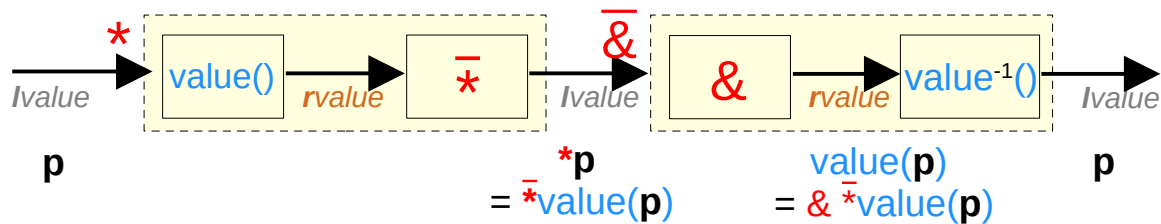
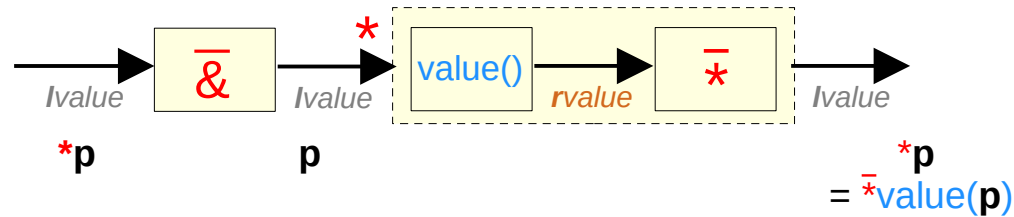
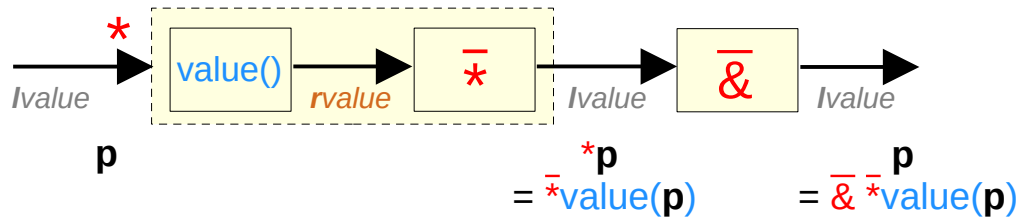
$$\bar{*}value(p) = *p$$

$$\&*value(p) = \&*p = value(p)$$

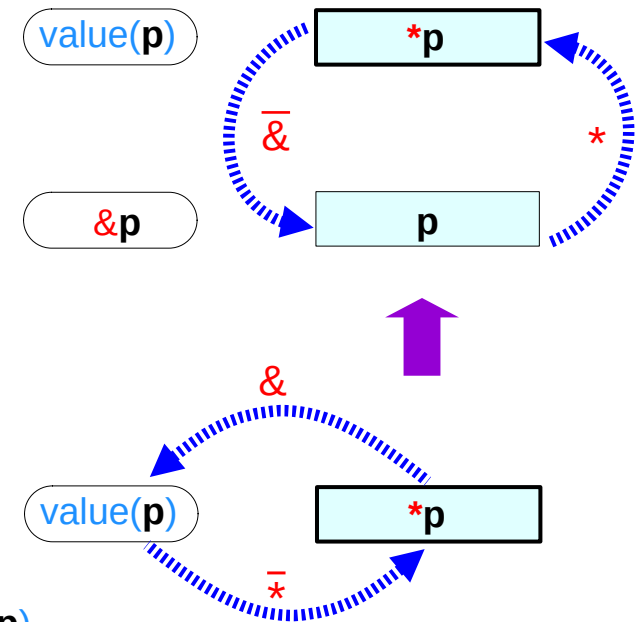
$$*\&*value(p) = *\&*p = *\&*p = *p$$



Inverse operators of & and *



$$\begin{aligned} \&* \text{value}(p) &= *p \\ \&* p &= p & p &= \&* \text{value}(p) \\ * \&p &= *p & \text{value}(p) &= \&* \text{value}(p) \end{aligned}$$



Inverse operators in pointer referencing (1)

$$\begin{aligned}\overline{*}&\&*p &= *p \\ \overline{*}&\&p &= p \\ \overline{*}&\&a &= a\end{aligned}$$

$$\begin{aligned}\&\overline{*} \text{value}(p) &= \text{value}(p) \\ \&\overline{*} \&p &= \&p \\ \&\overline{*} \&a &= \&a\end{aligned}$$

$\overline{*}&$

$*p$
 p
 a

$\overline{\&*}$

p

$\&\overline{*}$

$\text{value}(p)$
 $\&p$
 $\&x$

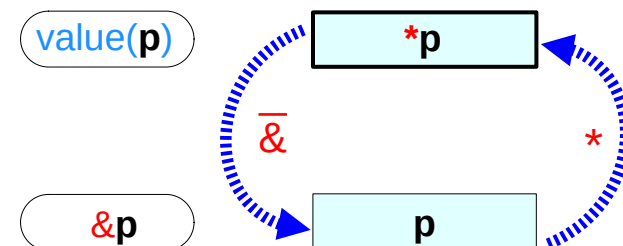
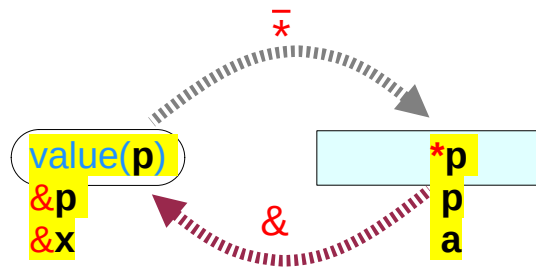
$*\overline{\&}$

$*p$

$$\overline{\&*} p = p$$

$$\&\overline{*} *p = *p$$

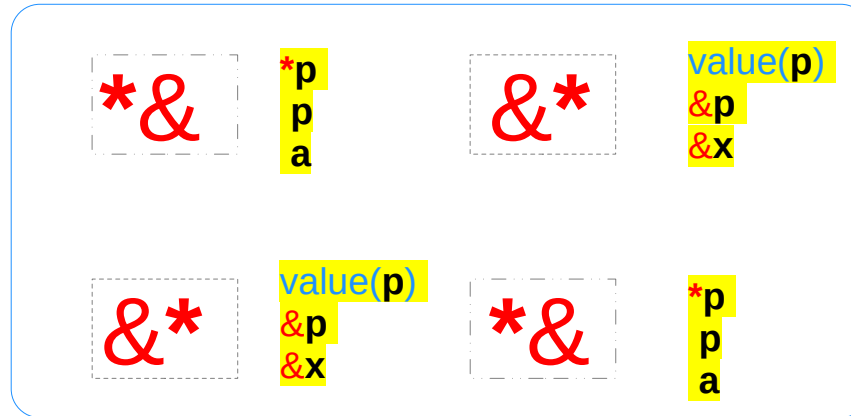
Math expressions



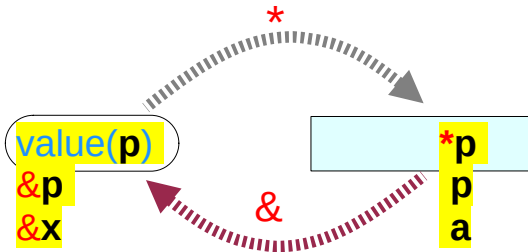
Inverse operators in pointer referencing (2)

`*&*p = *p`
`*&p = p`
`*&a = a`

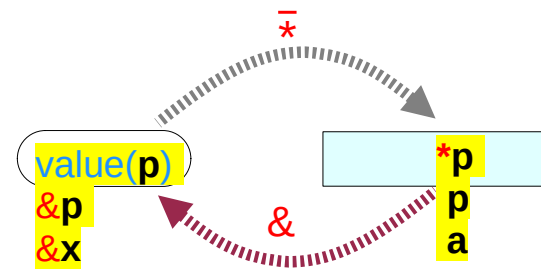
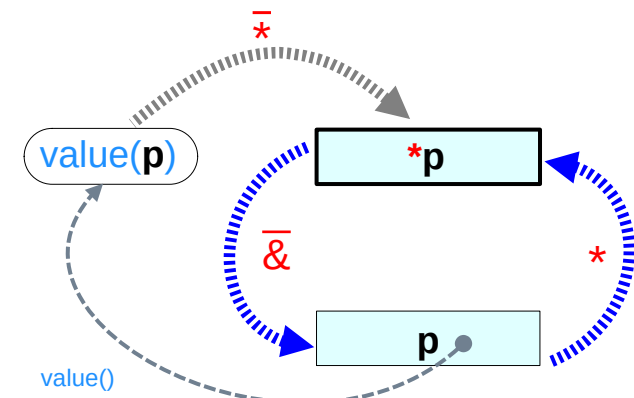
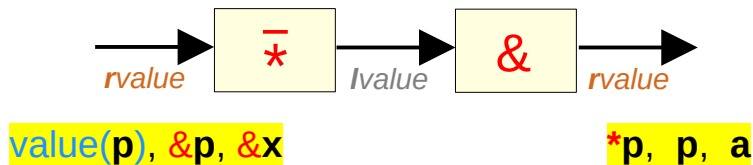
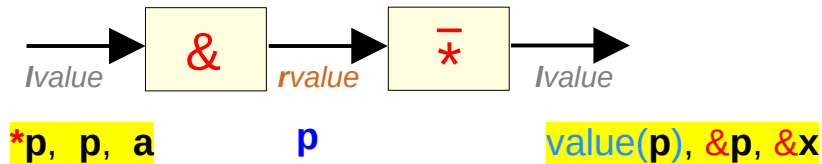
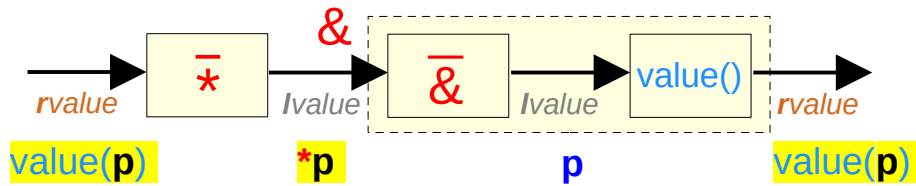
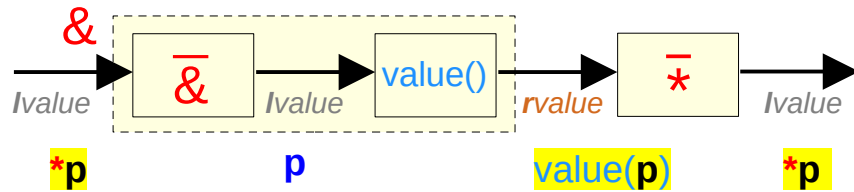
`&*value(p) = value(p)`
`&*p = &p`
`&*a = &a`



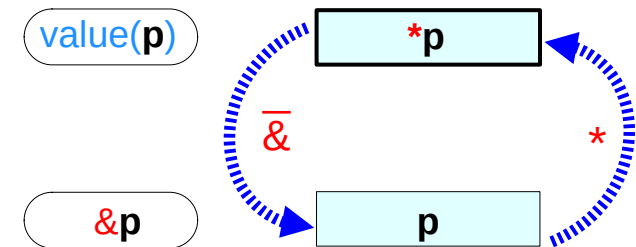
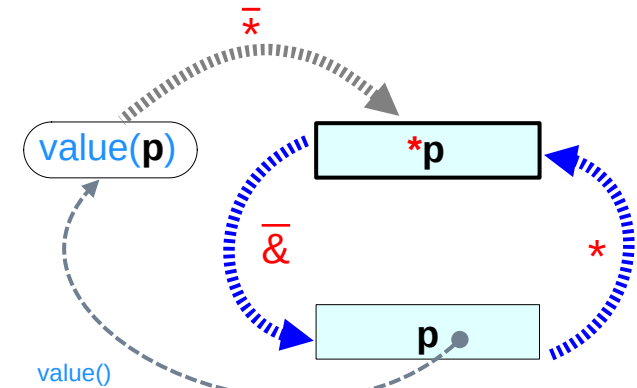
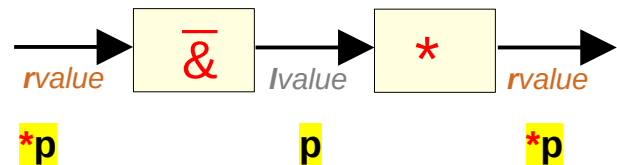
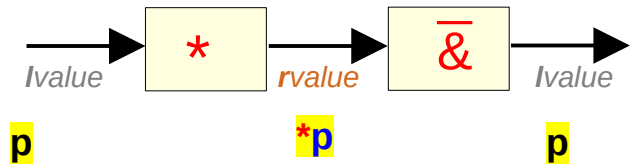
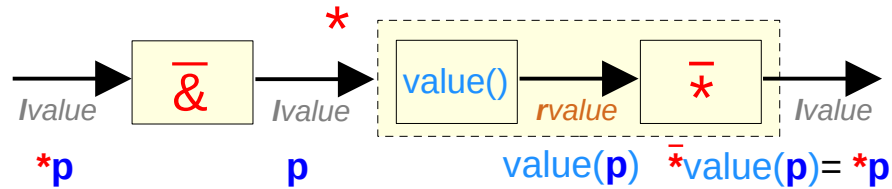
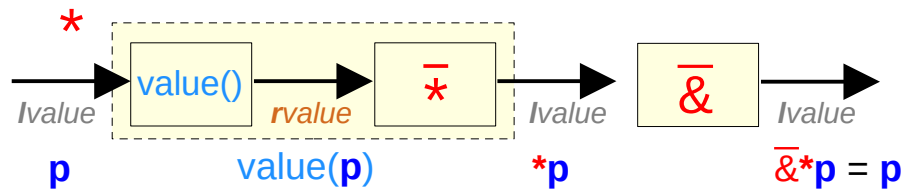
C expressions



Inverse operators in pointer referencing (3)



Inverse operators in pointer referencing (3)



References

- [1] Essential C, Nick Parlante
- [2] Efficient C Programming, Mark A. Weiss
- [3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
- [4] C Language Express, I. K. Chun