

Truth Table (A2)

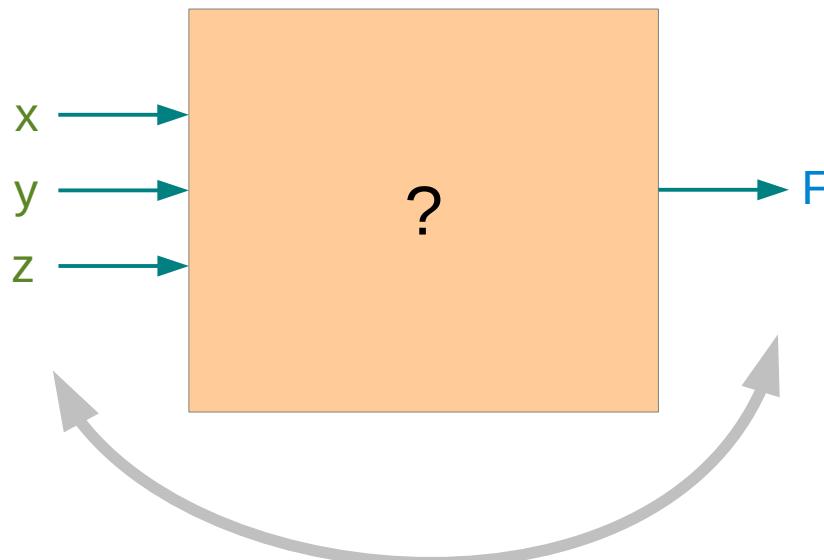
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Truth Table

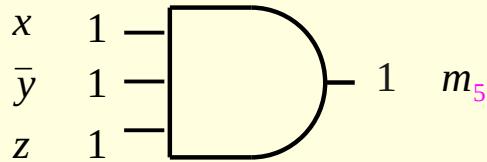


x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

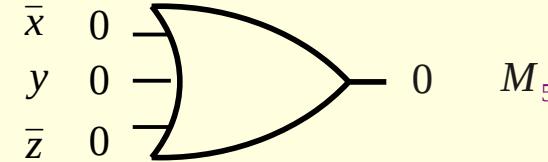
inputs output

Finding minterms and Maxterms

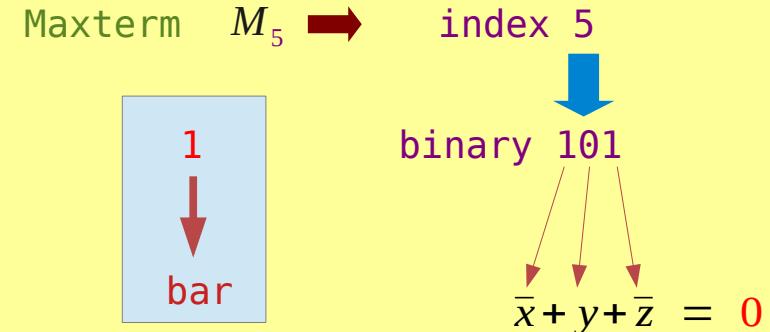
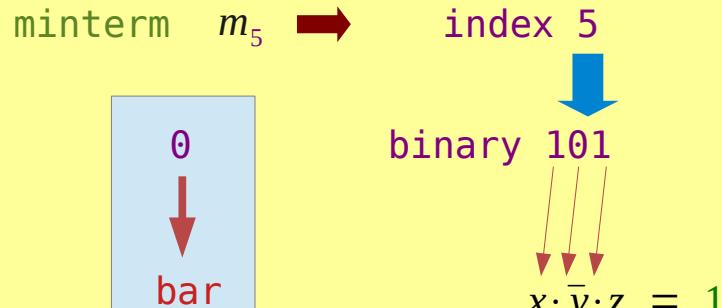
Consider the case when $x=1$ and $y=0$ and $z=1$ index = 5



$$\begin{aligned}(x=1) \wedge (y=0) \wedge (z=1) &\iff \\(x=1) \wedge (\bar{y}=1) \wedge (z=1) &\iff m_5=1\end{aligned}$$



$$\begin{aligned}(x=1) \wedge (y=0) \wedge (z=1) &\iff \\(\bar{x}=0) \wedge (y=0) \wedge (\bar{z}=0) &\iff M_5=0\end{aligned}$$



Boolean Function F with minterms and Maxterms

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

minterm

The truth table shows the function F for inputs x, y, z. The minterms are highlighted in red for rows 1, 3, and 4. To the right, a logic circuit diagram shows three AND gates (labeled 1, 2, 3) followed by an OR gate. Inputs x, y, and z enter gates 1, 2, and 3 respectively. The outputs of gates 1 and 2 enter the OR gate, along with input z. The final output is labeled F.

$$\begin{aligned} F &= m_1 + m_3 + m_4 \\ &= \bar{x}\bar{y}z + \bar{x}yz + xy\bar{z} \end{aligned}$$

	x	y	z	F
⇒ 0	0	0	0	0
1	0	0	1	1
⇒ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
⇒ 5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Maxterm

The truth table shows the function F for inputs x, y, z. The Maxterms are highlighted in red for rows 0, 2, 4, 5, 6, and 7. To the right, a logic circuit diagram shows six NOT gates (labeled 0 through 5) followed by an OR gate. Inputs x, y, and z enter gates 0, 1, and 2 respectively. The outputs of gates 0, 2, 4, and 5 enter the OR gate. The outputs of gates 3 and 6 enter another OR gate, which then feeds into the final OR gate. The final output is labeled F.

$$\begin{aligned} F &= M_0 + M_2 + M_5 + M_6 + M_7 \\ &= (x+y+z) \cdot (x+\bar{y}+z) \cdot (\bar{x}+y+\bar{z}) \cdot \\ &\quad (\bar{x}+\bar{y}+z) \cdot (\bar{x}+\bar{y}+\bar{z}) \end{aligned}$$

Boolean Function \bar{F} with minterms and Maxterms

	x	y	z	F	\bar{F}
→ 0	0	0	0	0	1
1	0	0	1	1	0
→ 2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	1	0
→ 5	1	0	1	0	1
→ 6	1	1	0	0	1
→ 7	1	1	1	0	1

minterm

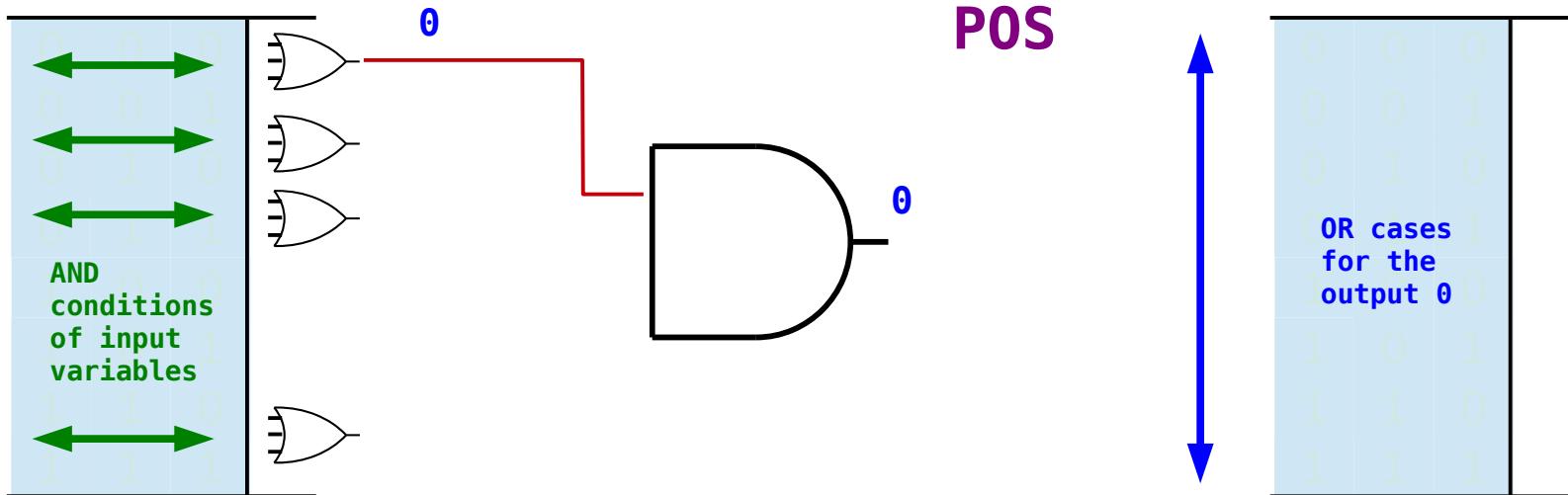
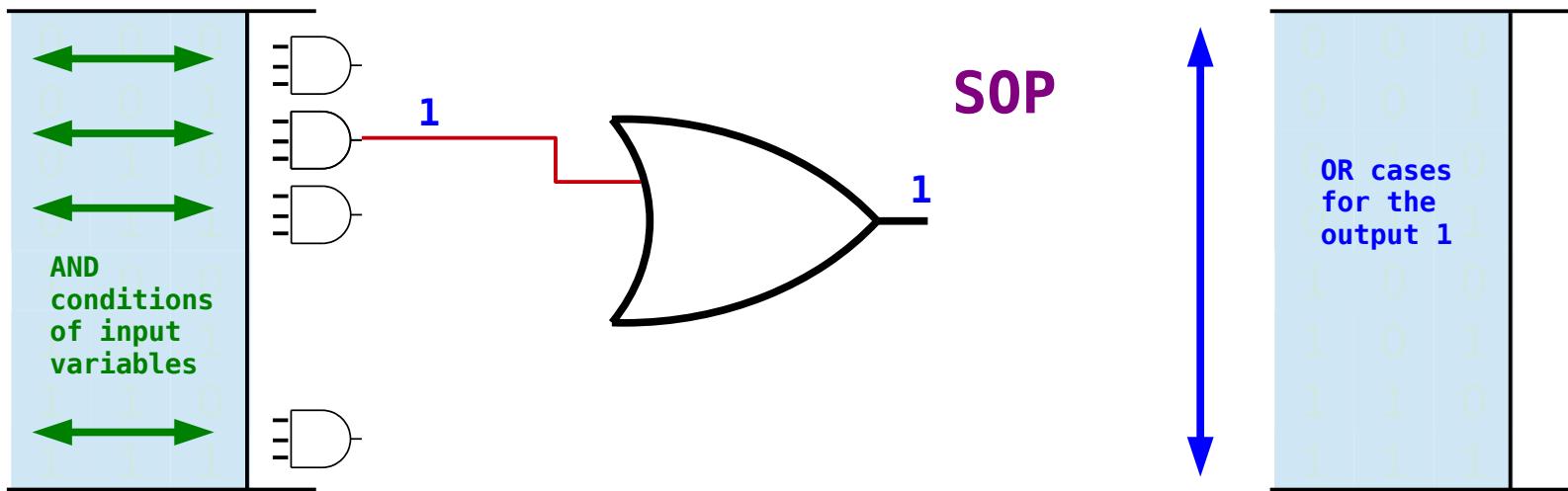
	x	y	z	F	\bar{F}
0	0	0	0	0	1
→ 1	0	0	1	1	0
2	0	1	0	0	1
→ 3	0	1	1	1	0
→ 4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	0	1

Maxterm

$$\begin{aligned}
 \bar{F} &= m_0 + m_2 + m_5 + m_6 + m_7 \\
 &= \bar{x} \cdot \bar{y} \cdot \bar{z} + \bar{x} \cdot y \cdot \bar{z} + x \cdot \bar{y} \cdot z \\
 &\quad + x \cdot y \cdot \bar{z} + x \cdot y \cdot z
 \end{aligned}$$

$$\begin{aligned}
 \bar{F} &= M_1 + M_3 + M_4 \\
 &= (x+y+\bar{z}) \cdot (x+\bar{y}+\bar{z}) \cdot (\bar{x}+y+z)
 \end{aligned}$$

SOP and POS



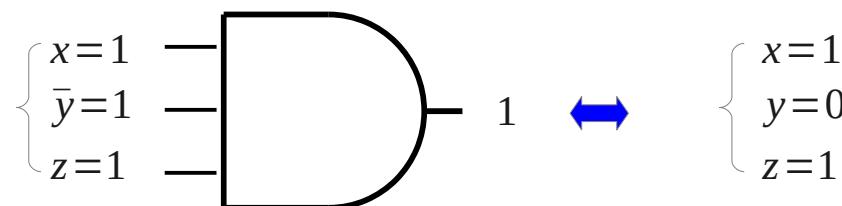
Truth Table and minterms (1)

x	y	z			
0	0	0	the case when x=0 and y=0 and z=0	\leftrightarrow	$\bar{x}\bar{y}\bar{z} = 1$
0	0	1	the case when x=0 and y=0 and z=1	\leftrightarrow	$\bar{x}\bar{y}z = 1$
0	1	0	the case when x=0 and y=1 and z=0	\leftrightarrow	$\bar{x}y\bar{z} = 1$
0	1	1	the case when x=0 and y=1 and z=1	\leftrightarrow	$\bar{x}yz = 1$
1	0	0	the case when x=1 and y=0 and z=0	\leftrightarrow	$x\bar{y}\bar{z} = 1$
1	0	1	the case when x=1 and y=0 and z=1	\leftrightarrow	$x\bar{y}z = 1$
1	1	0	the case when x=1 and y=1 and z=0	\leftrightarrow	$xy\bar{z} = 1$
1	1	1	the case when x=1 and y=1 and z=1	\leftrightarrow	$xyz = 1$

inputs

All possible combination of inputs

$$x\bar{y}z = 1 \quad \leftrightarrow$$



For the output of an **and** gate to be 1,
all inputs must be 1

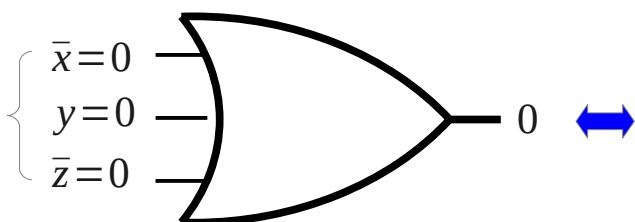
Truth Table and MAXterms (1)

x	y	z			
0	0	0	→ the case when x=0 and y=0 and z=0	↔	$x + y + z = 0$
0	0	1	→ the case when x=0 and y=0 and z=1	↔	$x + y + \bar{z} = 0$
0	1	0	→ the case when x=0 and y=1 and z=0	↔	$x + \bar{y} + z = 0$
0	1	1	→ the case when x=0 and y=1 and z=1	↔	$x + \bar{y} + \bar{z} = 0$
1	0	0	→ the case when x=1 and y=0 and z=0	↔	$\bar{x} + y + z = 0$
1	0	1	→ the case when x=1 and y=0 and z=1	↔	$\bar{x} + y + \bar{z} = 0$
1	1	0	→ the case when x=1 and y=1 and z=0	↔	$\bar{x} + \bar{y} + z = 0$
1	1	1	→ the case when x=1 and y=1 and z=1	↔	$\bar{x} + \bar{y} + \bar{z} = 0$

→ All possible combination of inputs

inputs

$\bar{x} + y + \bar{z} = 0$ ↔



$\begin{cases} \bar{x}=0 \\ y=0 \\ \bar{z}=0 \end{cases}$ → 0 ↔ $\begin{cases} x=1 \\ y=0 \\ z=1 \end{cases}$

For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and MAXterms (2)

	x	y	z
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

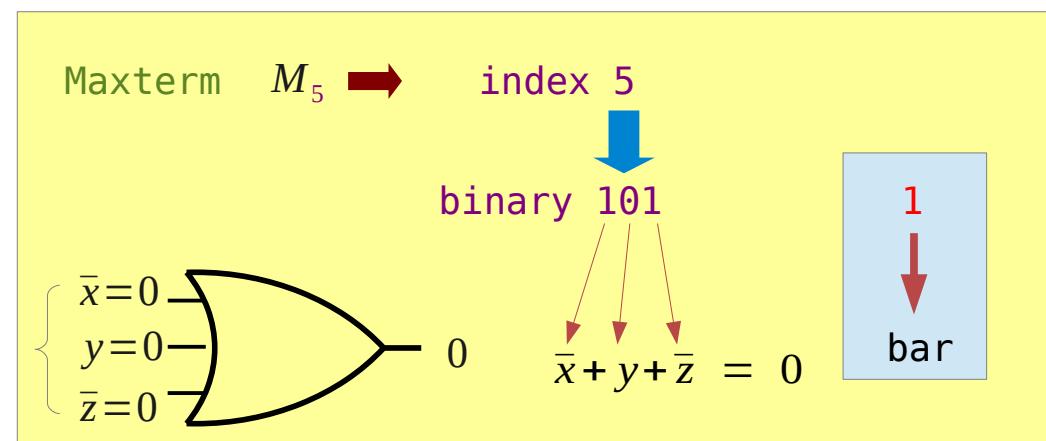
index

inputs

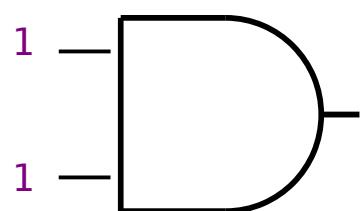
All possible combination of inputs

- the case when the MAXterm

$M_0 = x + y + z = 0$
$M_1 = x + y + \bar{z} = 0$
$M_2 = x + \bar{y} + z = 0$
$M_3 = x + \bar{y} + \bar{z} = 0$
$M_4 = \bar{x} + y + z = 0$
$M_5 = \bar{x} + y + \bar{z} = 0$
$M_6 = \bar{x} + \bar{y} + z = 0$
$M_7 = \bar{x} + \bar{y} + \bar{z} = 0$

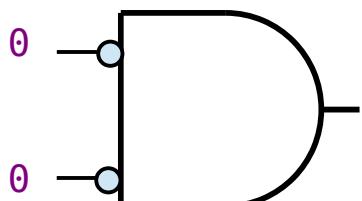
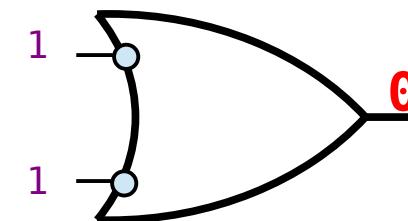


Maxterm and minterm Conditions



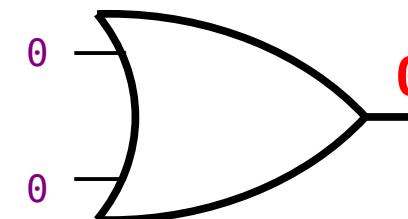
	x	y	xy
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

	\bar{x}	\bar{y}	$\bar{x}+\bar{y}$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0



	\bar{x}	\bar{y}	$\bar{x}\bar{y}$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0

	x	y	x+y
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Boolean Function with minterms (1)

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index { } inputs output

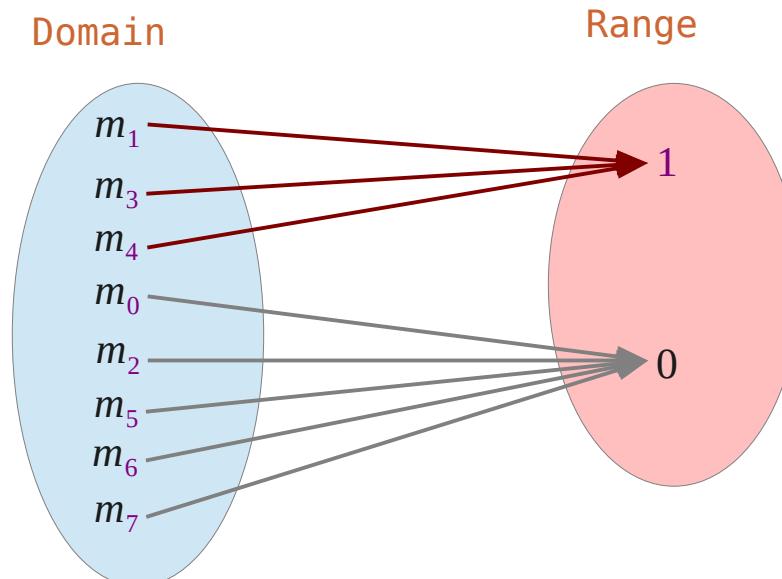
All possible combination of inputs

The output F becomes 1, for one of the three following cases

(the case when x=0 and y=0 and z=1) $\leftrightarrow m_1 = \bar{x}\bar{y}z = 1$

or (the case when x=0 and y=1 and z=1) $\leftrightarrow m_3 = \bar{x}yz = 1$

or (the case when x=1 and y=0 and z=0) $\leftrightarrow m_4 = x\bar{y}\bar{z} = 1$



Boolean Function with minterms (2)

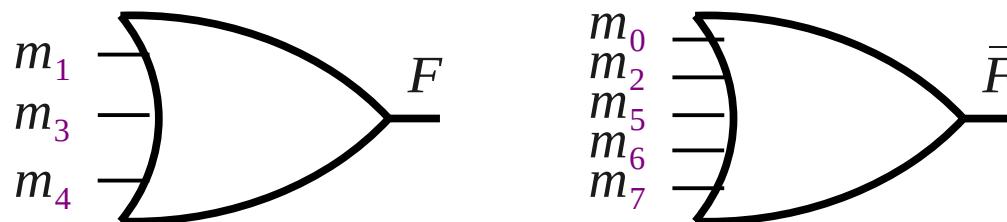
	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index { } inputs output

All possible combination of inputs

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \leftrightarrow \quad F = 1$$
$$\leftrightarrow \quad F = m_1 + m_3 + m_4$$



For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with minterms (3)

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index



inputs output

All possible combination of inputs

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \leftrightarrow \quad F = 1$$

$$\leftrightarrow \quad F = m_1 + m_3 + m_4$$

The output F becomes 0,
either $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$$m_0 + m_2 + m_5 + m_6 + m_7 = 1 \quad \leftrightarrow \quad F = 0$$

$$\leftrightarrow \quad \bar{F} = m_0 + m_2 + m_5 + m_6 + m_7$$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with Maxterms (1)

	x	y	z	F	
→ 0	0	0	0	0	The output F becomes 0, for one of the <u>five</u> following cases (the case when x=0 and y=0 and z=0) ↔ $x + y + z = 0$
1	0	0	1	1	
→ 2	0	1	0	0	or (the case when x=0 and y=1 and z=0) ↔ $x + \bar{y} + z = 0$
3	0	1	1	1	
4	1	0	0	1	
→ 5	1	0	1	0	or (the case when x=1 and y=0 and z=1) ↔ $\bar{x} + y + \bar{z} = 0$
→ 6	1	1	0	0	or (the case when x=1 and y=1 and z=0) ↔ $\bar{x} + \bar{y} + z = 0$
→ 7	1	1	1	0	or (the case when x=1 and y=1 and z=1) ↔ $\bar{x} + \bar{y} + \bar{z} = 0$

index { inputs } { output }

All possible combination of inputs

Domain Range

Boolean Function with Maxterms (2)

	x	y	z	F
→ 0	0	0	0	0
1	0	0	1	1
→ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
→ 5	1	0	1	0
→ 6	1	1	0	0
→ 7	1	1	1	0

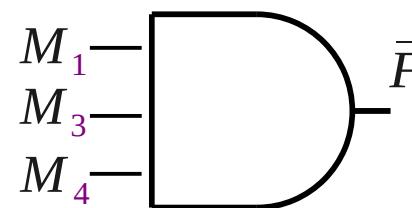
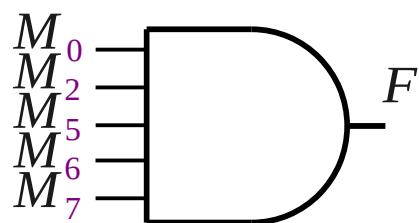
index inputs output

All possible combination of inputs

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \leftrightarrow \quad F = 0$$

$$\leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$



For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with Maxterms (2)

	x	y	z	F
→ 0	0	0	0	0
1	0	0	1	1
→ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
→ 5	1	0	1	0
→ 6	1	1	0	0
→ 7	1	1	1	0

index

inputs

output

All possible
combination of inputs

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \leftrightarrow \quad F = 0$$

$$\leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,
either $M_1=0$ or $M_3=0$ or $M_4=0$

$$M_1 \cdot M_3 \cdot M_4 = 0 \quad \leftrightarrow \quad F = 1$$

$$\leftrightarrow \quad \overline{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

Complimentary Relations

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

{ } { } { }

inputs output

All possible combination of inputs

$$m_i = \overline{M}_i$$

$$M_i = \overline{m}_i$$

$$F(x, y, z) = m_1 + m_3 + m_4$$

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

For the output of an **or** gate to be 1,
at least one must be 1

$$\bar{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7$$

$$\begin{aligned} \Leftrightarrow F(x, y, z) &= \overline{m_0 + m_2 + m_5 + m_6 + m_7} \\ &= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_5} \cdot \overline{m_6} \cdot \overline{m_7} \end{aligned}$$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 1,

for the cases

1) when $m_1=1$ or $m_3=1$ or $m_4=1$

$$F(x, y, z) = m_1 + m_3 + m_4 \rightarrow F=1$$

2) when $M_1=0$ or $M_3=0$ or $M_4=0$

$$\bar{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \rightarrow F=1 (\bar{F}=0)$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 0,

for the cases

1) when $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$$\bar{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \rightarrow F=0 (\bar{F}=1)$$

2) when $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \rightarrow F=0$$

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$F(x, y, z) = m_1 + m_3 + m_4 \rightarrow F=1$$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \rightarrow F=0$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\bar{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \rightarrow F=0 \ (\bar{F}=1)$$

$$\bar{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \rightarrow F=1 \ (\bar{F}=0)$$

Truth Table

References

[1] <http://en.wikipedia.org/>