Multi-dimensional Arrays (1A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.
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Assumption

assume that

value(c) returns the hexadecimal number
that is obtained by printf("%p", c),
when the variable c contains
an address as its value

```
#include <stdio.h>
int main(void) {
  int c[3] ;
  printf ("c= %p \n", &c);
}

c= 0x7fffd923487c
```

type(c) can be determined
by the warning messsage of printf("%d", c),
when the variable c contains
an address as its value

```
#include <stdio.h>
int main(void) {
  int c[3] ;
  printf ("c= %d \n", &c);
}

t.c: In function 'main':
t.c:5:16: warning: format '%d' expects argument of type 'int',
but argument 2 has type 'int (*)[3]' [-Wformat=]
   printf ("c= %d \n", &c);
```

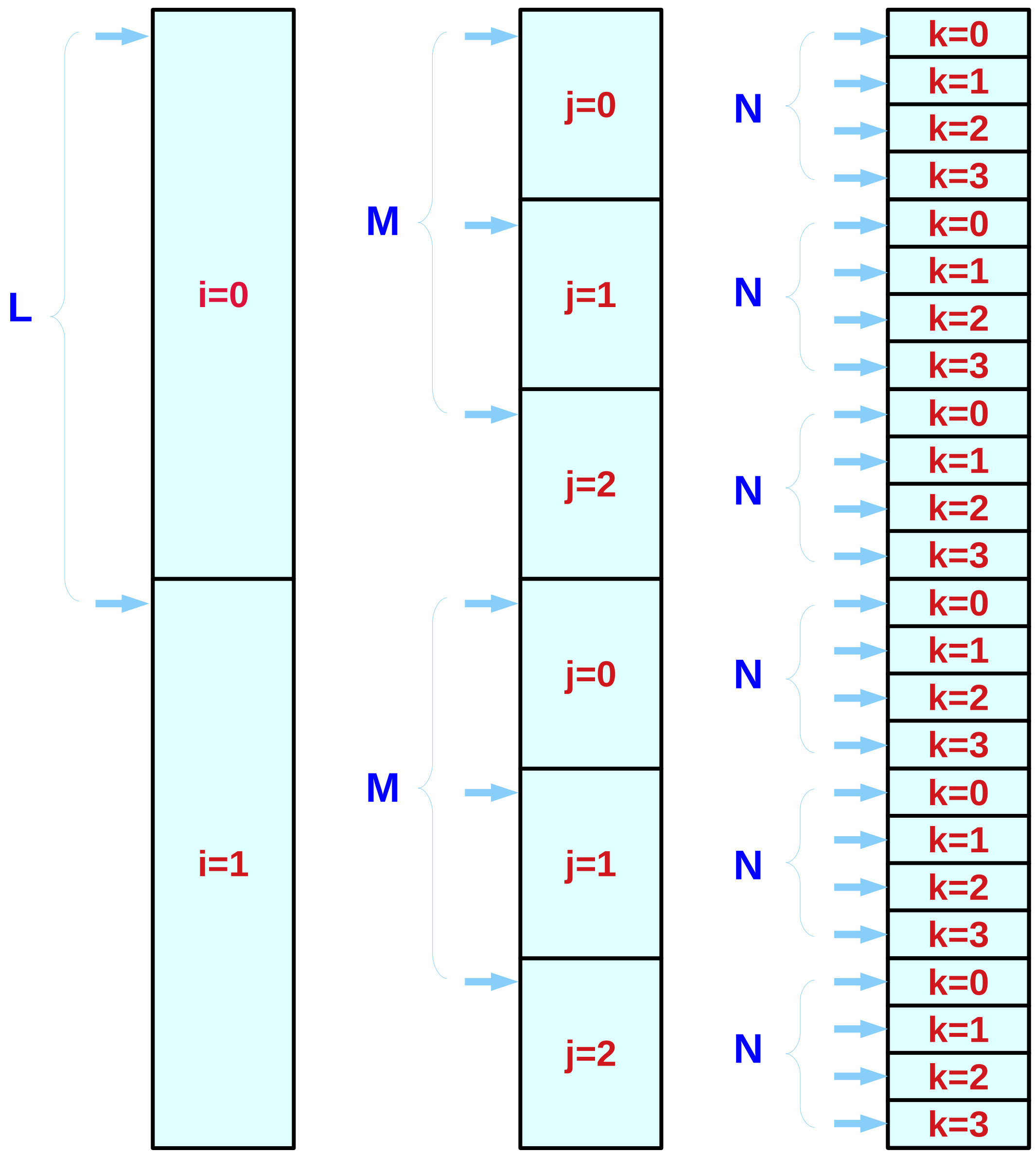
3-d Array Index

c [**i**][**j**][**k**]

interpret as recursive indirections
interpret as hierarchical sub-arrays

L, M, N – the number of index values

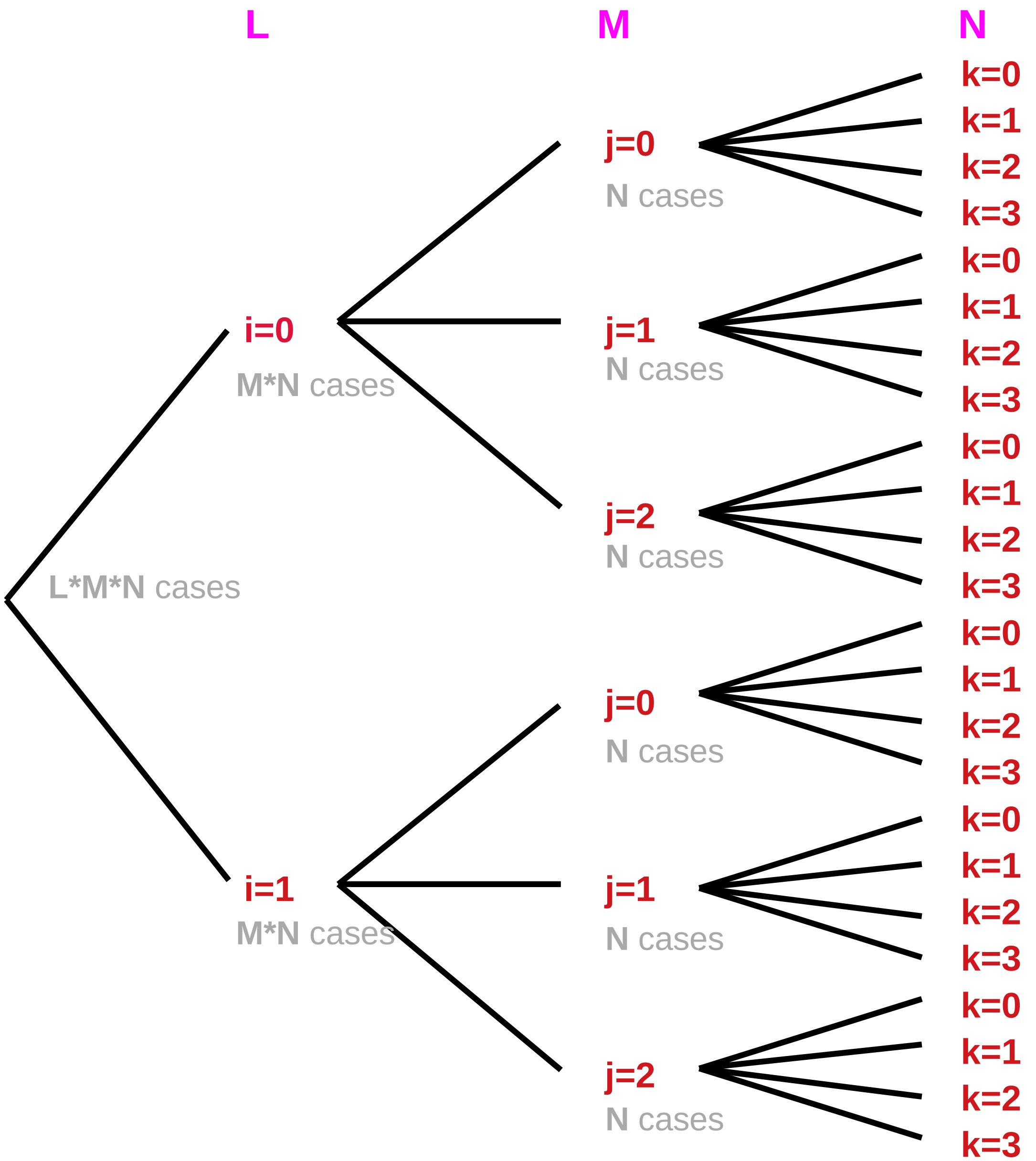
```
int     c [L][M][N];

        c [i][j][k]
```



Index value tree – all possible combinations

int c [L][M][N];

c [i][j][k]



The number of elements of subarrays

```
int     c [L][M][N];

        c [i][j][k]
```

c ……………. L*M*N elements
c[i] ……………. M*N elements
c[i][j] ……………. N elements

array names | covering elements

From a **3-d** index to a **1-d** index

```
int     c [L][M][N];

        c [i][j][k]
```

L **M** **N**

[i] **[j]** **[k]**

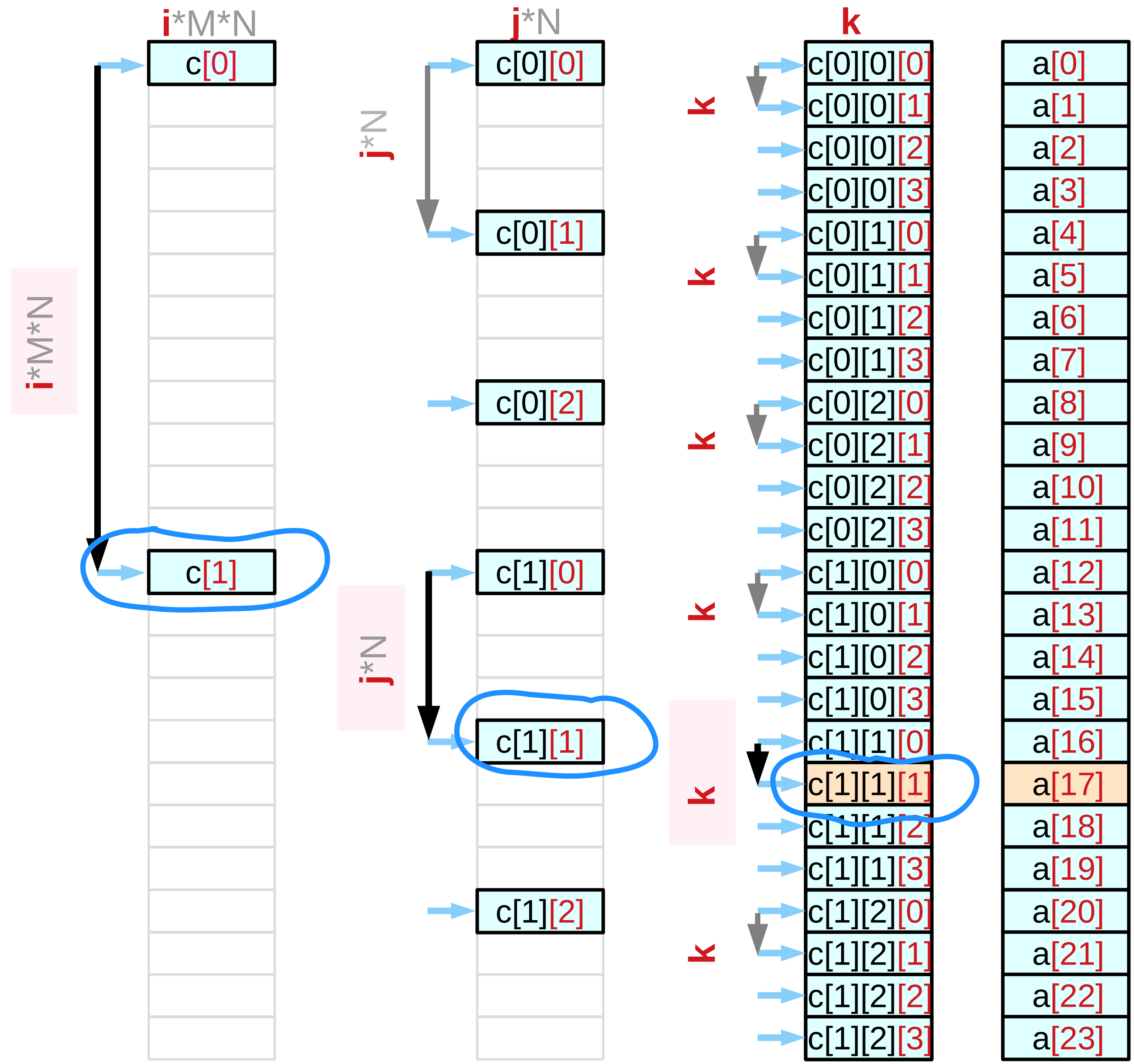
$i*M*N + j*N + k$

$(i*M + j)*N + k$

i*M*N, j*N, k – index offset values

int c [L][M][N];

c [i][j][k]

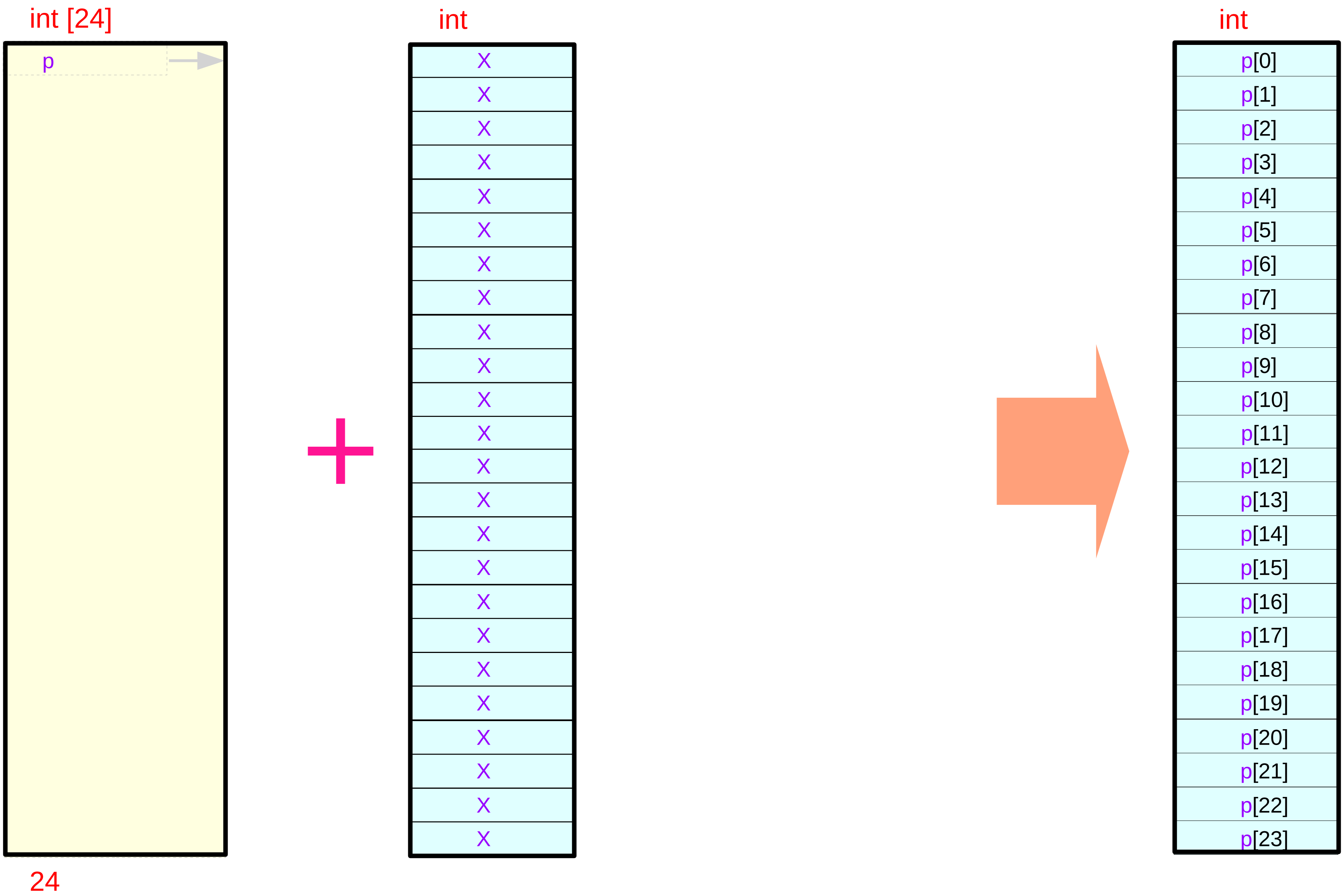


c [1][1][1]

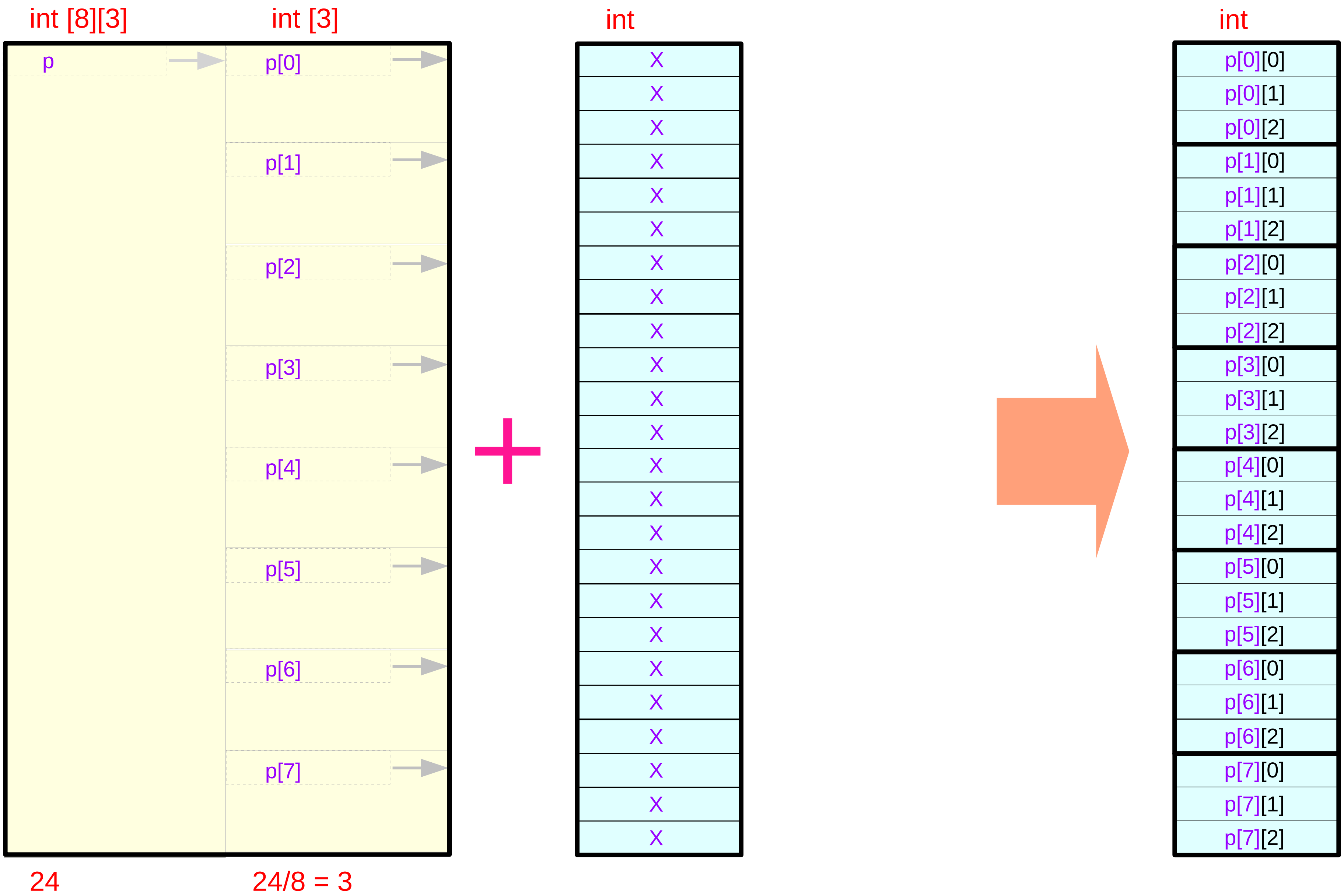
i=1	j=1	k=1

a [(1*3 + 1)*4 + 1]

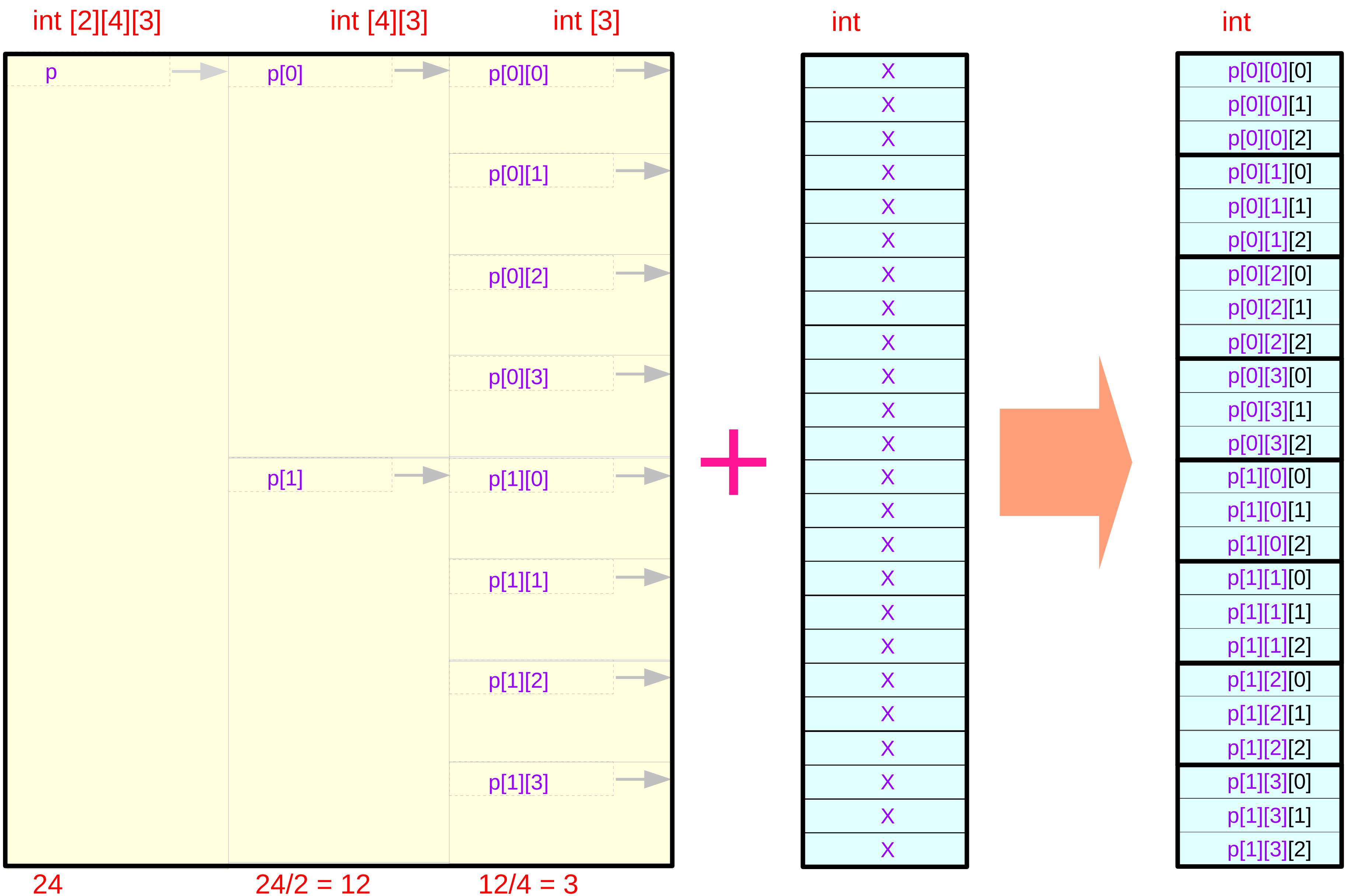
Indexing in int p[24]



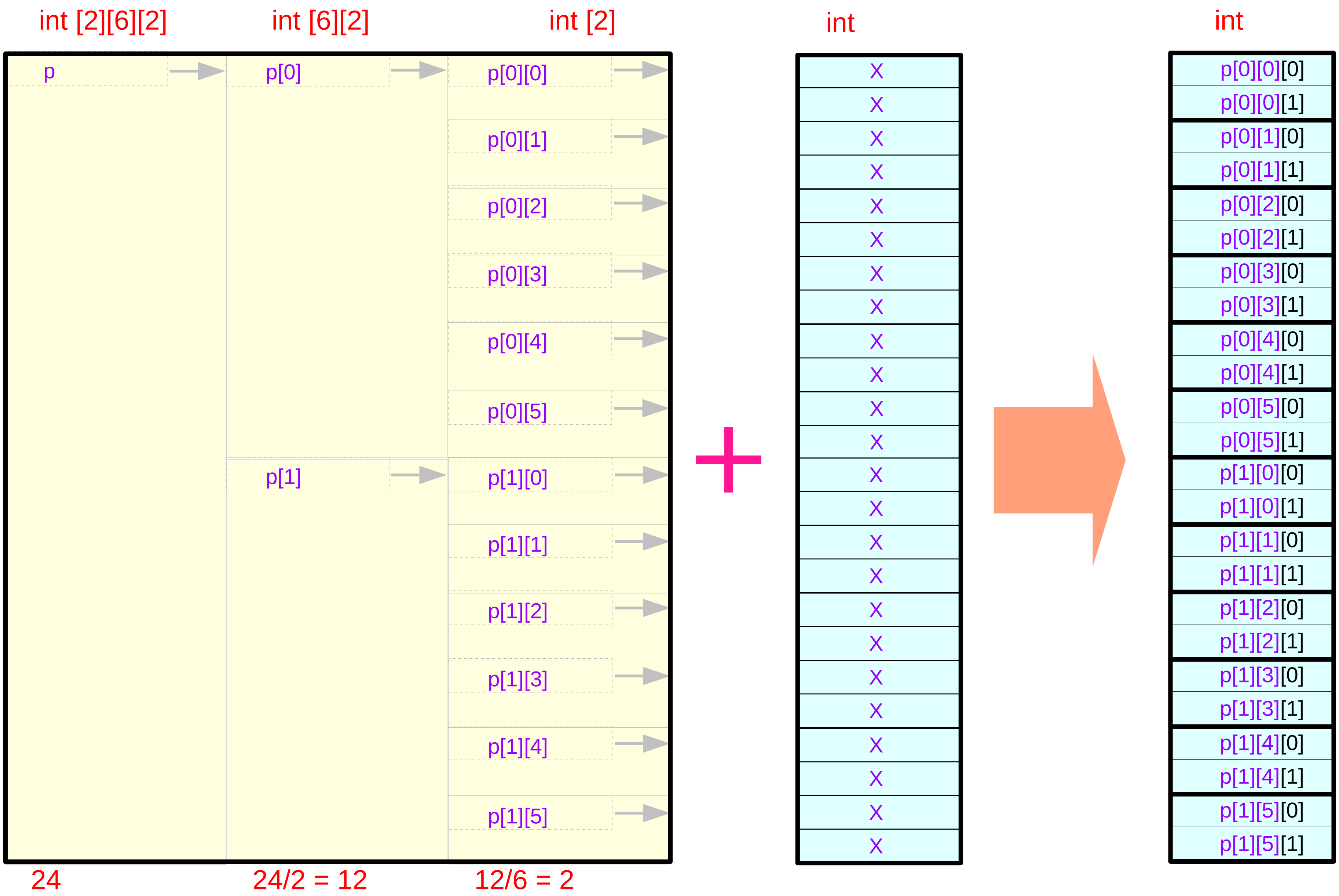
Indexing in int p[8][3]



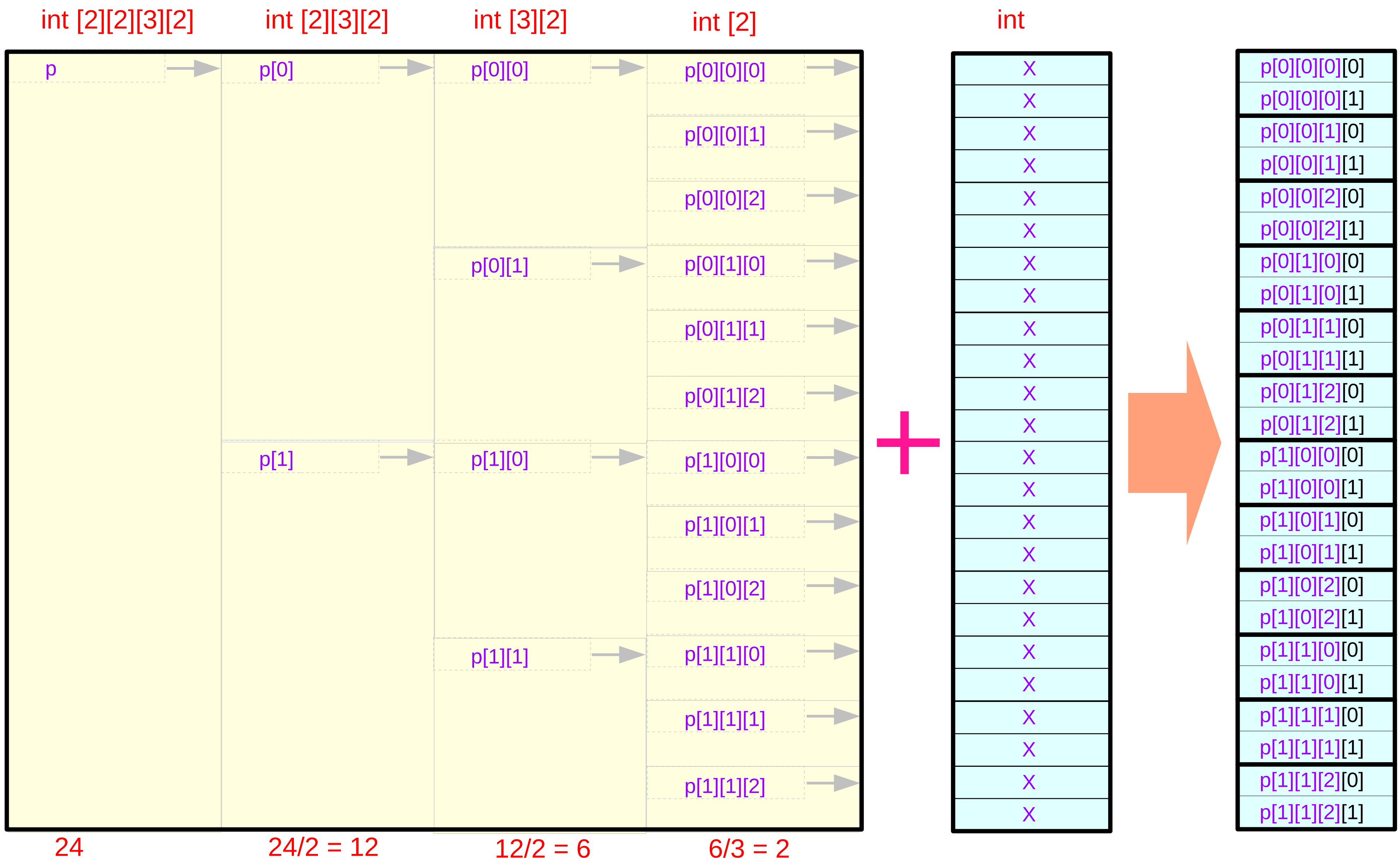
Indexing in int p[2][4][3]



Indexing in int p[2][6][2]



Indexing in int p[2][2][3][2]



Indexing in muti-dimension arrays

int p[24]	int p[8][3]	int p[2][4][3]	int p[2][6][2]	int p[2][2][3][2]
1-d array	2-d array	3-d array	3-d array	4-d array
p[0]	p[0][0]	p[0][0][0]	p[0][0][0]	p[0][0][0][0]
p[1]	p[0][1]	p[0][0][1]	p[0][0][1]	p[0][0][0][1]
p[2]	p[0][2]	p[0][0][2]	p[0][1][0]	p[0][0][1][0]
p[3]	p[1][0]	p[0][1][0]	p[0][1][1]	p[0][0][1][1]
p[4]	p[1][1]	p[0][1][1]	p[0][2][0]	p[0][0][2][0]
p[5]	p[1][2]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
p[9]	p[3][0]	p[0][3][0]	p[0][4][1]	[illegible]
p[10]	p[3][1]	p[0][3][1]	p[0][5][0]	p[0][1][2][0]
p[11]	p[3][2]	p[0][3][2]	p[0][5][1]	p[0][1][2][1]
p[12]	p[4][0]	p[1][0][0]	p[1][0][0]	p[1][0][0][0]
p[13]	p[4][1]	p[1][0][1]	p[1][0][1]	p[1][0][0][1]
p[14]	p[4][2]	p[1][0][2]	p[1][1][0]	p[1][0][1][0]
p[15]	p[5][0]	p[1][1][0]	p[1][1][1]	p[1][0][1][1]
p[16]	p[5][1]	p[1][1][1]	p[1][2][0]	p[1][0][2][0]
p[17]	p[5][2]	p[1][1][2]	p[1][2][1]	p[1][0][2][1]
p[18]	p[6][0]	p[1][2][0]	p[1][3][0]	p[1][1][0][0]
p[19]	p[6][1]	p[1][2][1]	p[1][3][1]	p[1][1][0][1]
p[20]	p[6][2]	p[1][2][2]	p[1][4][0]	p[1][1][1][0]
p[21]	p[7][0]	p[1][3][0]	p[1][4][1]	p[1][1][1][1]
p[22]	p[7][1]	p[1][3][1]	p[1][5][0]	p[1][1][2][0]
p[23]	p[7][2]	p[1][3][2]	p[1][5][1]	p[1][1][2][1]

Subarray sizes of 24 element multi-dimensional arrays (1)

int p[2][2][3][2]

p sizeof(p)	int [2][2][3][2] = 2*2*3*2 =24 integers
p[0] sizeof(p[0])	int [2][3][2] = 2*3*2 =12 integers
p[0][0] sizeof(p[0][0])	int [3][2] = 3*2 =6 integers
p[0][0][0] sizeof(p[0][0][0])	int [2] = 2 integers
p[0][0][0][0] sizeof(p[0][0][0][0])	int = 1 integers

int p[2][6][2]

p sizeof(p)	int [2][6][2] = 2*6*2 =24 integers
p[0] sizeof(p[0])	int [6][2] = 6*2 =12 integers
p[0][0] sizeof(p[0][0])	int [2] = 2 integers
p[0][0][0] sizeof(p[0][0][0])	int = 1 integers

int p[2][4][3]

p sizeof(p)	int [2][4][3] = 2*4*3 =24 integers
p[0] sizeof(p[0])	int [4][3] = 4*3 =12 integers
p[0][0] sizeof(p[0][0])	int [3] = 3 integers
p[0][0][0] sizeof(p[0][0][0])	int = 1 integers

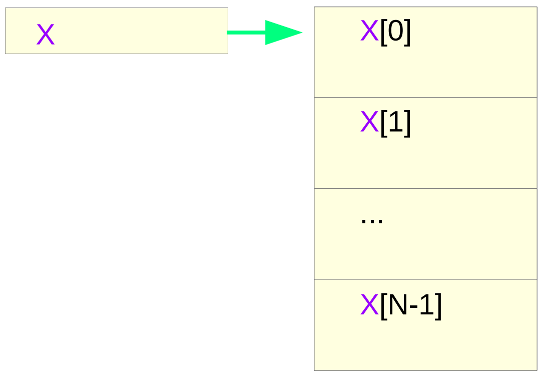
Subarray sizes of 24 element multi-dimensional arrays (1)

int p[8][3]

p sizeof(p)	int [8][3] = 8*3 =24 integers
p[0] sizeof(p[0])	int [3] = 3 integers
p[0][0] sizeof(p[0][0])	int = 1 integers

int p[24]

p sizeof(p)	int [24] = 24 integers
p[0] sizeof(p[0])	int = 1 integers



sizeof(X) = N * sizeof(X[0])

```
int p[2][3][4];
```

(1) **typedef int atype [2][3][4] ;** (atype ≡ int [2][3][4])

atype p; (int p [2][3][4])

(2) **typedef int btype [3][4] ;** (btype ≡ int [3][4])

btype p[2]; (int p[2] [3][4])

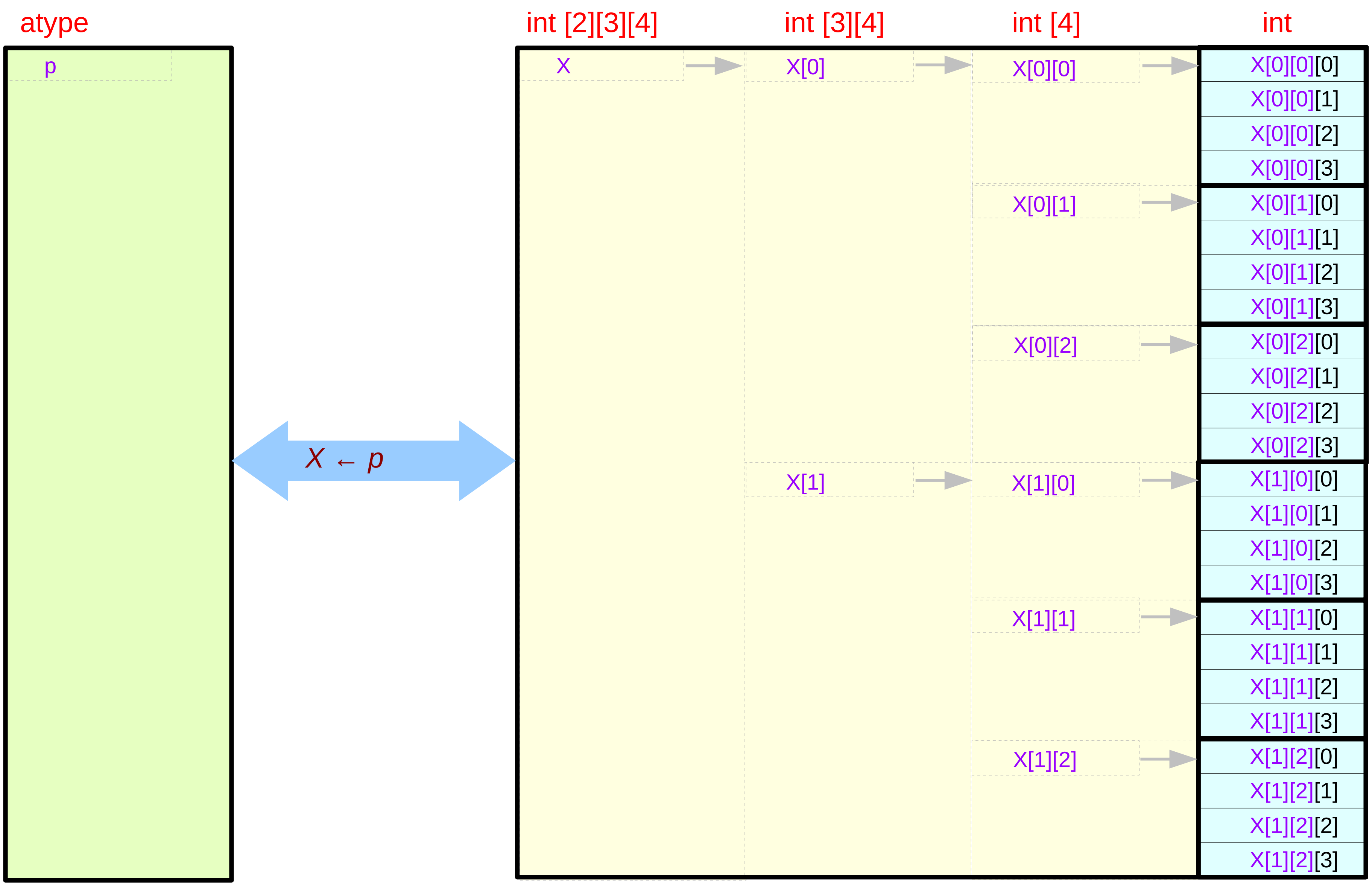
(3) **typedef int ctype [4] ;** (ctype ≡ int [4])

ctype p[2][3]; (int p[2][3] [4])

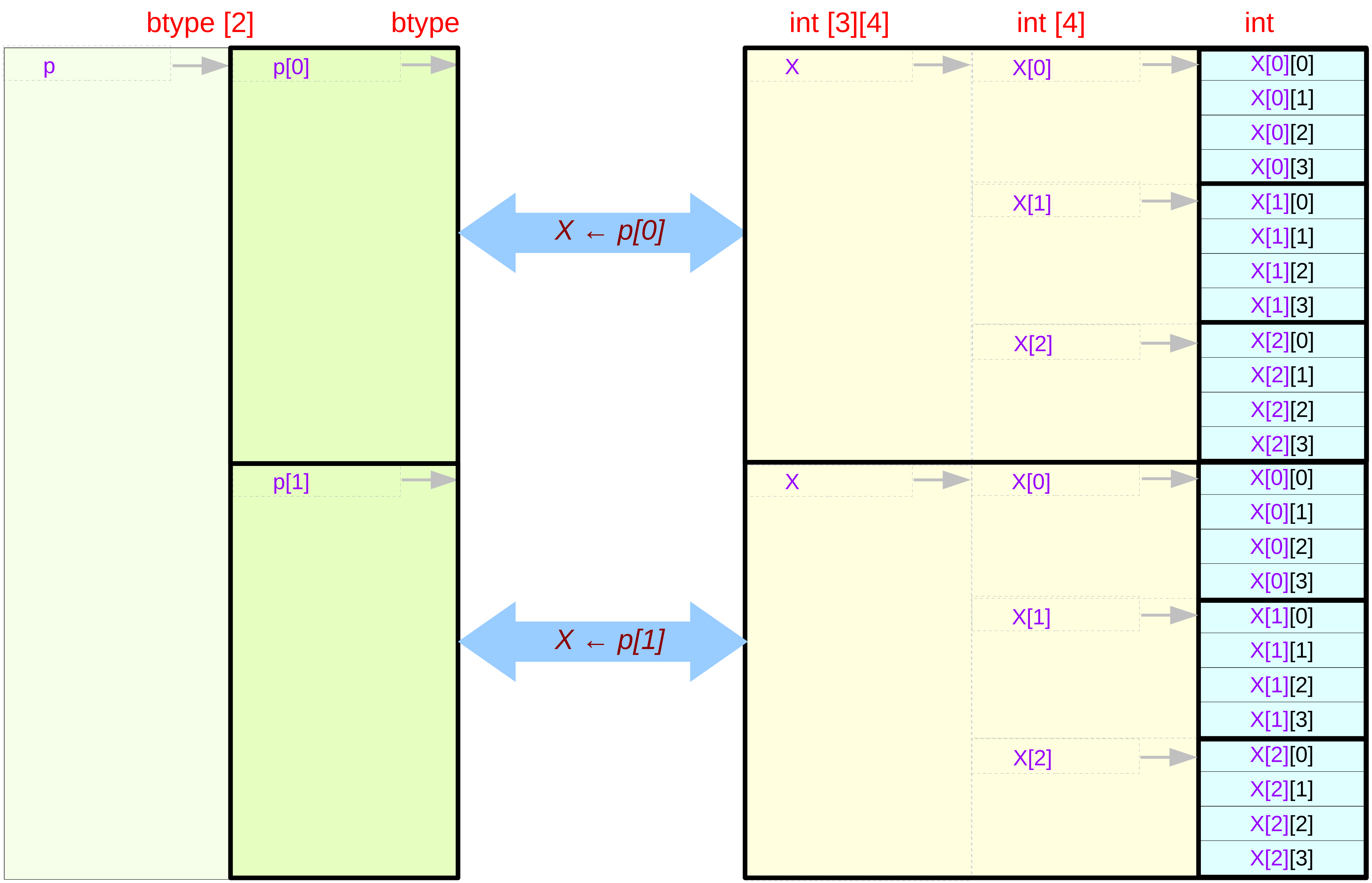
(4) **typedef int dtype ;** (dtype ≡ int)

dtype p[2][3][4]; (int p[2][3][4])

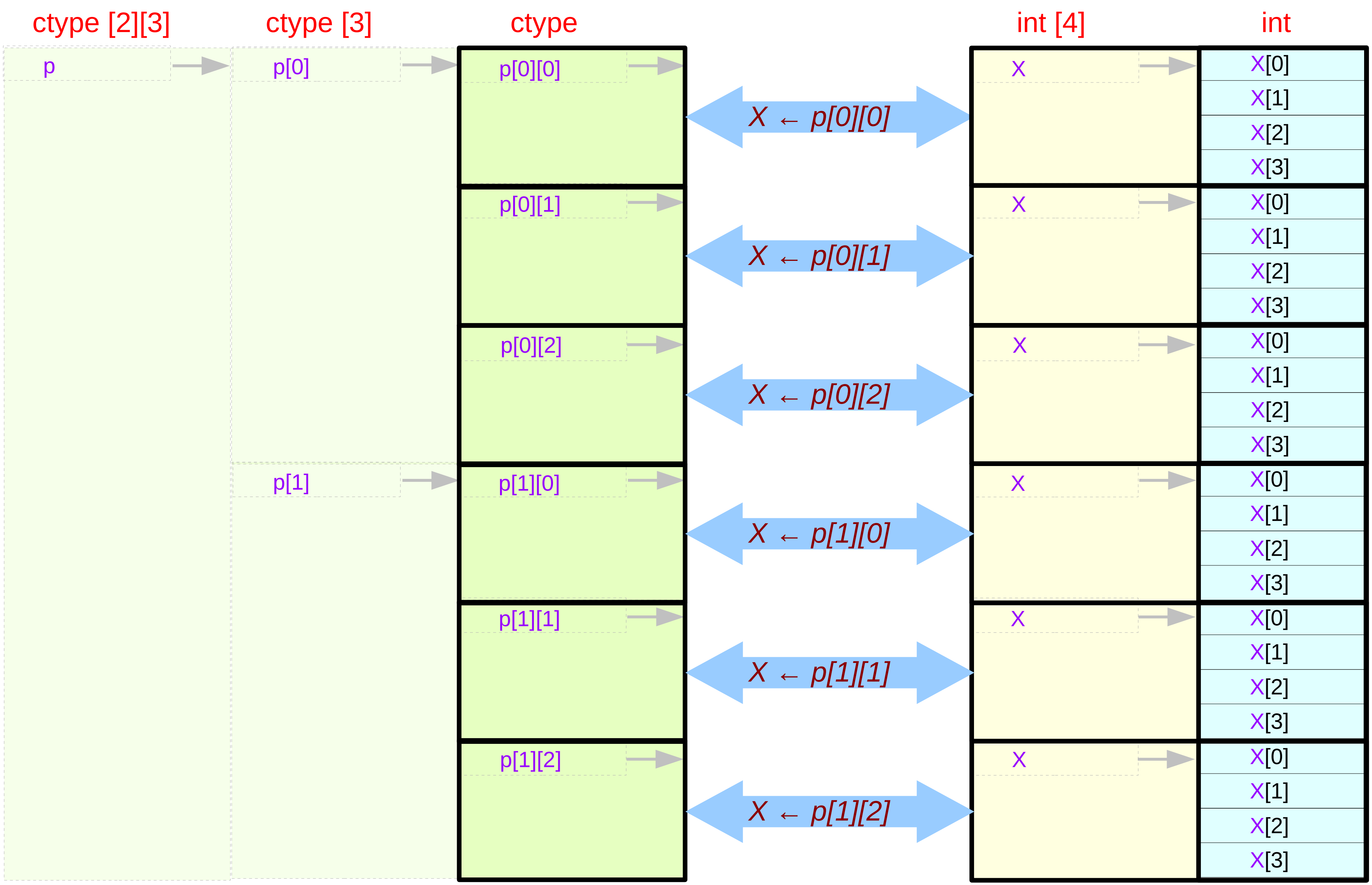
(1) atype p; where typedef int atype [2][3][4] ;



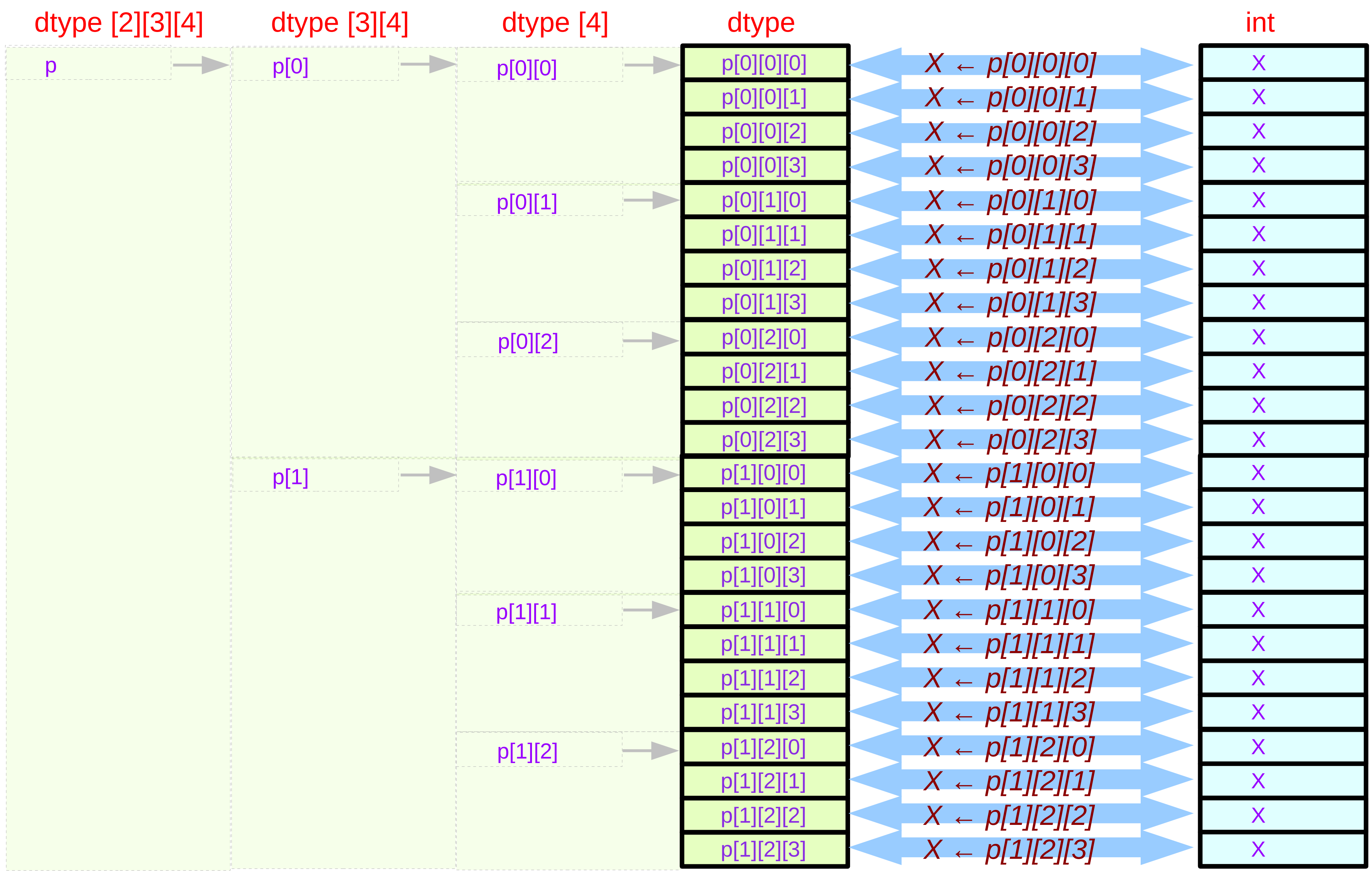
(2) btype p[2]; where typedef int btype [3][4] ;



(3) ctype p[2][3]; where typedef int ctype [4] ;



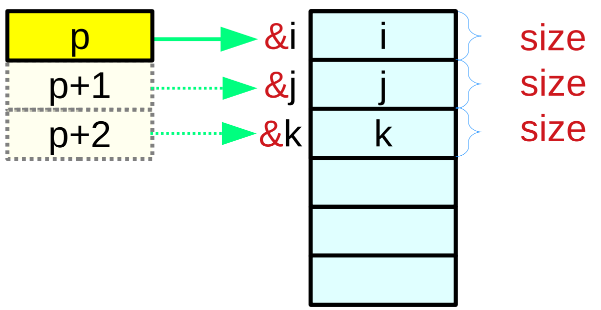
(4) dtype p[2][3][4]; where typedef int ctype ;



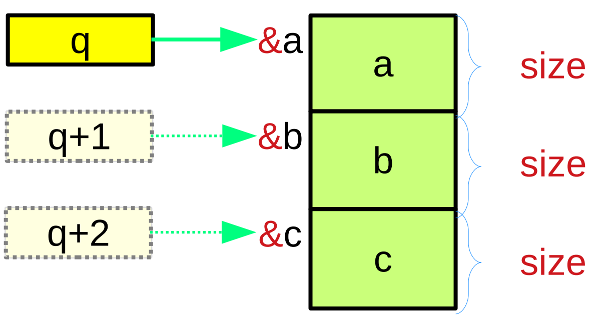
Virtual array pointers in a multi-dimensional array

Pointers to various data types

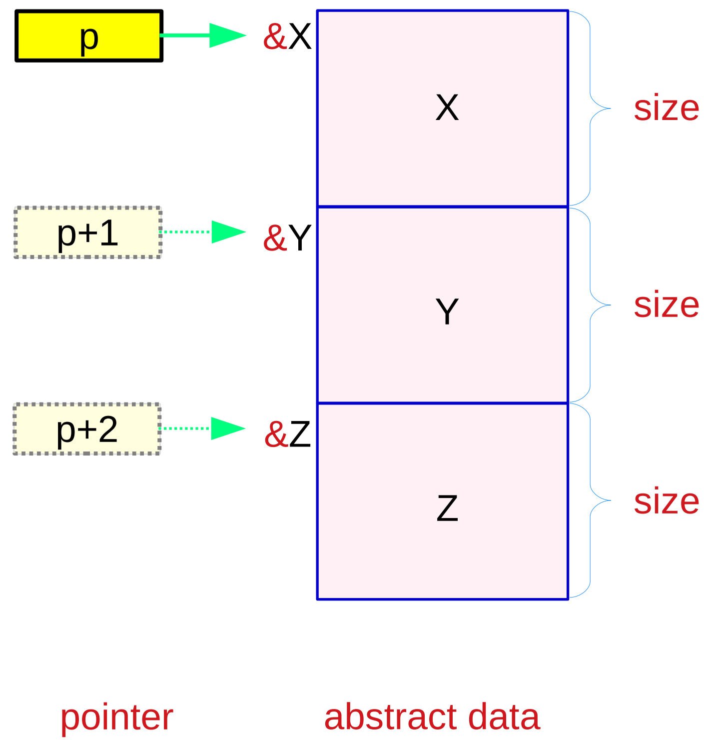
int *p;   int i, j, k;



double *q;   double a, b, c;

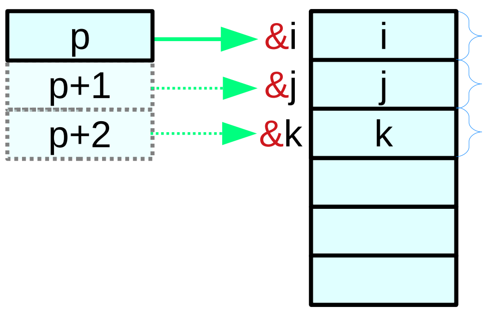


T *p;   T X, Y, Z;



Pointers to primitive data

int *p;   **int i, j, k;**   **sizeof(int) = 4 bytes**



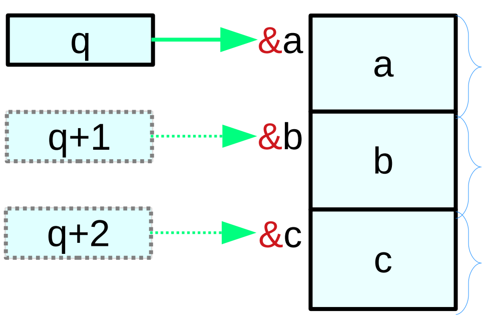
size = sizeof(i) = sizeof(*p) ≠ sizeof(p)

size = sizeof(j) = sizeof(*(p+1)) ≠ sizeof(p+1)

size = sizeof(k) = sizeof(*(p+2)) ≠ sizeof(p+2)

pointer size
4 or 8 bytes

double *q;   **double a, b, c;**   **sizeof(double) = 8 bytes**



size = sizeof(a) = sizeof(*q) ≠ sizeof(q)

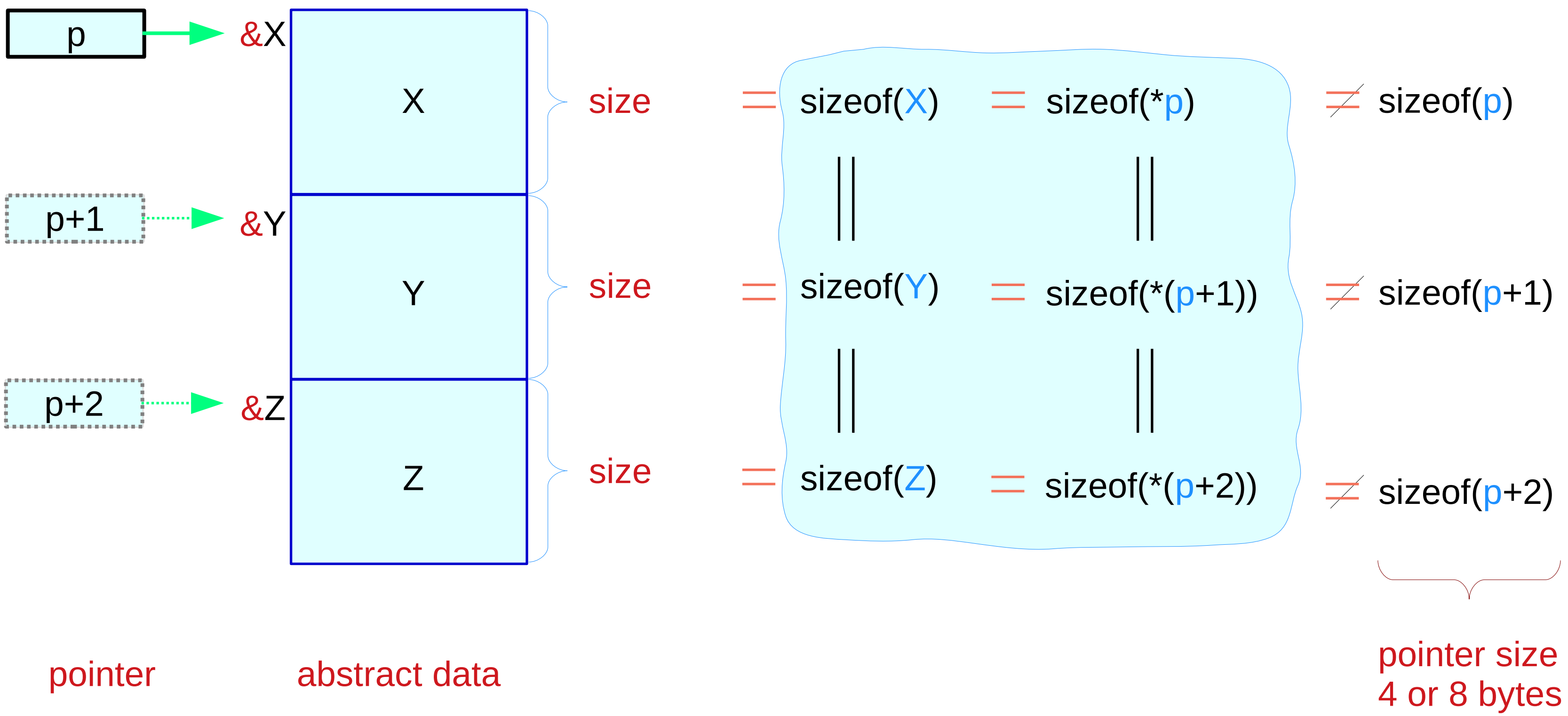
size = sizeof(b) = sizeof(*(q+1)) ≠ sizeof(q+1)

size = sizeof(c) = sizeof(*(q+2)) ≠ sizeof(q+2)

pointer size
4 or 8 bytes

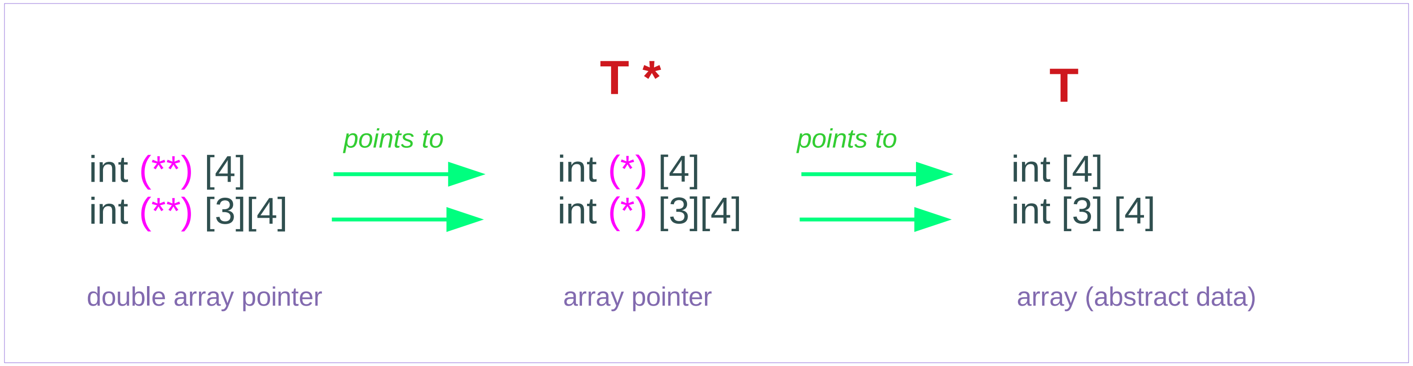
Pointers to abstract data

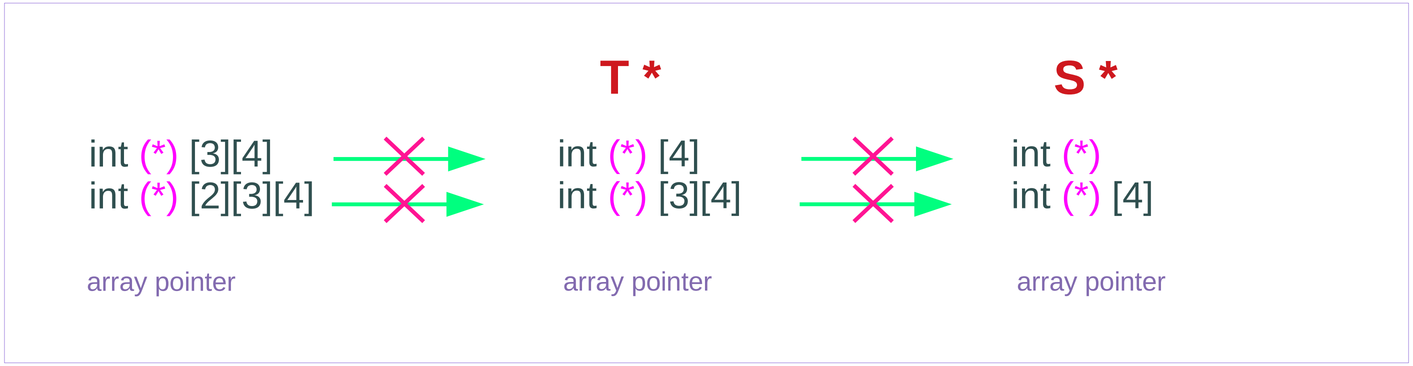
T *p; **T X, Y, Z;** **sizeof(T)**



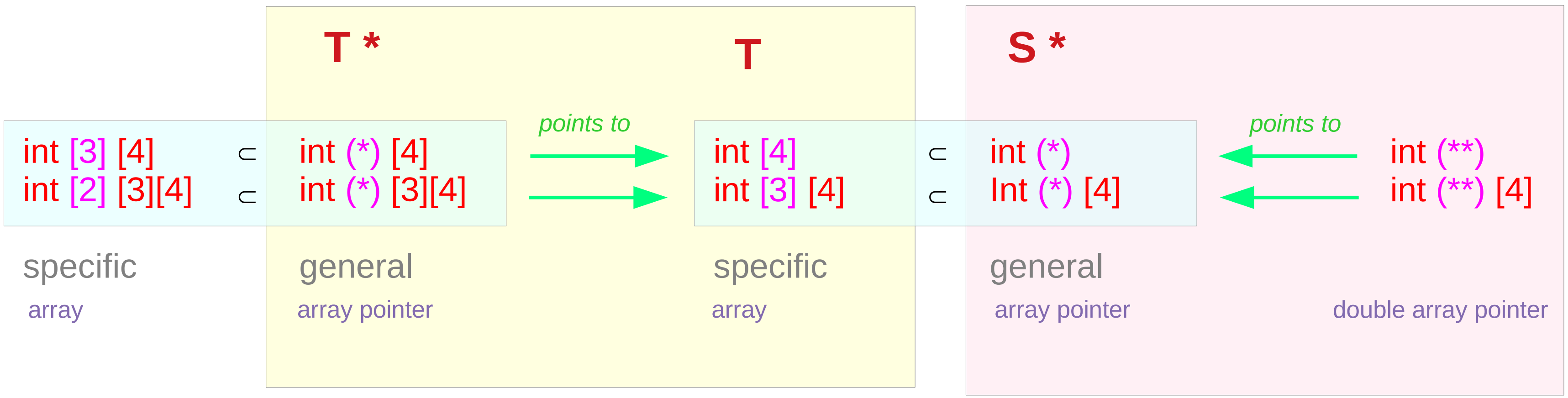
type -------- array
value -------- start address
increment size -------- size

Array pointer types v.s. array types





General array pointer types v.s. specific array types



Array pointers have augmented dimensions

```
typedef int (*T1) [4];          typedef int T2[4];
typedef int (*T1) [3][4];       typedef int T2[3][4];

    int (*) [4]                     int [4]
    int (*) [3][4]                  int [3] [4]
    general                         specific
```

```
T1  a;
T2  b;
```

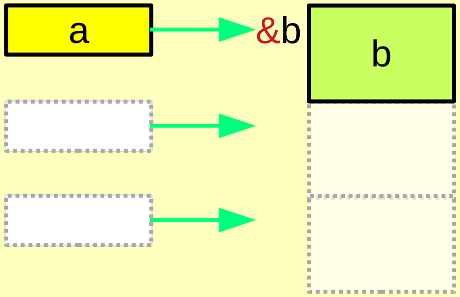
T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

```
 a  = &b;
*a  =  b;
```

a references **b**

b is the dereference of **a**

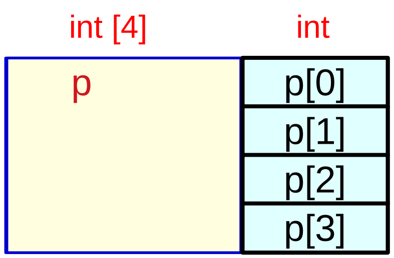
```
  (a+1) = ?
* (a+1) = ?

  (a+2) = ?
* (a+2) = ?
```



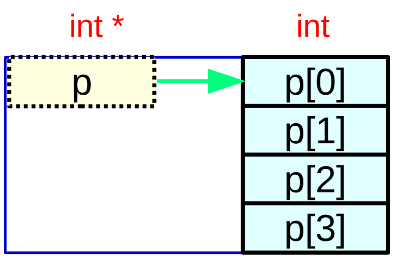
Dual types in an array of integers

int p[3];

p is an abstract data (array)



p can also be viewed as a pointer



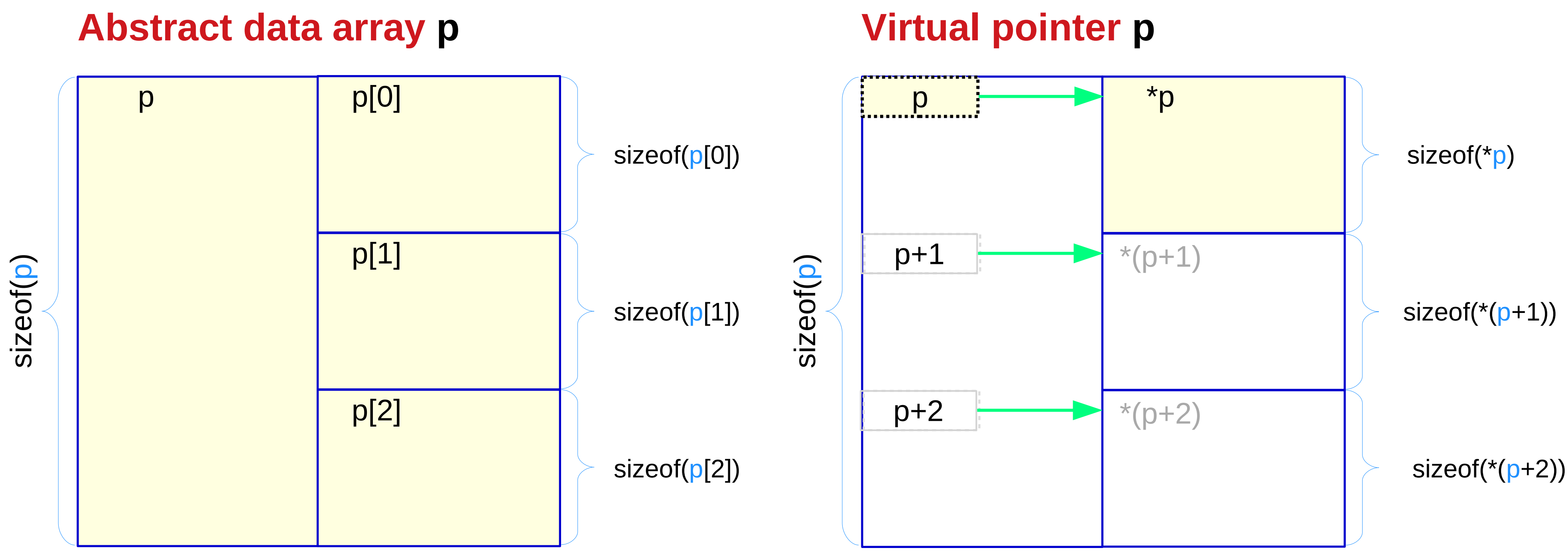
p is the name of an array
p has the size of the whole array

p has an array type (abstract data)

p also has pointer characteristics
p has the value of the starting address

p is a virtual pointer

Dual types in an array of abstract data

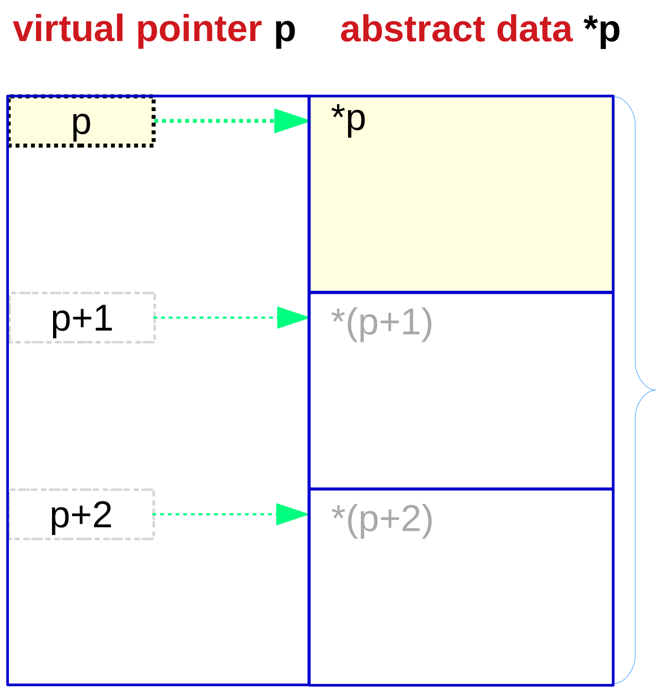


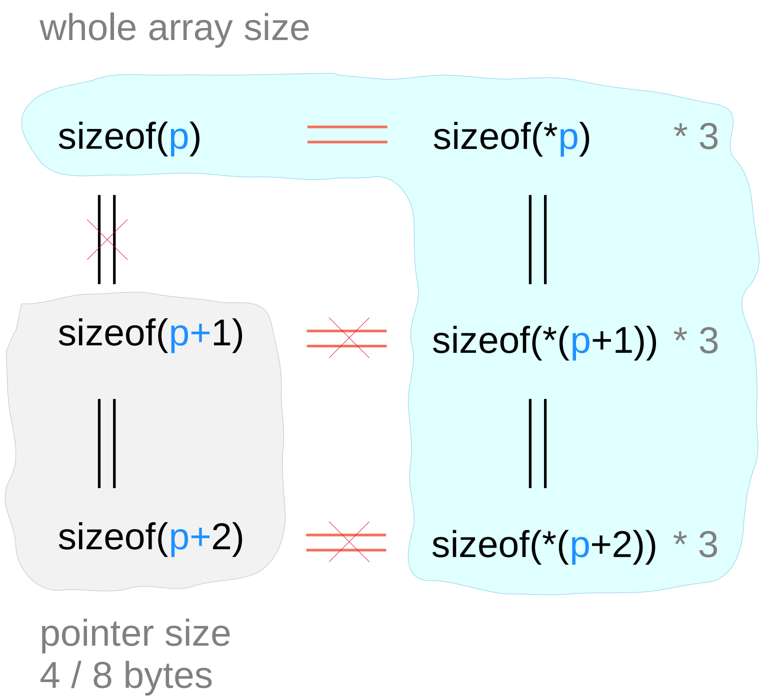
p has an array type (abstract data element)
p is the name of an array
p has the size of the whole array

p also has a pointer type
p has the value of the starting address

p is a virtual array pointer

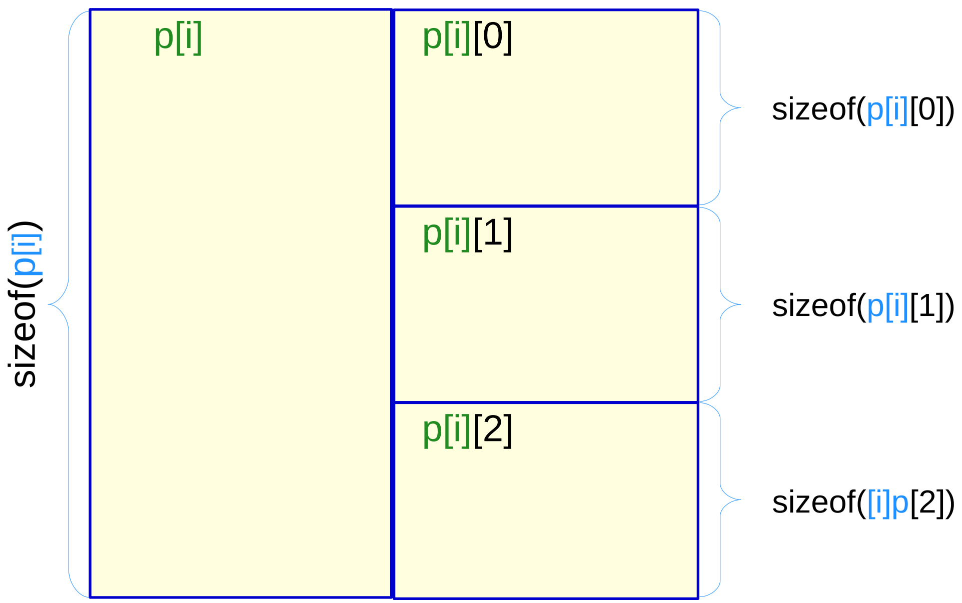
Virtual pointer to abstract data



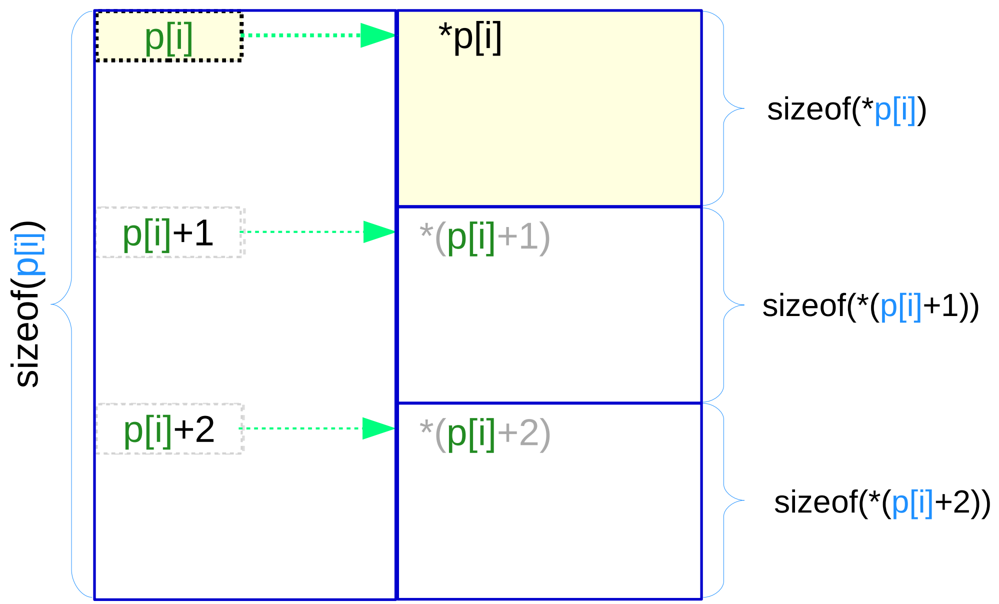


Dual types in a multi-dimensional array

Abstract data (array) p[i]



Virtual array pointer p[i]



p[i] has an array type (abstract data)
p[i] is the name of an array
p[i] has the size of the whole array

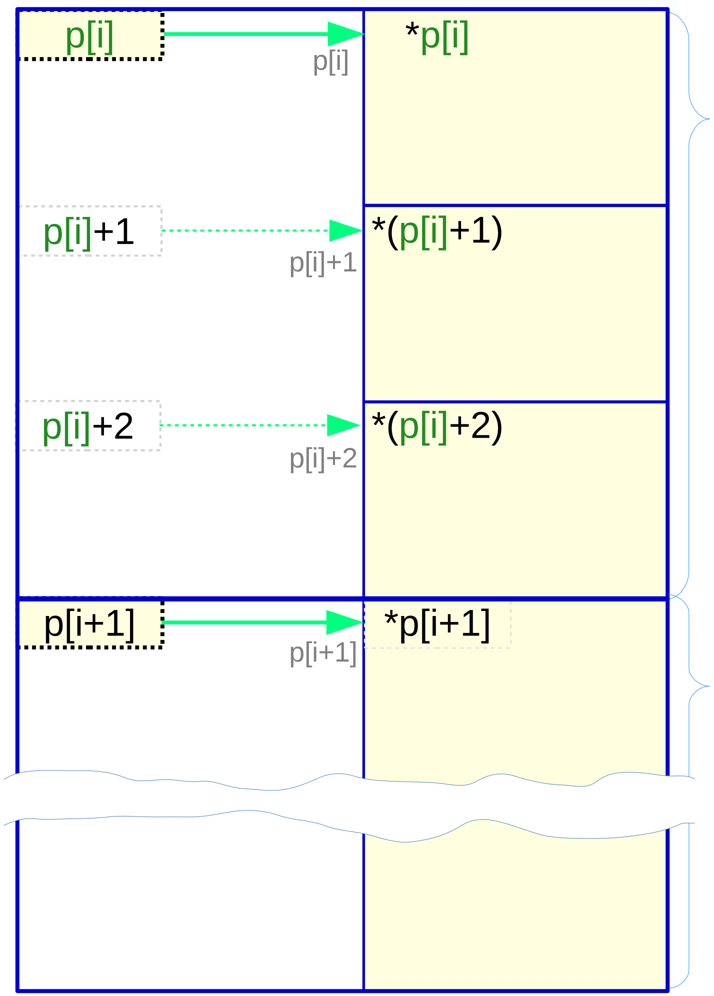
p[i] also has an array pointer type
p[i] has the value of the starting address

p[i] is a virtual array pointer

Virtual pointers to sub-arrays

p[i] :: T *　　　　***p[i], *p[i+1] :: T**

array pointer type　　　　array type



sizeof(p[i])

= sizeof(*p[i]) * N
sizeof(p[i][0]) * N

size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

||

size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

sizeof(p[i+1])

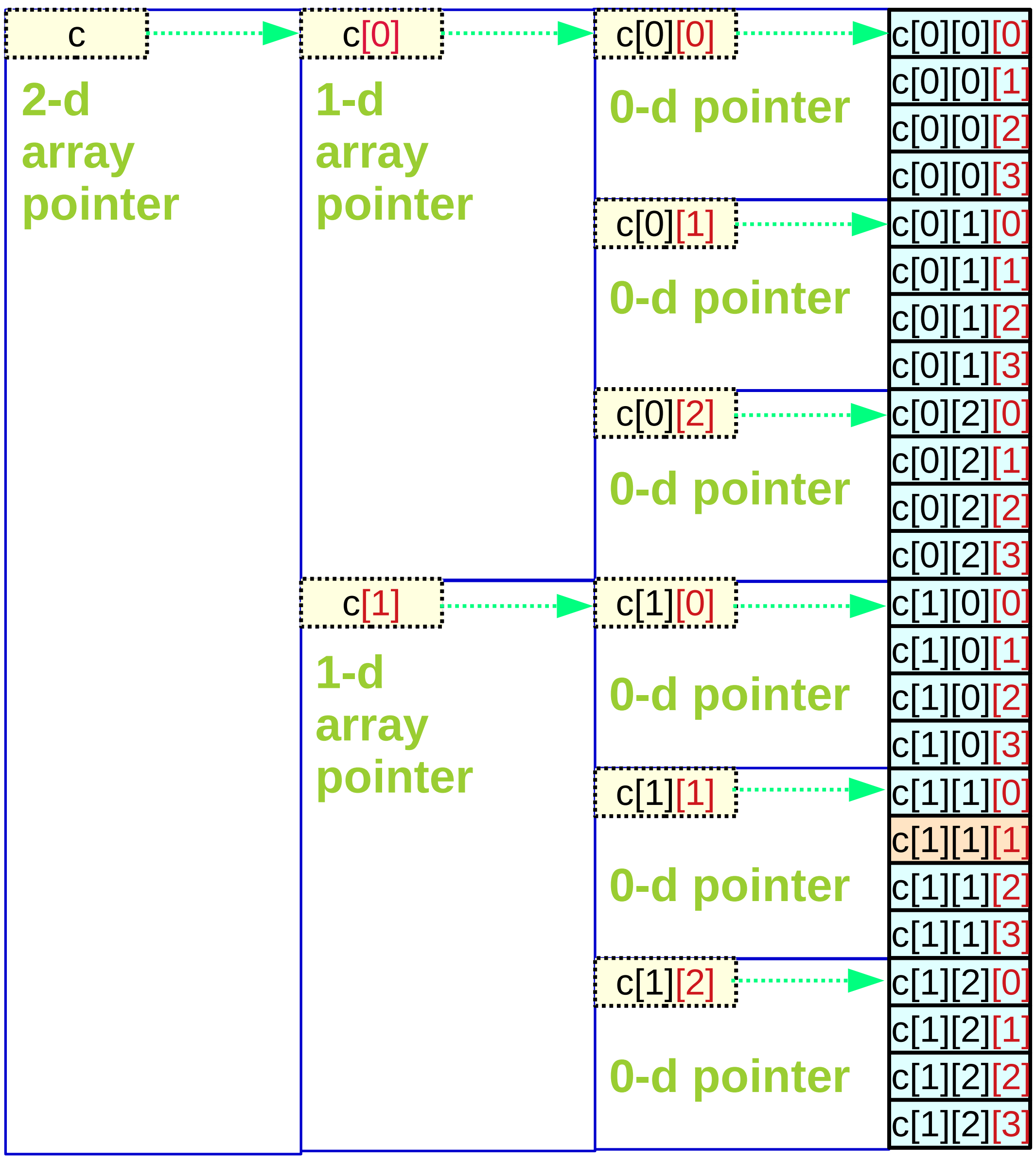
= sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

3-d array structure – virtual pointer representation

int c[2][3][4];

((*(c +i) +j) +k)

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

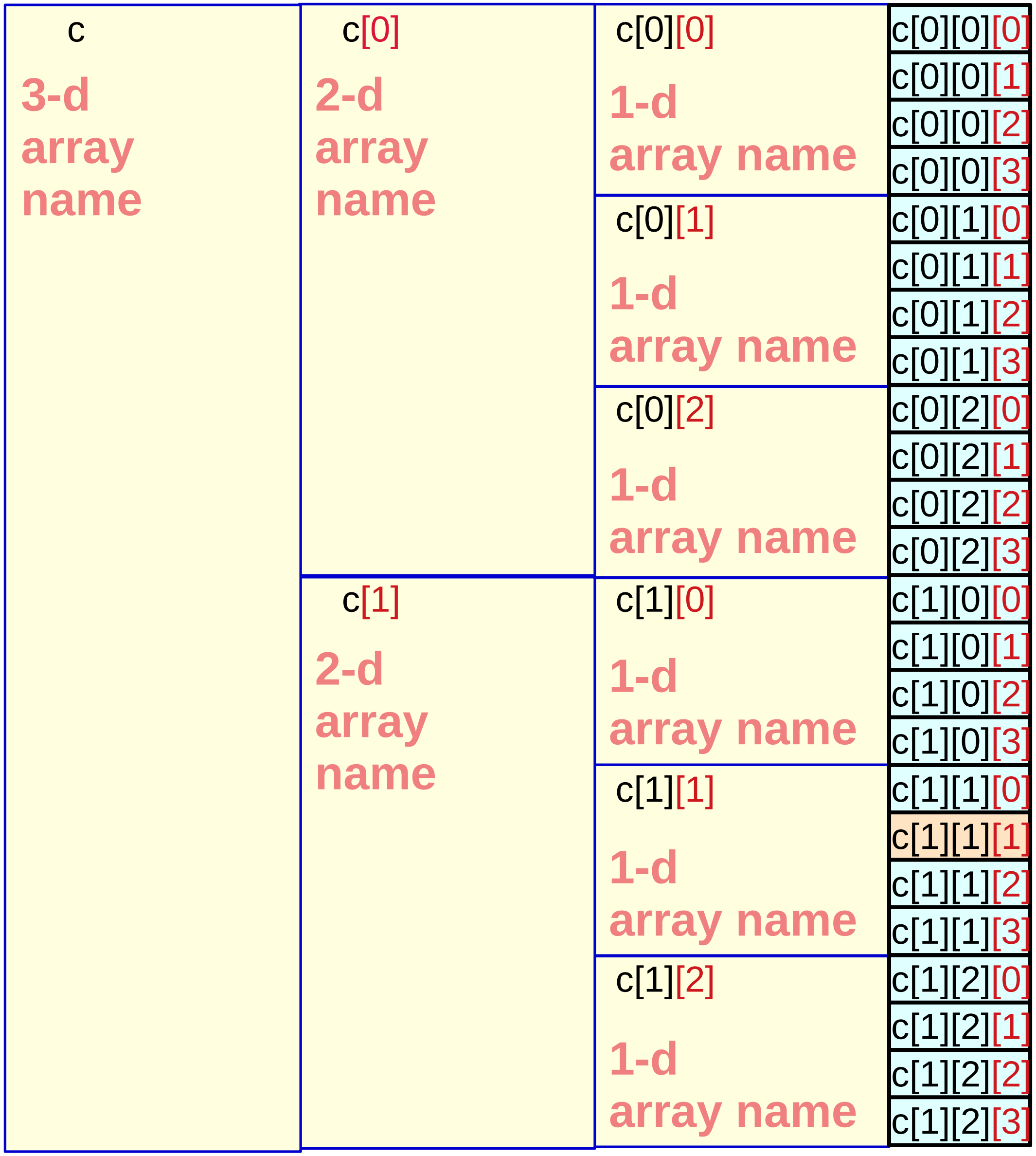


3-d array structure – abstract data representation

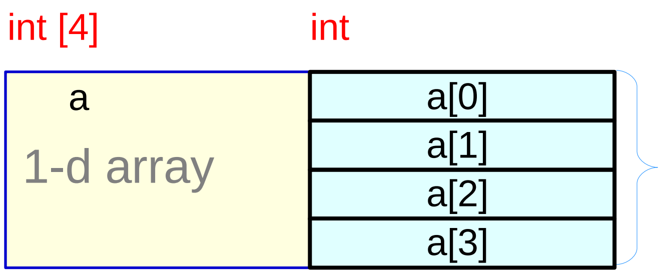
int c[2][3][4];

((c [i])[j])[k]

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

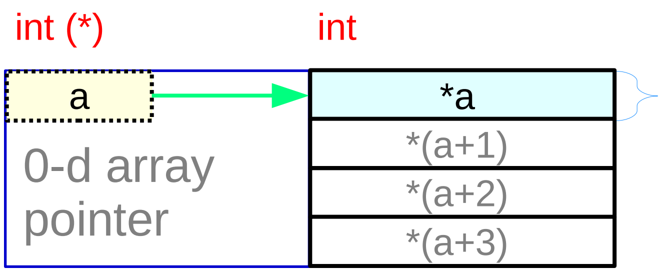


Array **a** and virtual pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

a is the name of a 1-d array
a also has a pointer type
a has the size of the array
a has the value of the starting address

a is a virtual array pointer

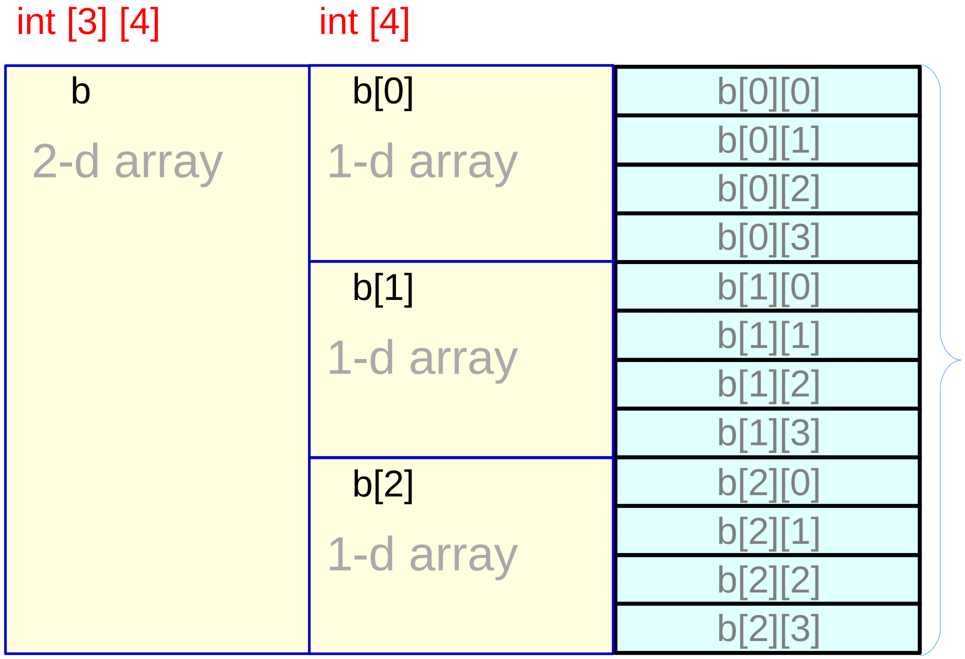
Array **b** and virtual pointer **b**

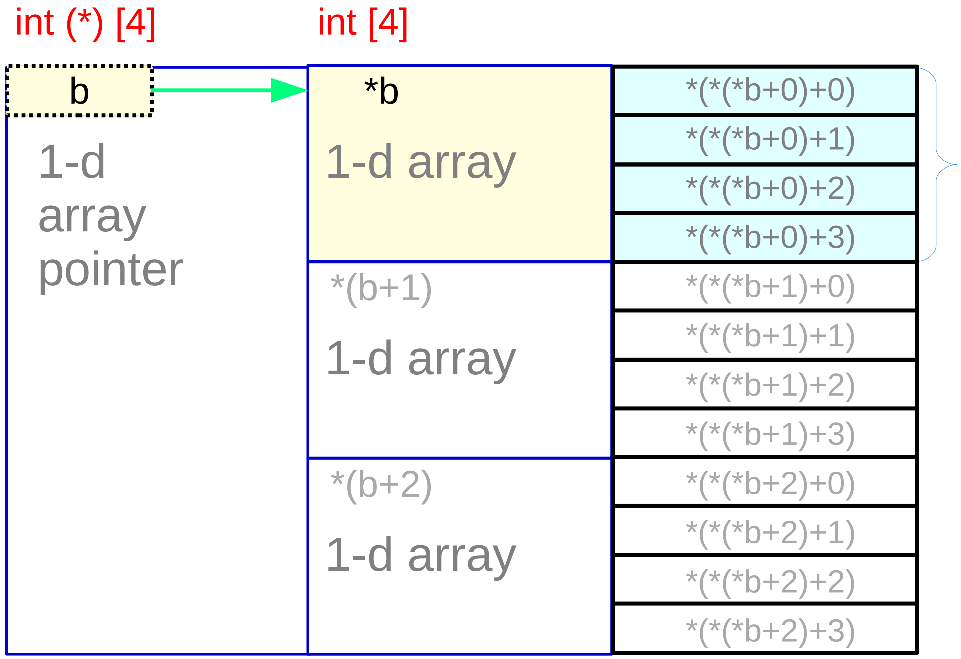
2-d array b specific array type

sizeof(b)

1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3





b is the name of a 2-d array
b has the size of the array

b also has a 1-d array pointer type
b has the value of the starting address

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

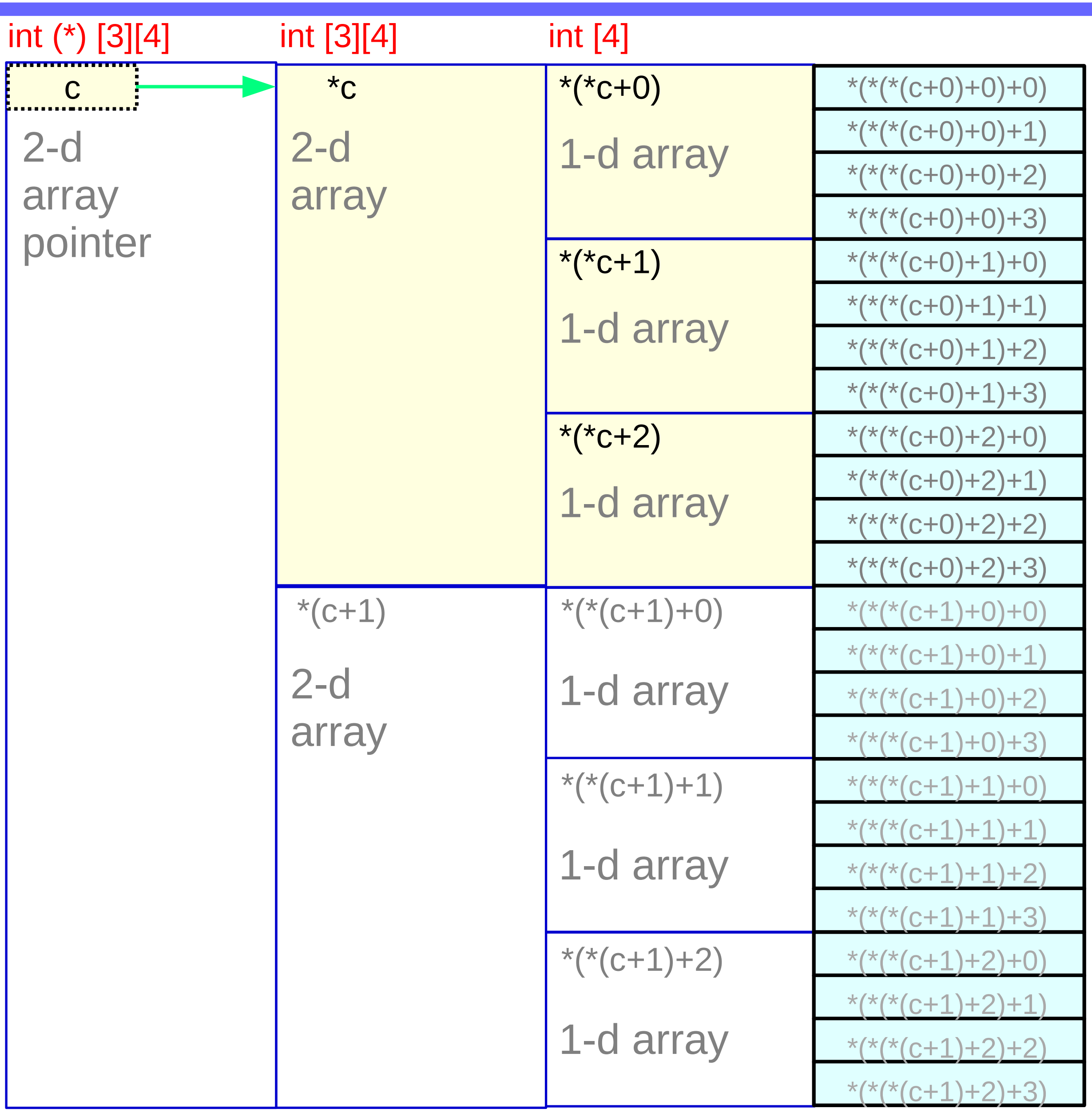
c is the name of a 3-d array
c has the size of the array

Virtual pointer **c**

2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2



c also has a 2-d array pointer type
c has the value of the starting address

c is a virtual array pointer

Finding dual types in a **3-d** array

int c[2][3][4];

The type of c

dual types
- **Int [2][3][4]** abstract data type
- **Int [ ][3][4]** relaxing the 1st dimension
- **Int (*)[3][4]** virtual pointer type - gcc displaying type

int c[2][3][4];

The type of c[i]

dual types
- **int [3][4]** abstract data type
- **int [ ][4]** relaxing the 1st dimension
- **int (*)[4]** virtual pointer type - gcc displaying type

int c[2][3][4];

The type of c[i][j]

dual types
- **int [4]** abstract data type
- **Int [ ]** relaxing the 1st dimension
- **int (*)** virtual pointer type - gcc displaying type

Types of virtual array pointers in a **3-d** array

int c[2][3][4];

	array type (name)	array pointer type
c[i][j][k]	int	int
c[i][j] [k]	int [4] [k]	int (*) [k]
c[i] [j] [k]	int [3][4] [j] [k]	int (*)[4] [j] [k]
c [i] [j] [k]	int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]

Sizes of virtual array pointers in a **3-d** array

```
int c[2][3][4];
```

sizeof(**c[i][j][k]**)	=	sizeof(int)	
sizeof(**c[i][j]**) [k]	=	sizeof(int)	* 4 [k]
sizeof(**c[i]**) [j] [k]	=	sizeof(int)	* 3 * 4 [j] [k]
sizeof(**c**) [i] [j] [k]	=	sizeof(int)	* 2 * 3 * 4 [i] [j] [k]

Element Size

Addresses of virtual array pointers in a **3-d** array

int c[2][3][4];

			[i] [3] [4]	[j] [4]	[k]
&c[i][j][k]	=	&c[0][0][0]	+ i*3*4*4	+ j*4*4	+ k*4
&c[i][j] [4]	=	&c[0][0][0]	+ i*3*4*4	+ j*4*4	sizeof(*c[i][j]) = sizeof(c[i][j][0]) = sizeof(int)
&c[i] [3] [4]	=	&c[0][0][0]	+ i*3*4*4	sizeof(*c[i]) = sizeof(c[i][0]) = sizeof(int) * 4 [k]	
&c [2] [3] [4]	=	&c[0][0][0]	sizeof(*c) = sizeof(c[0]) = sizeof(int) * 3 * 4 [j] [k]		
		Base Address			

Values of virtual array pointers in a **3-d** array

```
int c[2][3][4];
```

c[i][j][k] +0	=	&c[i][j][k]	=	the **value** of a virtual array pointer is its **address**
c[i][j] +k	=	&c[i][j][k]	=	&c[i][j][0] + k * sizeof(c[i][j][0]) sizeof(*c[i][j]) = sizeof(int)
c[i] +j	=	&c[i][j]	=	&c[i][0][0] + j * sizeof(c[i][0]) sizeof(*c[i]) = sizeof(int) * 4 [k]
c +i	=	&c[i]	=	&c[0][0][0] + i * sizeof(c[0]) sizeof(*c) = sizeof(int) * 3 * 4 [j] [k]

Start Address

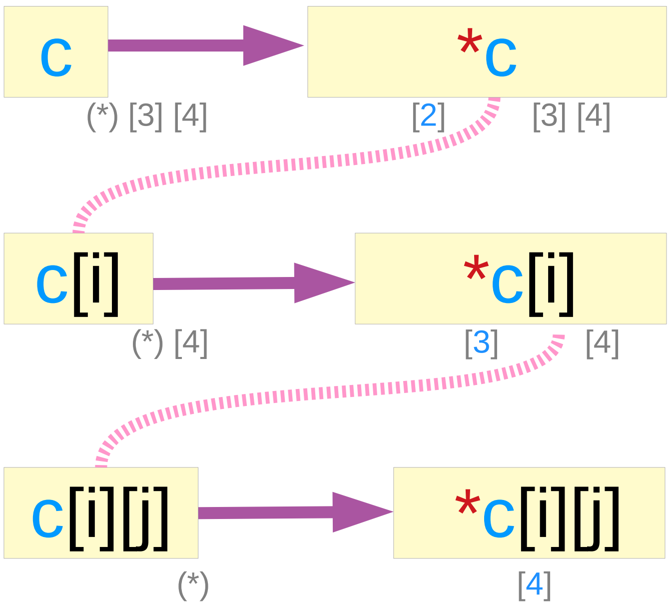
Virtual array pointers and abstract data in a **3-d** array

int c [2][3][4];

2-d array pointer — **int (*) [3][4]**

1-d array pointer — **int (*) [4]**

0-d array pointer — **int (*)**



int [3][4] — **2-d** array

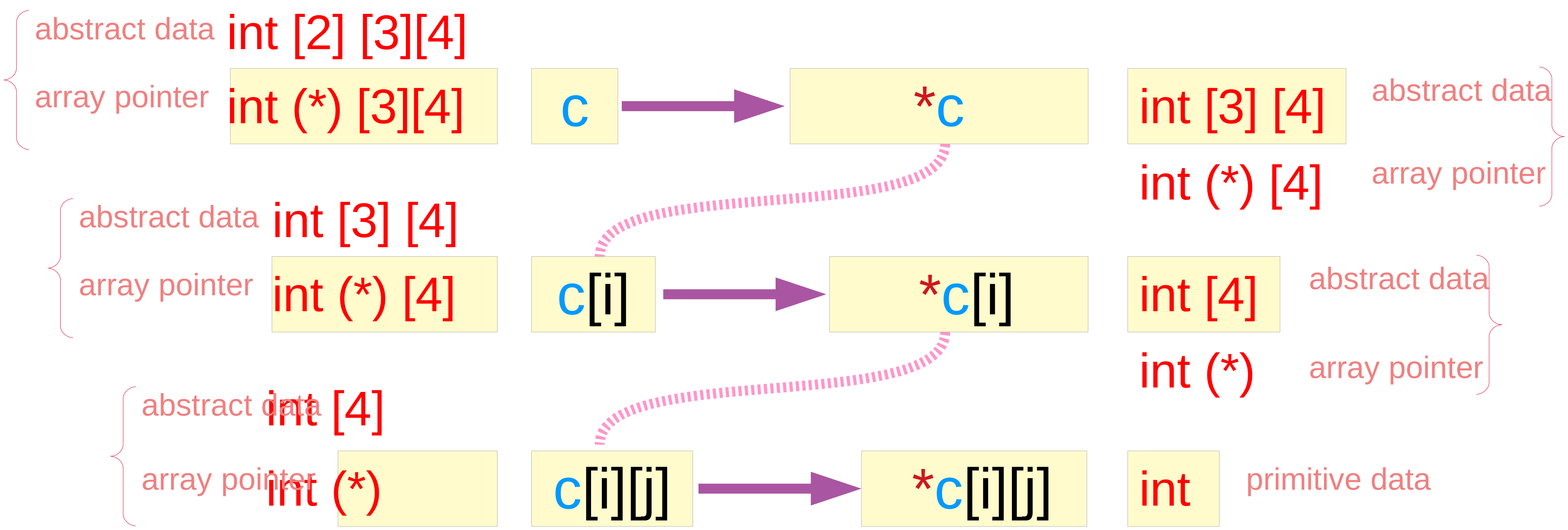
int [4] — **1-d** array

int — **0-d** array

Virtual Array Pointers **Abstract Data (Array)**

Dual types in a **3-d** array

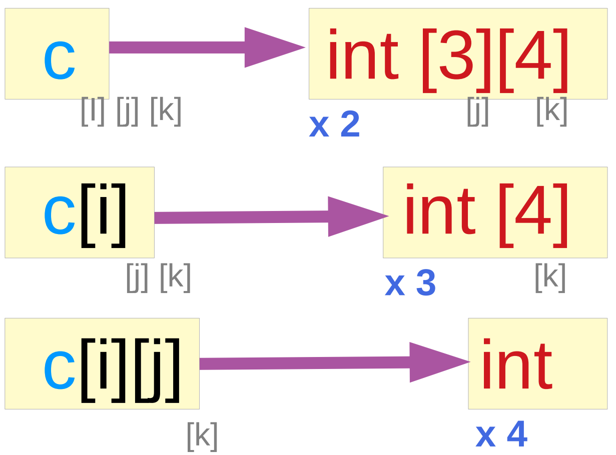
```
int     c [2][3][4];
```



Pointed array sizes in a **3-d** array

```
int      c [2][3][4];
```

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types



sizeof(*c) = sizeof(int [3][4])

sizeof(*c[i]) = sizeof(int [4])

sizeof(*c[i][j]) = sizeof(int)

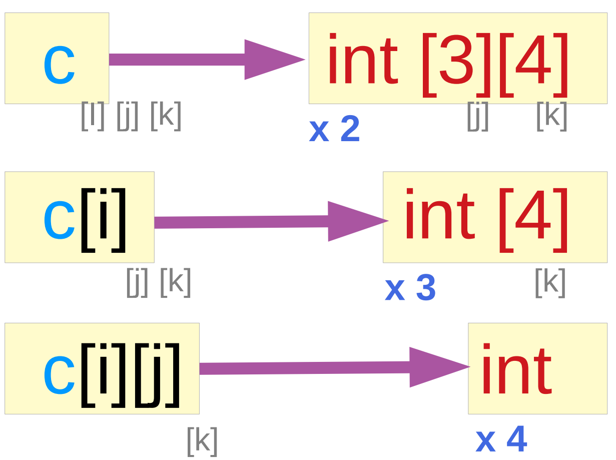
all are sizes of arrays

c, c[i], c[i][j] [k] are virtual array pointers
and they are also abstract data (arrays)

when sizes are considered,
view them as abstract data (arrays)

Virtual array pointer sizes in a **3-d** array

int c [2][3][4];



size of a virtual array pointer = size of the pointed abstract data type * the number of such data

sizeof(c) = **2** * sizeof(*c)

sizeof(c[i]) = **3** * sizeof(*c[i])

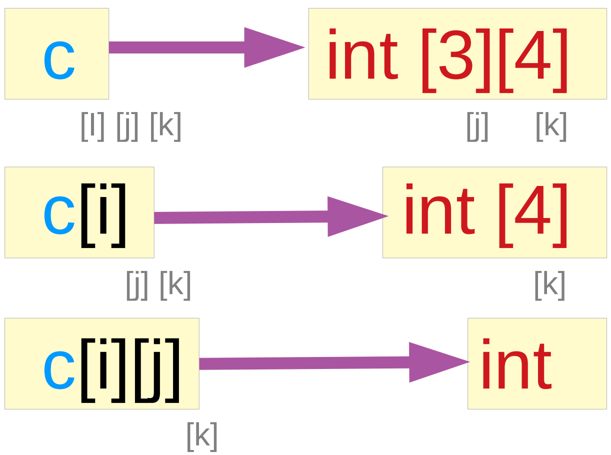
sizeof(c[i][j]) = **4** * sizeof(*c[i][j])

sizeof(Virtual Array Pointer) =
sizeof(Array of the dual type)

Virtual pointer sizes are subarray sizes

```
int      c [2][3][4];
```

sizeof(Virtual Array Pointer) =
sizeof(Array of the dual type)



sizeof(int [2] [3][4]) = sizeof(c) = 2*3*4 * 4
sizeof(int (*) [3][4]) = pointer size = 4 or 8

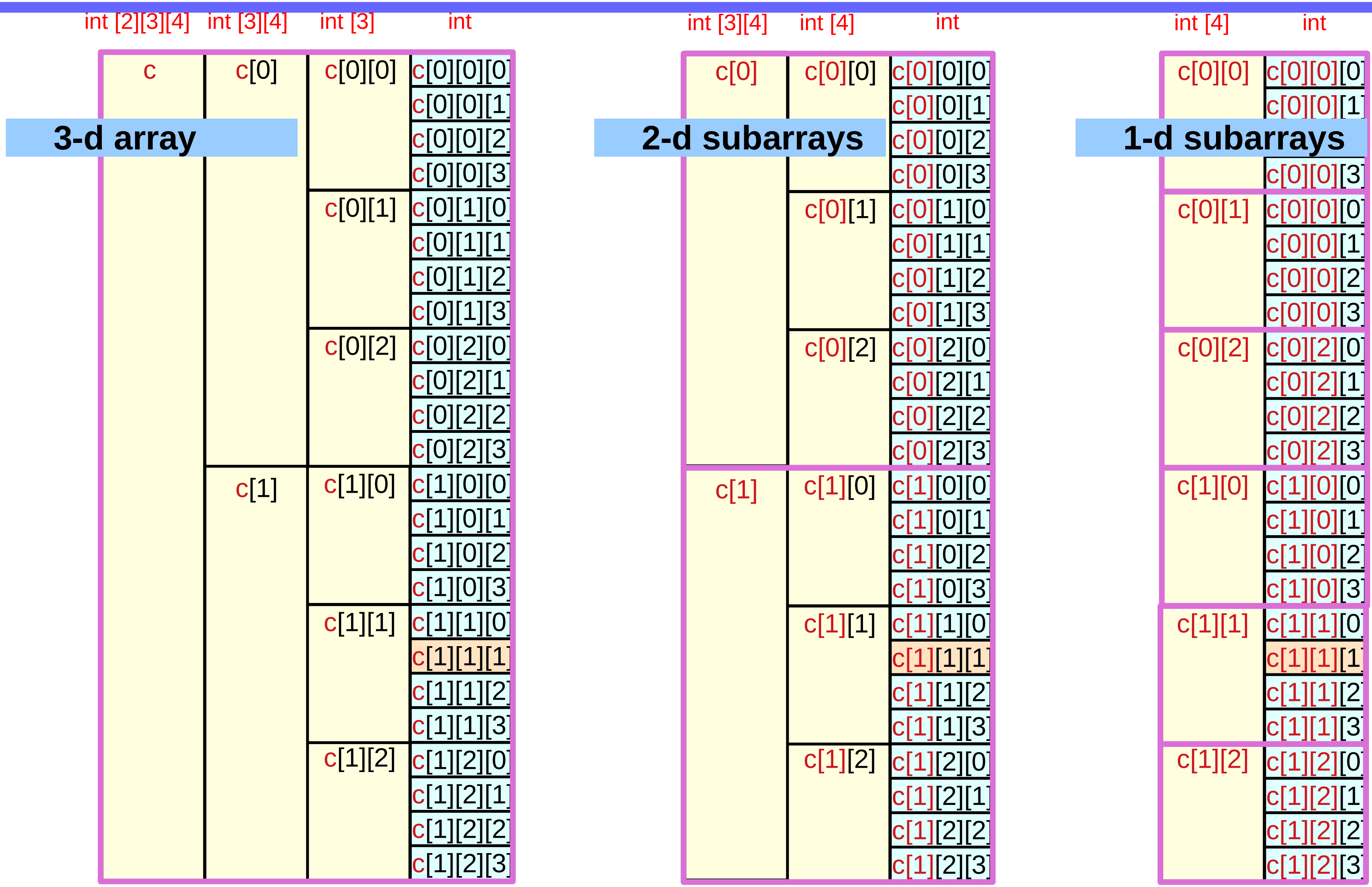
sizeof(int [3] [4]) = sizeof(c[i]) = 3*4 * 4
sizeof(int (*) [4]) = pointer size = 4 or 8

sizeof(int [4]) = sizeof(c[i][j]) = 4 * 4
sizeof(int [4]) = pointer size = 4 or 8

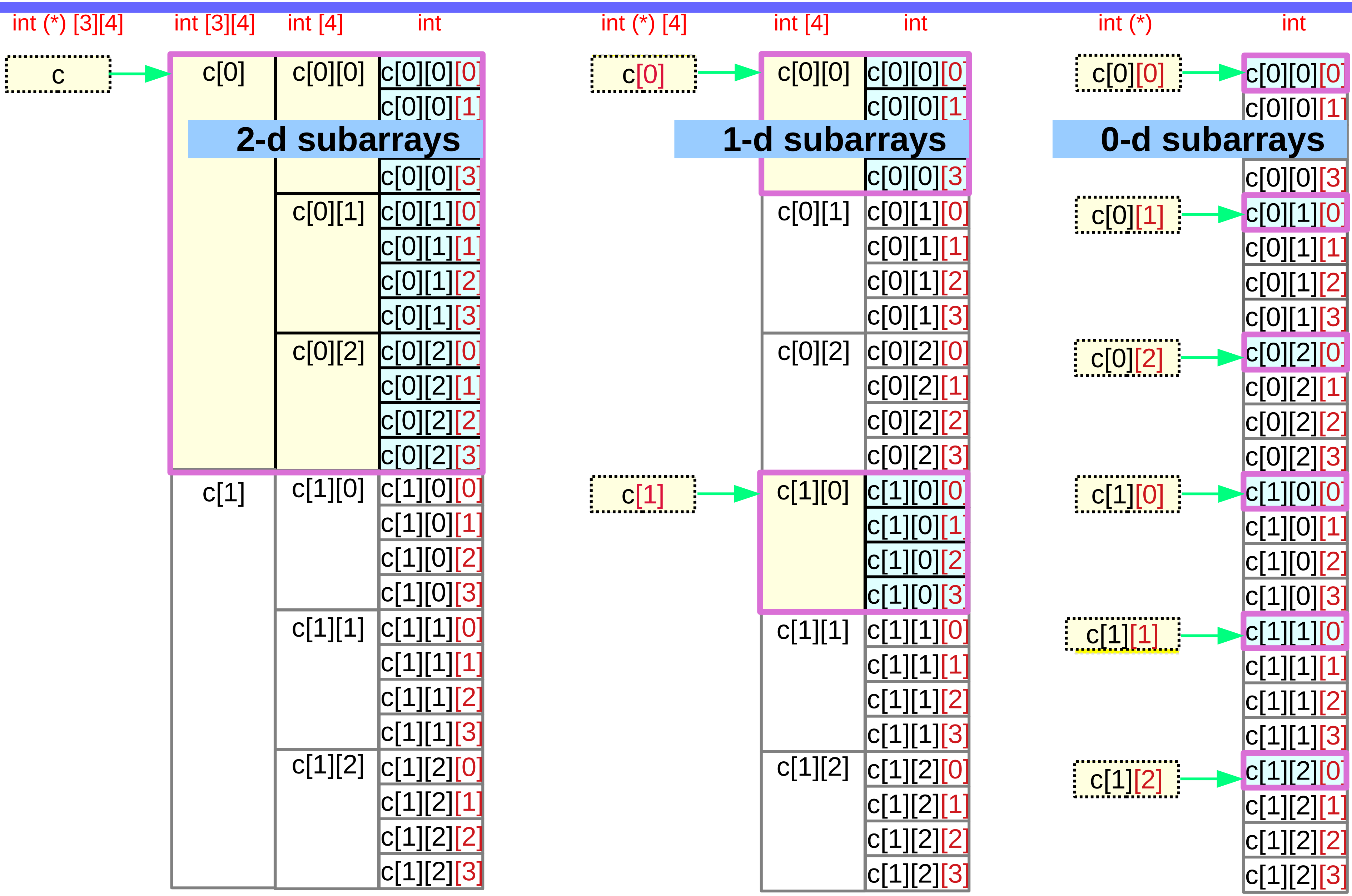
4 bytes for 32-bit machines
8 bytes for 64-bit machines

not real array pointers
virtual array pointers

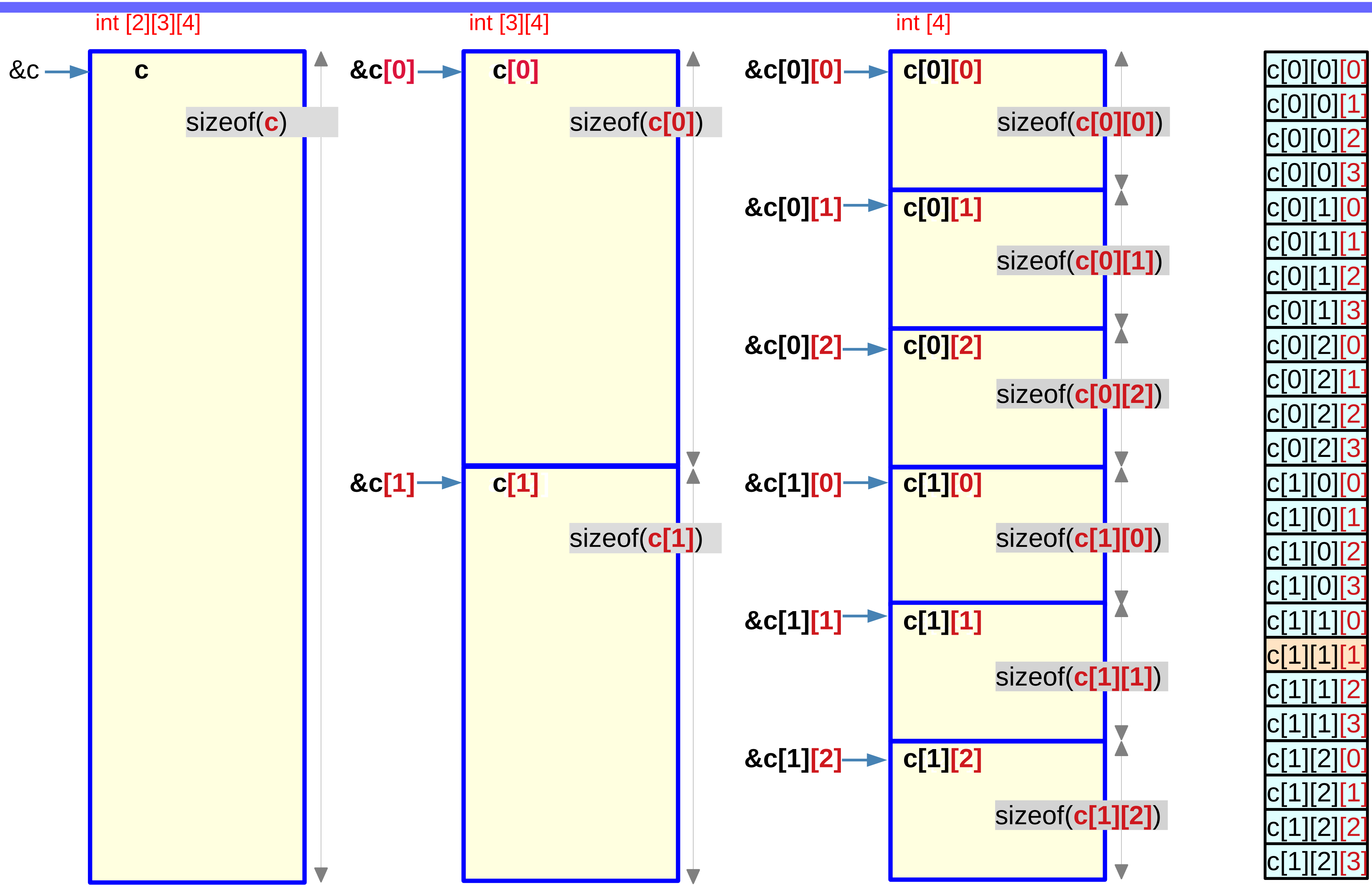
Subarrays **c**, **c[i]**, **c[i][j]** in a **3-d** array



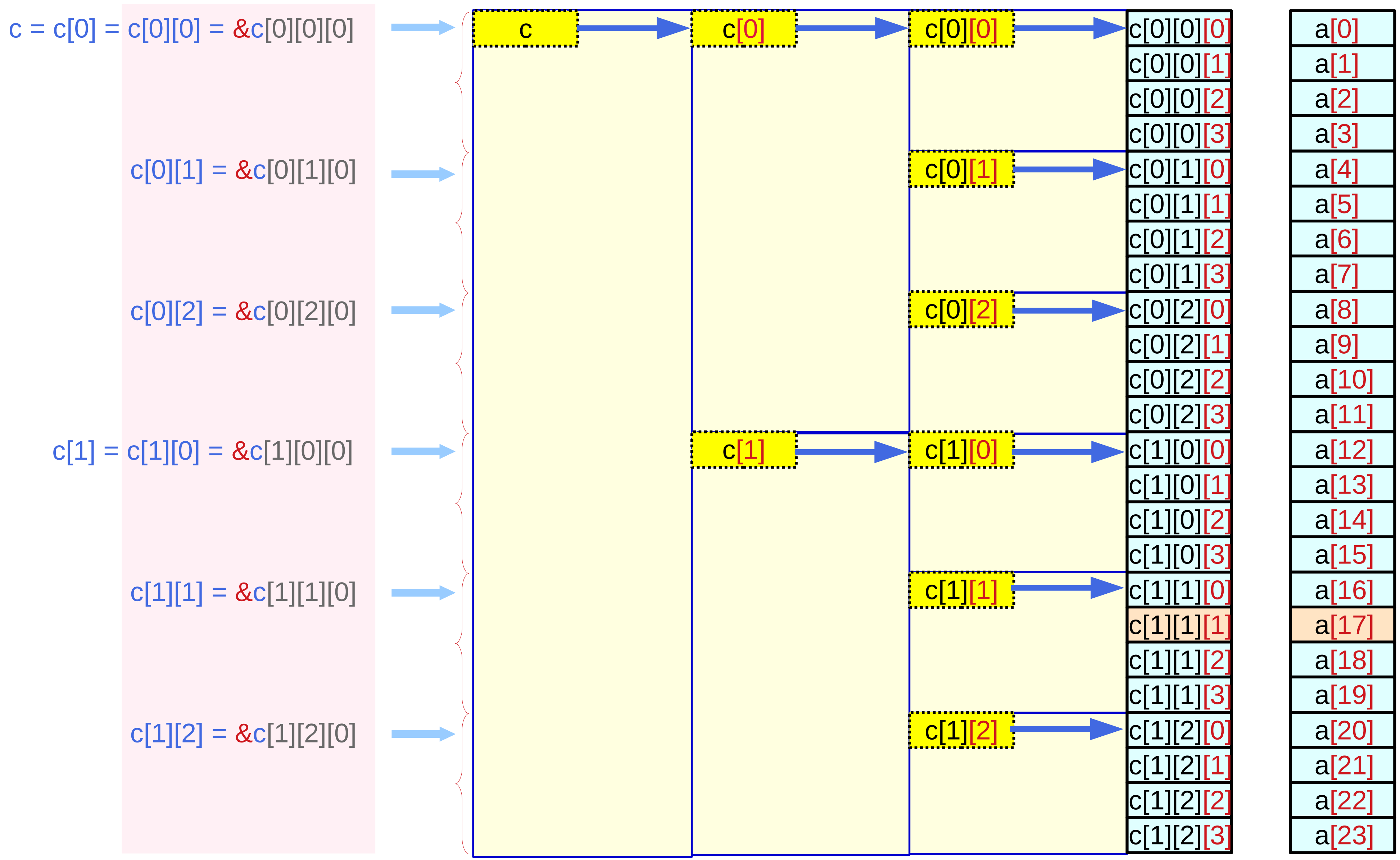
Virtual array pointers **c**, **c[i]**, **c[i][j]** in a **3-d** array



Abstract data **c**, **c[i]**, **c[i][j]** – start addresses and sizes



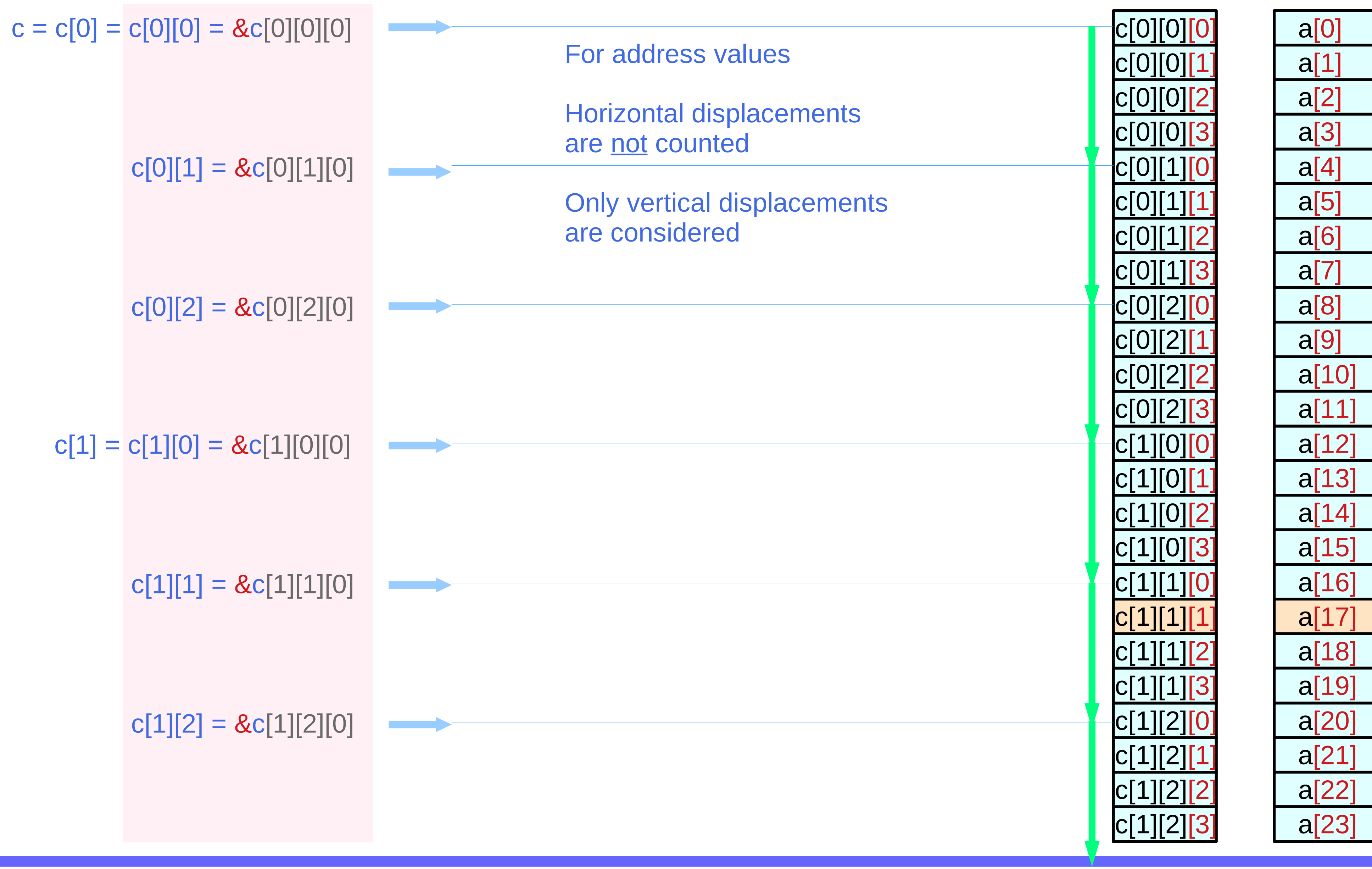
Virtual array pointer **c**, **c[i]**, **c[i][j]** – values (addresses)



Virtual array pointer **c**, **c[i]**, **c[i][j]** – values and types

c = c[0] = c[0][0] = &c[0][0][0]	means ⇒	value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0]) type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0]) int (*) [3][4] int (*) [4] int * int *
c[0][1] = &c[0][1][0]	means ⇒	value(c[0][1]) = value(&c[0][1][0]) type(c[0][1]) = type(&c[0][1][0]) int * int *
c[0][2] = &c[0][2][0]	means ⇒	value(c[0][2]) = value(&c[0][2][0]) type(c[0][2]) = type(&c[0][2][0]) int * int *
c[1] = c[1][0] = &c[1][0][0]	means ⇒	value(c[1]) = value(c[1][0]) = value(&c[1][0][0]) type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0]) int (*) [4] int * int *
c[1][1] = &c[1][1][0]	means ⇒	value(c[1][1]) = value(&c[1][1][0]) type(c[1][1]) = type(&c[1][1][0]) int * int *
c[1][2] = &c[1][2][0]	means ⇒	value(c[1][2]) = value(&c[1][2][0]) type(c[1][2]) = type(&c[1][2][0]) int * int *

Virtual array pointer **c**, **c[i]**, **c[i][j]** – vertical displacement



Virtual array pointers – types, sizes, and values

int c[2][3][4];	**c[i][j]**	**c[i][j][0]**	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * 4	= sizeof(int) * 4
value (address)	c[i][j] =	&c[i][j][0]	

int c[2][3][4];	**c[i]**	**c[i][0]**	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * 3	= sizeof(int) * 4 * 3
value (address)	c[i] =	&c[i][0][0]	

int c[2][3][4];	**c**	**c[0]**	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * 2	= sizeof(int) * 4 * 3 * 2
value (address)	c =	&c[0][0][0]	

Summary of virtual array pointers in a **3-d** array

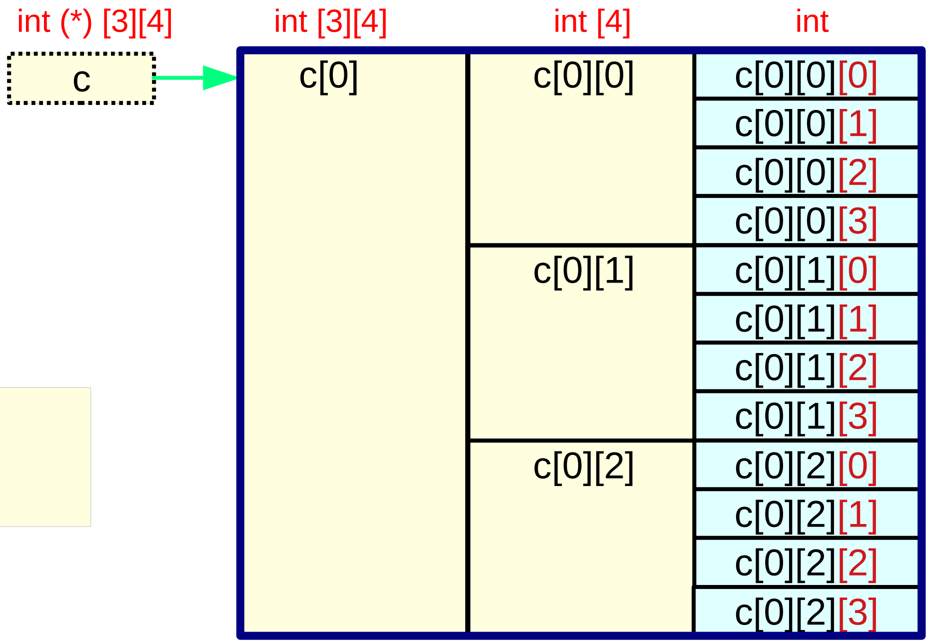
c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Dual type constraints in a multi-dimensional array

Virtual array pointers to subarrays in a 3-d array

virtual 2-d array pointer

sizeof(**c**) =
sizeof(**c[0]**) * **2**



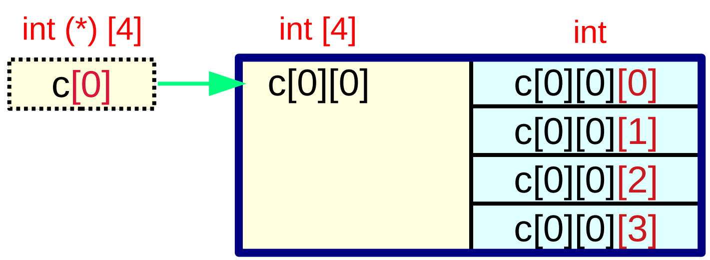
the first 2-d subarray

sizeof(**c[0]**) =
sizeof(**int [3][4]**)

```
int     c [2][3][4];
```

virtual 1-d array pointer

sizeof(**c[0]**) =
sizeof(**c[0][0]**) * **3**

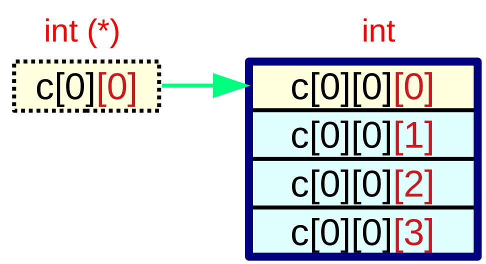


the first 1-d subarray

sizeof(**c[0][0]**) =
sizeof(**int [4]**)

virtual 0-d array pointer

sizeof(**c[0][0]**) =
sizeof(**c[0][0][0]**) * **4**



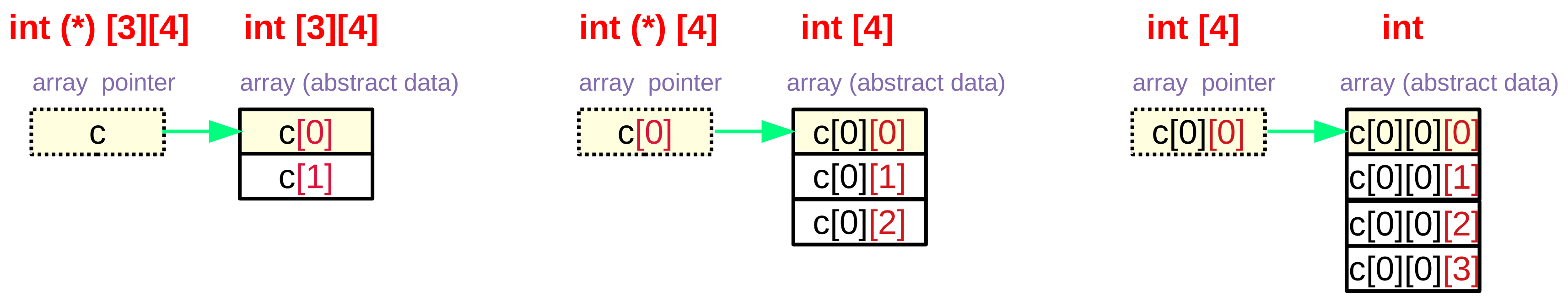
the first 0-d subarray

sizeof(**c[0][0][0]**) =
sizeof(**int**)

Virtual array pointer **c**, **c[0]**, **c[0][0]** – types and sizes

```
int    c [2][3][4];
```

Types – array pointers



Sizes – abstract data

sizeof(**c**)
sizeof(**int [2][3][4]**)
sizeof(**int**) * **2** * **3** * **4**

sizeof(**c[0]**)
sizeof(**int [3][4]**)
sizeof(**int**) * **3** * **4**

sizeof(**c[0][0]**)
sizeof(**int [4]**)
sizeof(**int**) * **4**

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

Contiguous subarrays **c[i]**, **c[i][j]**, **c[i][j][k]**

int c [2][3][4];

array pointer

c

int [2][3][4] **Contiguous 2*3*4 integers**

array pointer → array (abstract data)

c → c[0]

int (*) [3][4] **int [3][4]** **Contiguous 3*4 integers**

2 c[i]'s, contiguous

c+0	c[0]
c+1	c[1]

sizeof(**int [3][4]**)

for a given **i**

array pointer → array (abstract data)

c[i] → c[i][0]

int (*) [4] **int [4]** **Contiguous 4 integers**

3 c[i][j]'s, contiguous

c[i]+0	c[i][0]
c[i]+1	c[i][1]
c[i]+2	c[i][2]

sizeof(**int [4]**)

for a given **i, j**

array pointer → array (abstract data)

c[i][j] → c[i][j][0]

int (*) **int**

4 c[i][j][k]'s, contiguous

c[i][j]+0	c[i][j][0]
c[i][j]+1	c[i][j][1]
c[i][j]+2	c[i][j][2]
c[i][j]+3	c[i][j][3]

sizeof(**int**)

Dual types of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

c :: int (*) [3][4] — 2-d array pointers
c[i] :: int [3] [4] — 2-d arrays
c[i] :: int (*) [4] — 1-d array pointers
c[i][j] :: int [4] — 1-d arrays
c[i][j] :: int (*) — 0-d array pointers
c[i][j][k] :: int — 0-d arrays (integers)

c →	c[0] →	c[0][0] →	c[0][0][0] … c[0][0][3]
		c[0][1] →	c[0][1][0] … c[0][1][3]
		c[0][2] →	c[0][2][0] … c[0][2][3]
	c[1] →	c[1][0] →	c[1][0][0] … c[0][0][3]
		c[1][1] →	c[1][1][0] … c[0][1][3]
		c[1][2] →	c[1][2][0] … c[0][2][3]
int [2] [3][4]	int [3] [4]	int [4]	int … int
int (*) [3][4]	int (*) [4]	int (*)	int … int

Values of virtual array pointers **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

virtual array pointers have address values
in each row in the following figure
have the <u>same</u> <u>address value</u>

c	≡	c[0]	≡	c[0][0]	≡	&c[0][0][0]
				c[0][1]	≡	&c[0][1][0]
				c[0][2]	≡	&c[0][2][0]
		c[1]	≡	c[1][0]	≡	&c[1][0][0]
				c[1][1]	≡	&c[1][1][0]
				c[1][2]	≡	&c[1][2][0]

Horizontal displacements are <u>not</u> counted
only **vertical displacements** are considered
for address values

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Finding values of virtual array pointers **c**, **c[i]**, **c[i][j]**

```
int      c [2][3][4];
```

append [0] to the right

c	$\overset{+[0]}{=}$	c[0]	$\overset{+[0]}{=}$	c[0][0]	$\overset{+[0]}{=}$	&c[0][0][0]
				c[0][1]	$\overset{+[0]}{=}$	&c[0][1][0]
				c[0][2]	$\overset{+[0]}{=}$	&c[0][2][0]
		c[1]	$\overset{+[0]}{=}$	c[1][0]	$\overset{+[0]}{=}$	&c[1][0][0]
				c[1][1]	$\overset{+[0]}{=}$	&c[1][1][0]
				c[1][2]	$\overset{+[0]}{=}$	&c[1][2][0]
int (*) [3][4]		int (*) [4]		int [4]		int

c[0][0][0] **:** leading elements of **c**

&c[0][0][0]

c[i][0][0] **:** leading elements of **c[i]**

&c[0][0][0]
&c[1][0][0]

c[i][j][0] **:** leading elements of **c[i][j]**

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

Finding sub-arrays with the address &c[i][j][0]

```
int     c [2][3][4];
```

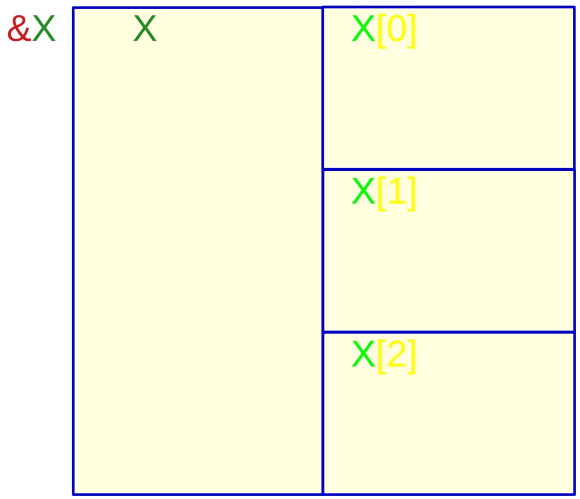
delete [0] from the right

&c[0][0][0]	= (-[0])	c[0][0]	= (-[0])	c[0]	= (-[0])	c
&c[0][1][0]	= (-[0])	c[0][1]				
&c[0][2][0]	= (-[0])	c[0][2]				
&c[1][0][0]	= (-[0])	c[1][0]	= (-[0])	c[1]		
&c[1][1][0]	= (-[0])	c[1][1]				
&c[1][2][0]	= (-[0])	c[1][2]				
int		int [4]		int (*) [4]		int (*) [3][4]

c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Dual types in a **3-d** array

Abstract data (array) X

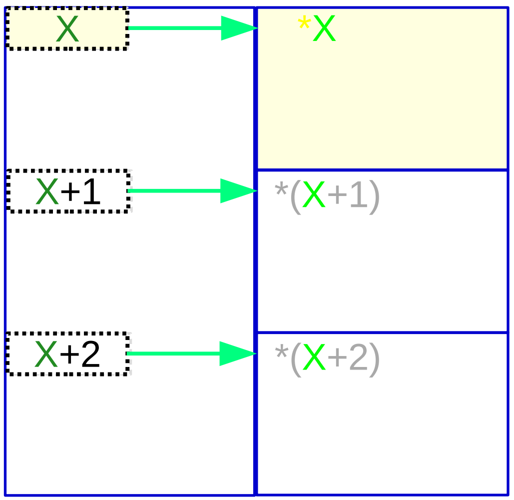


array (abstract data) | array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&c[i][j]	=	&c[i][j][0]
&c[i]	=	&c[i][0]
&c	=	&c[0]

Virtual array pointer X



array pointer | array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

c[i][j]	=	&c[i][j][0]
c[i]	=	&c[i][0]
c	=	&c[0]

Dual type constraints

&c[i][j][0]	= c[i][j]
&c[i][0]	= c[i]
&c[0]	= c

Virtual array pointer

array pointer — array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

X

+

&c[i][j][0]	= &c[i][j]
&c[i][0]	= &c[i]
&c[0]	= &c

Abstract data (array)

array (abstract data) — array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&X X

c[i][j]	= &c[i][j]
c[i]	= &c[i]
c	= &c

array (abstract data) — Address of an array pointer

c[i][j]	pointer value	=	pointer address	&c[i][j]
c[i]	pointer value	=	pointer address	&c[i]
c	pointer value	=	pointer address	&c
X				&X

c[0] = c[0][0] relation

int c [2][3][4];

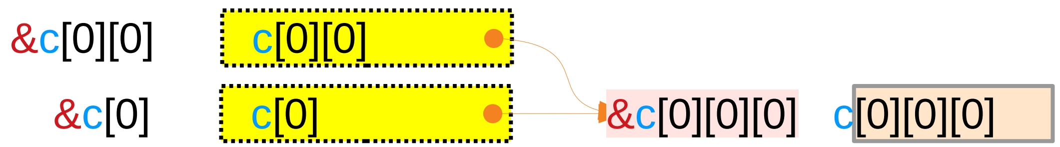
c = c[0] = c[0][0] = &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

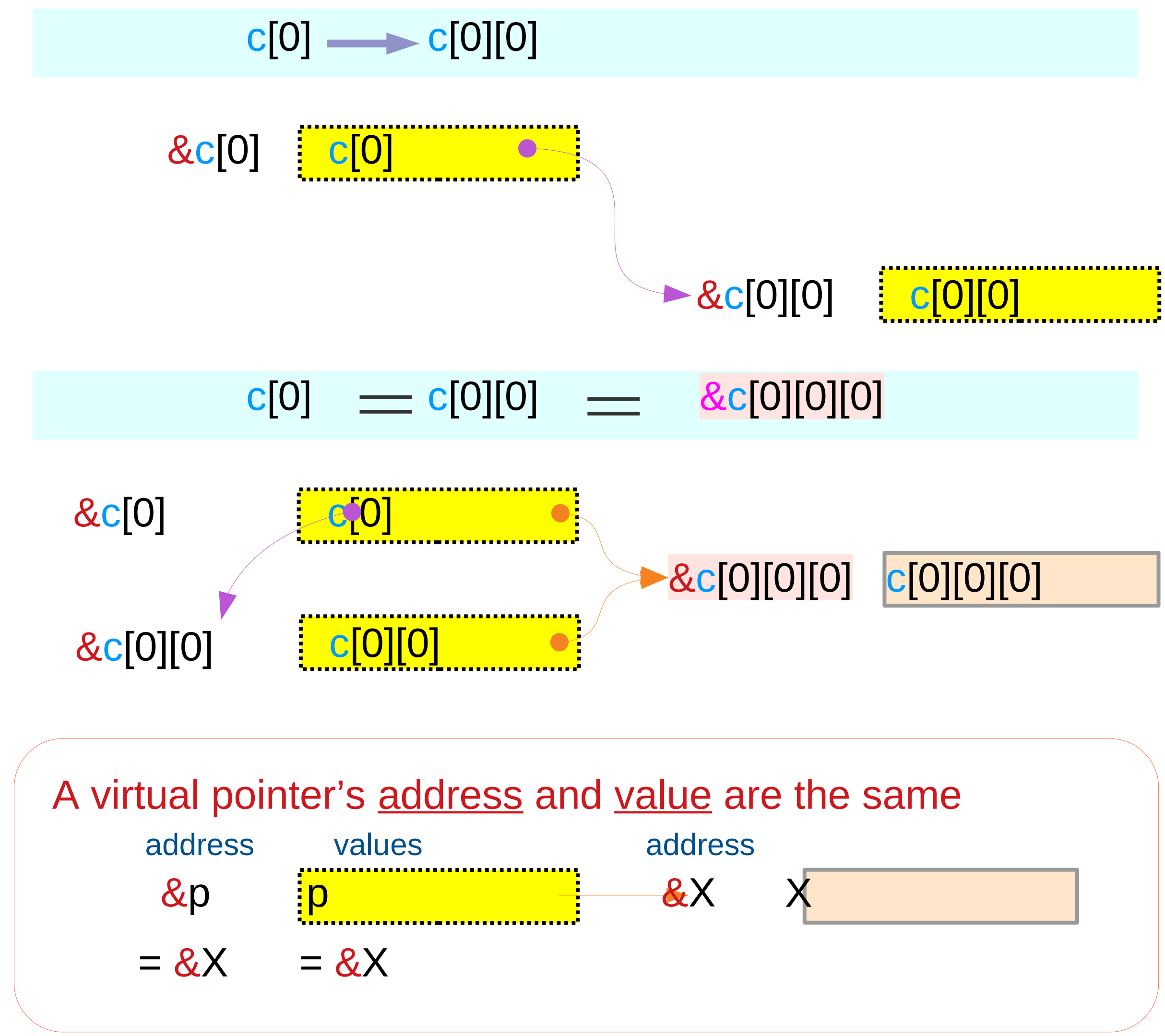
c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])

c[0] = c[0][0] *does **not** mean*
type(c[0]) = type(c[0][0])



Addresses and Values of **c[0]** and **c[0][0]**

```
int     c [2][3][4];
```

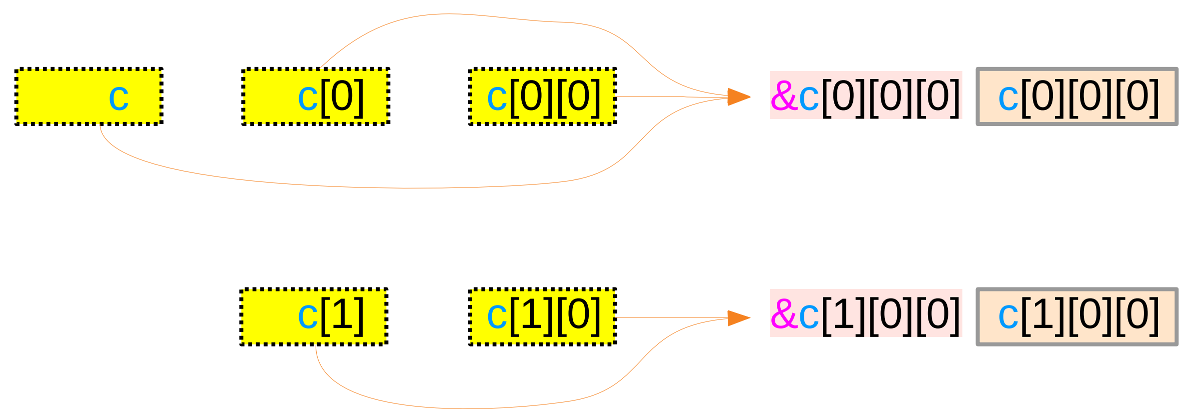


c[i] and c[i][0] point to the same c[i][0][0]

```
int c [2][3][4];
```

c =	c[0] =	c[0][0] =	&c[0][0][0]	← value
int(*)[3][4]	int(*)[4]	int(*)	int	← type

	c[1] =	c[1][0] =	&c[1][0][0]	← value
	int(*)[4]	int(*)	int	← type



These virtual pointers have different types
but the same value (address)

&c[i][0] and &c[i][0][0] – equivalence relations

int c [2][3][4];

int(*)[3][4] int(*)[4] int(*) int *

c = c[0] = c[0][0] = &c[0][0][0]

&

&c &c[0] &c[0][0]

equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]

c[1] = c[1][0] = &c[1][0][0]

&

&c[1] &c[1][0]

equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

Horizontal displacements are <u>not</u> counted
only **vertical displacements** are considered
for <u>**address values**</u>

equivalences

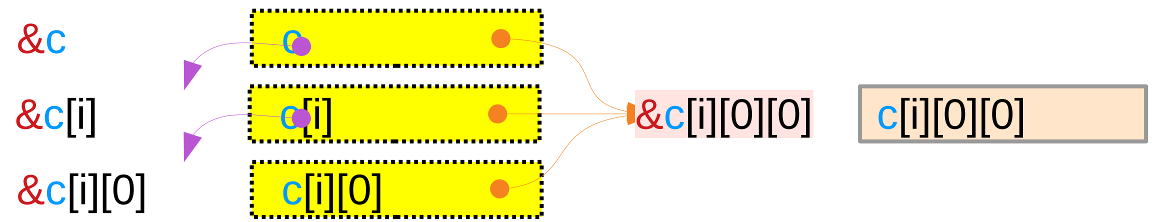
c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

```
int     c [2][3][4];
```

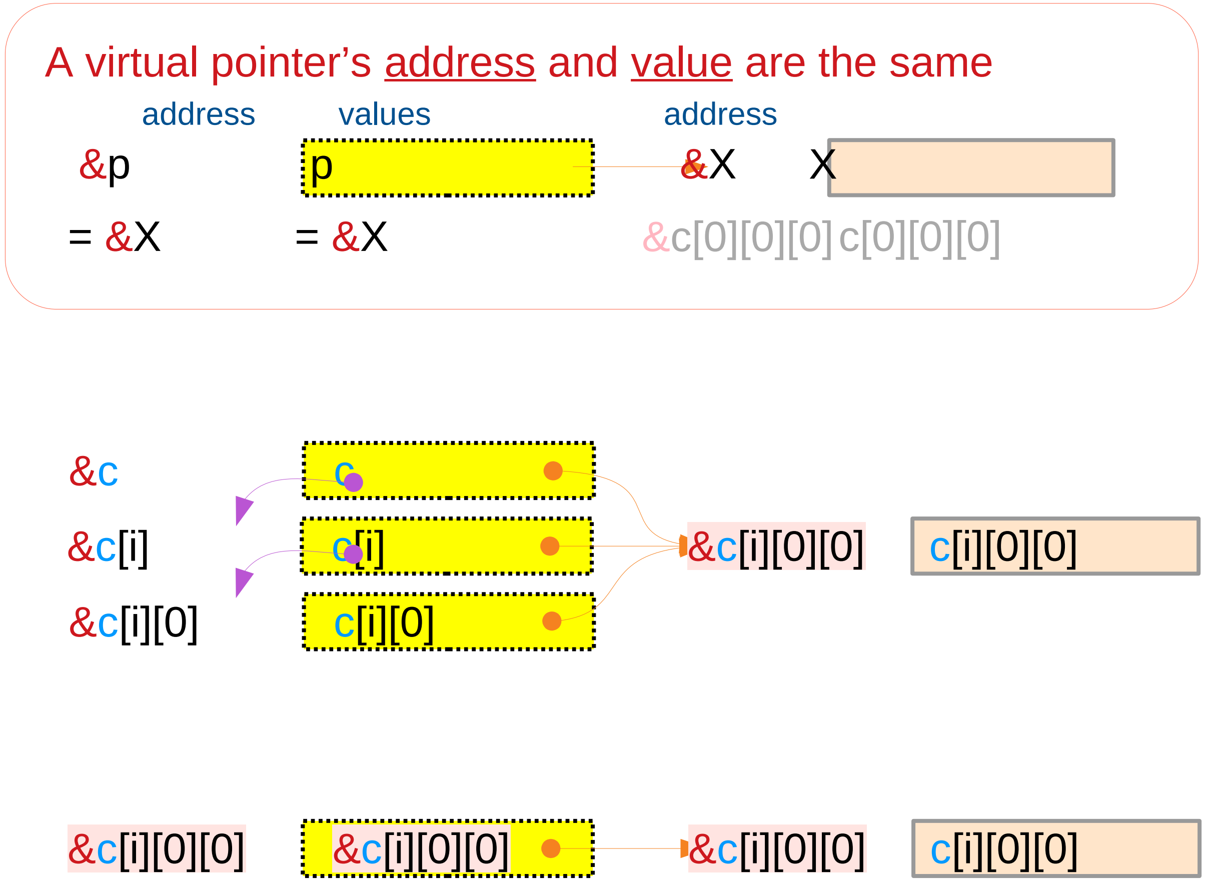
c = c[0] = c[0][0]= &c[0][0][0]
|| || ||
&c = &c[0] = &c[0][0]

c[1] = c[1][0] = &c[1][0][0]
|| ||
&c[1] = &c[1][0]



c[i] = &c[i] and c[i][0] = &c[i][0]

```
int      c [2][3][4];
```



Array Pointers to **c[i][0][0]**

&c[i][0][0] ≡ c[i][0]

&c[i][0] ≡ c[i]

&c[i] ≡ c+i

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

virtual pointers:
the address of a pointer is
the same as its value

delete [0] from the right

&c[0][0][0]	$\overset{-[0]}{=}$	c[0][0]	$\overset{-[0]}{=}$	c[0]	$\overset{-[0]}{=}$	c
&c[1][0][0]	$\overset{-[0]}{=}$	c[1][0]	$\overset{-[0]}{=}$	c[1]		

Array Pointers to **c[i][j][0]**

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	-[0] ═	c[0][0]	-[0] ═	c[0]	-[0] ═	c
&c[0][1][0]	-[0] ═	c[0][1]				
&c[0][2][0]	-[0] ═	c[0][2]				
&c[1][0][0]	-[0] ═	c[1][0]	-[0] ═	c[1]		
&c[1][1][0]	-[0] ═	c[1][1]				
&c[1][2][0]	-[0] ═	c[1][2]				

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun