Truth Table (A2)

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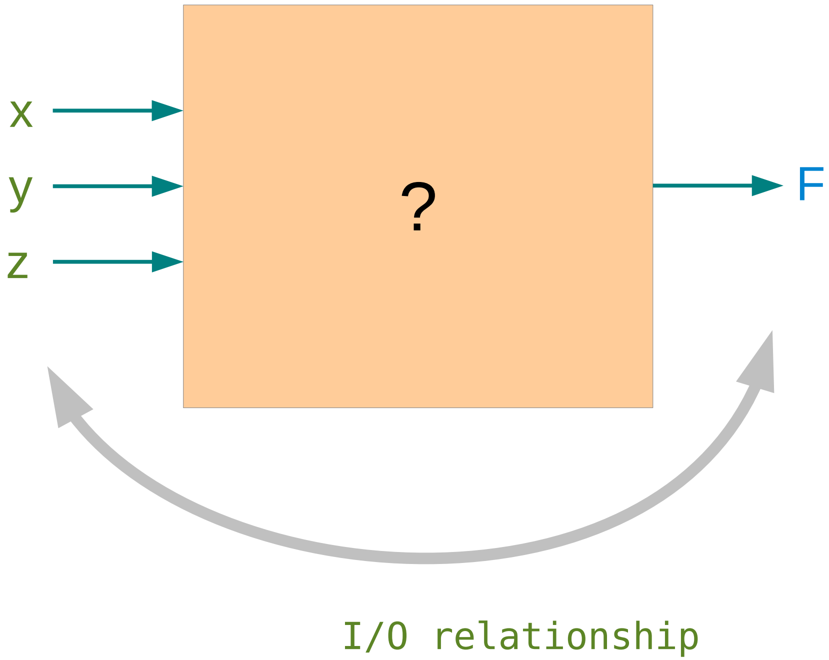
Truth Table

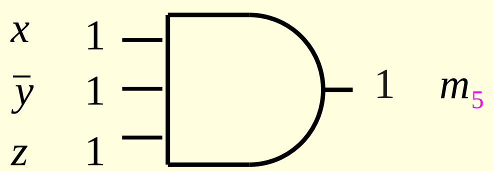


x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

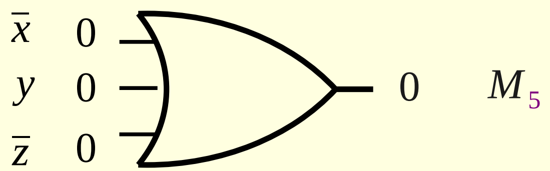
Finding minterms and Maxterms

Consider the case when x=1 **and** y=0 **and** z=1   index = 5



$(x=1) \wedge (y=0) \wedge (z=1) \Leftrightarrow$

$(x=1) \wedge (\bar{y}=1) \wedge (z=1) \Leftrightarrow m_5=1$



$(x=1) \wedge (y=0) \wedge (z=1) \Leftrightarrow$

$(\bar{x}=0) \wedge (y=0) \wedge (\bar{z}=0) \Leftrightarrow M_5=0$

minterm m_5 ➡ index 5

binary 101

0 → bar

$x \cdot \bar{y} \cdot z = 1$

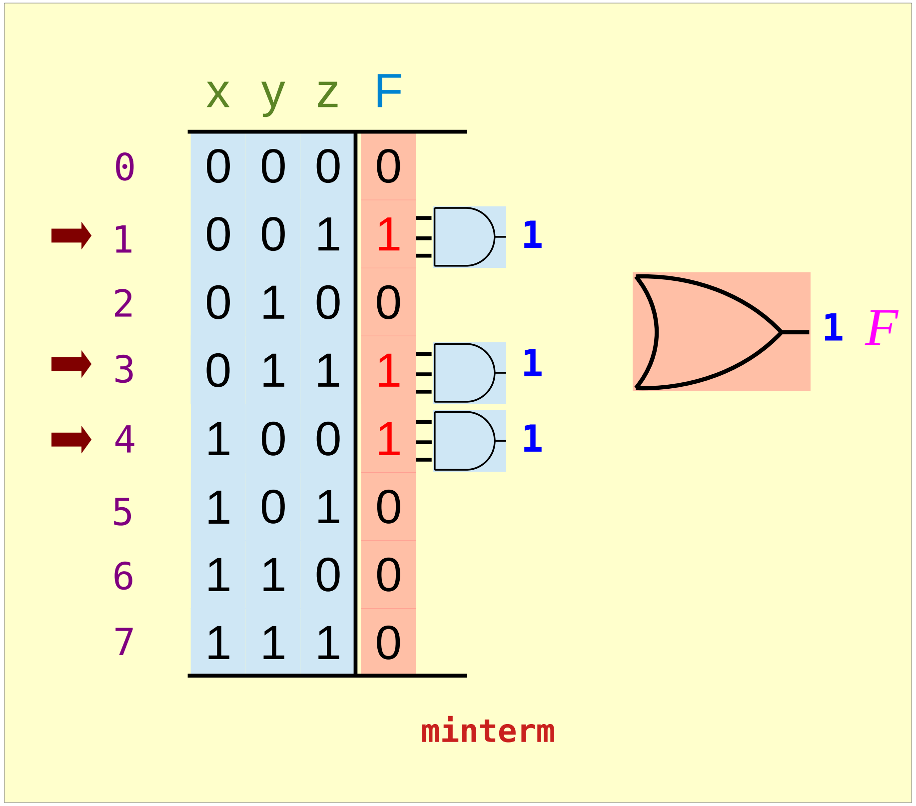
Maxterm M_5 ➡ index 5

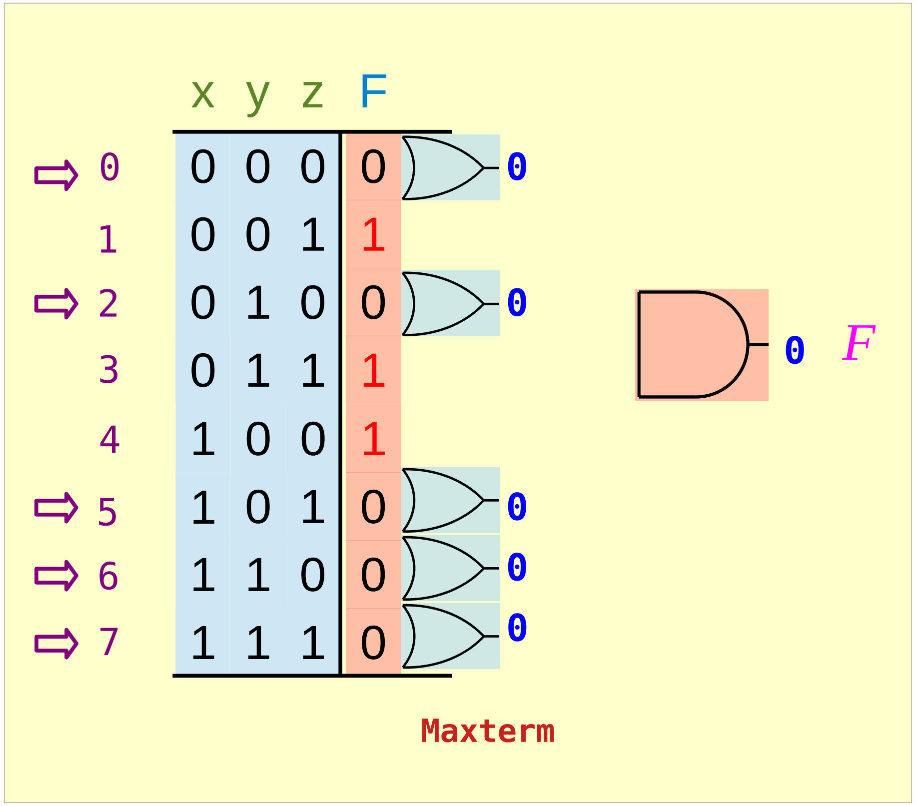
binary 101

1 → bar

$\bar{x} + y + \bar{z} = 0$

Boolean Function **F** with minterms and Maxterms

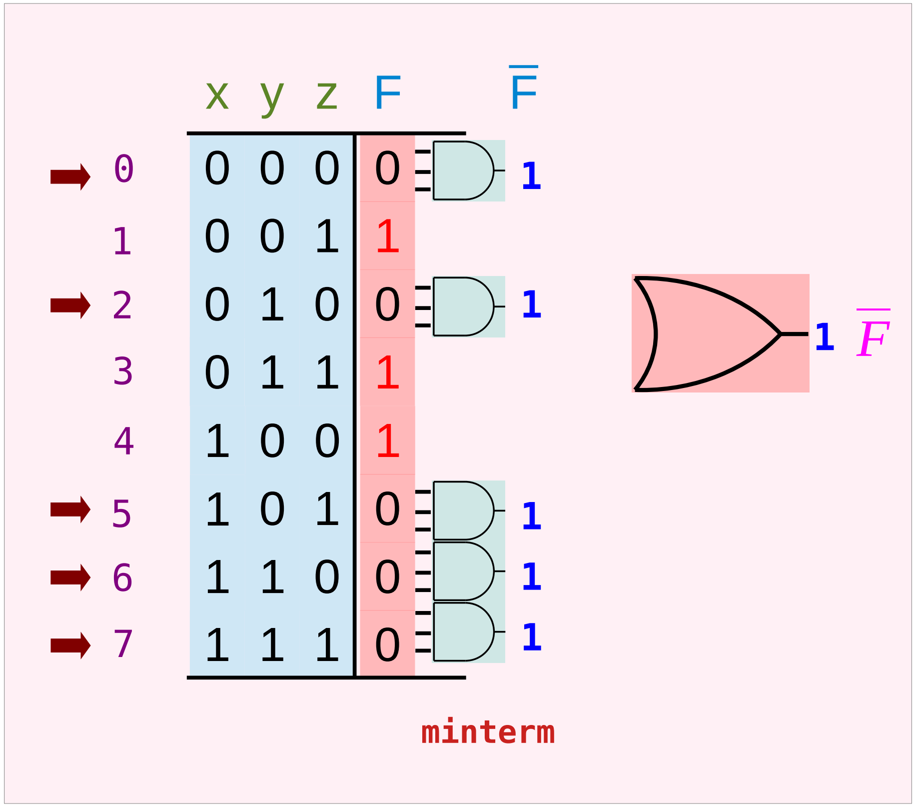




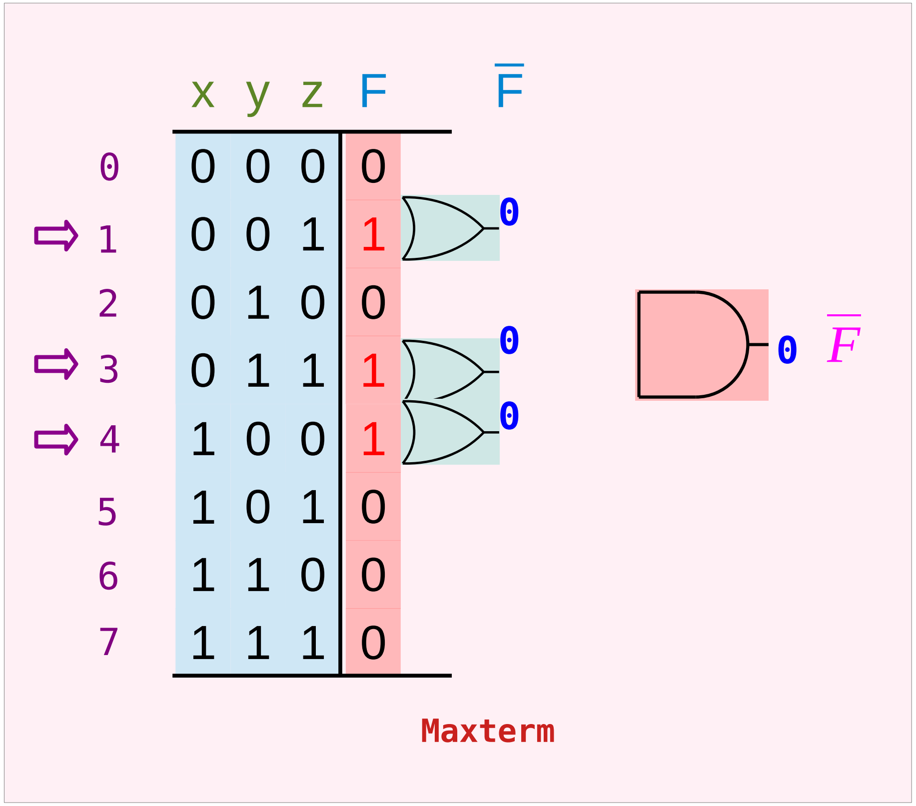
$$F = m_1 + m_3 + m_4 = \bar{x}\,\bar{y}\,z + \bar{x}\,y\,z + x\,\bar{y}\,\bar{z}$$

$$F = M_0 + M_2 + M_5 + M_6 + M_7 = (x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+y+\bar{z})\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{x}+\bar{y}+\bar{z})$$

Boolean Function $\overline{\mathbf{F}}$ with minterms and Maxterms



	x	y	z	F	$\overline{F}$
➡ 0	0	0	0	0	1
1	0	0	1	1	
➡ 2	0	1	0	0	1
3	0	1	1	1	
4	1	0	0	1	
➡ 5	1	0	1	0	1
➡ 6	1	1	0	0	1
➡ 7	1	1	1	0	1

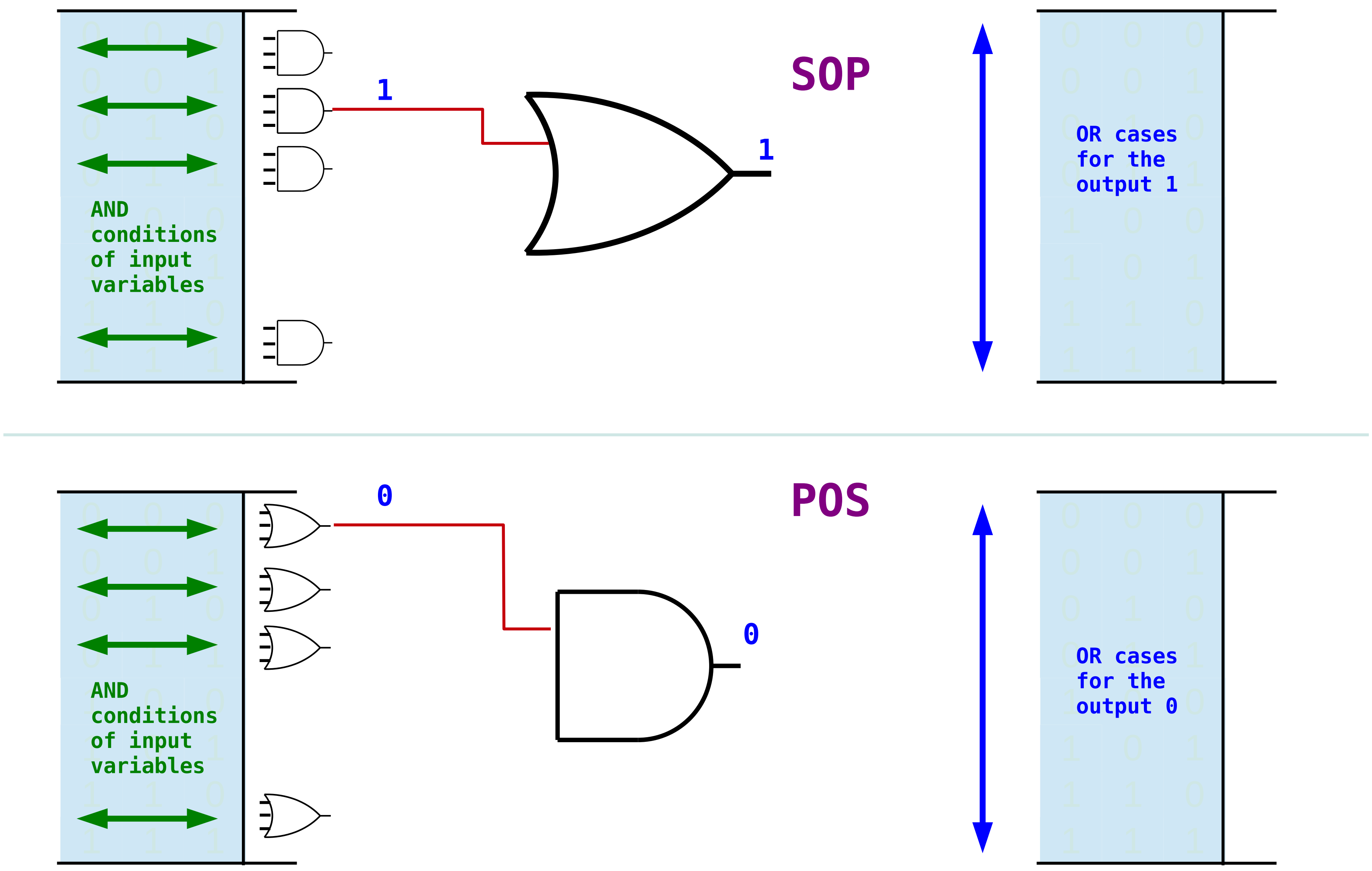


	x	y	z	F	$\overline{F}$
0	0	0	0	0	
⇨ 1	0	0	1	1	0
2	0	1	0	0	
⇨ 3	0	1	1	1	0
⇨ 4	1	0	0	1	0
5	1	0	1	0	
6	1	1	0	0	
7	1	1	1	0	

$$\overline{F} = m_0 + m_2 + m_5 + m_6 + {}_7$$
$$= \overline{x}\cdot\overline{y}\cdot\overline{z} + \overline{x}\cdot y\cdot\overline{z} + x\cdot\overline{y}\cdot z$$
$$+ x\cdot y\cdot\overline{z} + x\cdot y\cdot z$$

$$\overline{F} = M_1 + M_3 + M_4$$
$$= (x+y+\overline{z})\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+z)$$

SOP and POS



1
1
SOP
OR cases
for the
output 1
AND
conditions
of input
variables
0
0
POS
OR cases
for the
output 0
AND
conditions
of input
variables

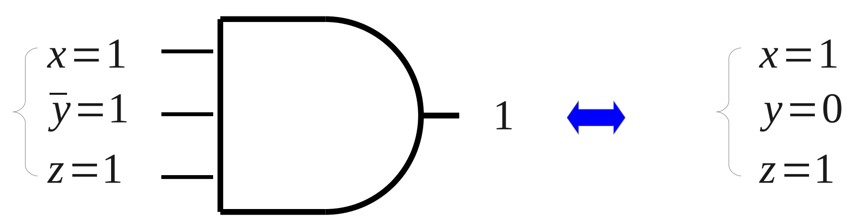
Truth Table and minterms (1)

x	y	z		
0	0	0	the case when x=0 **and** y=0 **and** z=0 ⟺	$\bar{x}\,\bar{y}\,\bar{z} = 1$
0	0	1	the case when x=0 **and** y=0 **and** z=1 ⟺	$\bar{x}\,\bar{y}\,z = 1$
0	1	0	the case when x=0 **and** y=1 **and** z=0 ⟺	$\bar{x}\,y\,\bar{z} = 1$
0	1	1	the case when x=0 **and** y=1 **and** z=1 ⟺	$\bar{x}\,y\,z = 1$
1	0	0	the case when x=1 **and** y=0 **and** z=0 ⟺	$x\,\bar{y}\,\bar{z} = 1$
1	0	1	the case when x=1 **and** y=0 **and** z=1 ⟺	$x\,\bar{y}\,z = 1$
1	1	0	the case when x=1 **and** y=1 **and** z=0 ⟺	$x\,y\,\bar{z} = 1$
1	1	1	the case when x=1 **and** y=1 **and** z=1 ⟺	$x\,y\,z = 1$

inputs

All possible combination of inputs

$x\,\bar{y}\,z = 1$ ⟺



For the output of an **and** gate to be 1, all inputs must be 1

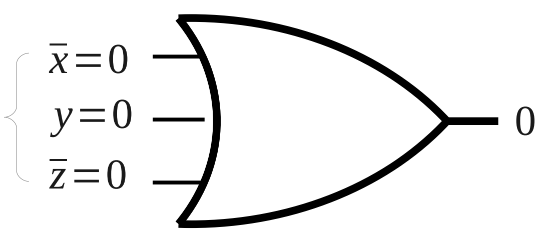
Truth Table and MAXterms (1)

x	y	z			
0	0	0	the case when x=0 **and** y=0 **and** z=0	⟺	$x+y+z = 0$
0	0	1	the case when x=0 **and** y=0 **and** z=1	⟺	$x+y+\bar{z} = 0$
0	1	0	the case when x=0 **and** y=1 **and** z=0	⟺	$x+\bar{y}+z = 0$
0	1	1	the case when x=0 **and** y=1 **and** z=1	⟺	$x+\bar{y}+\bar{z} = 0$
1	0	0	the case when x=1 **and** y=0 **and** z=0	⟺	$\bar{x}+y+z = 0$
1	0	1	the case when x=1 **and** y=0 **and** z=1	⟺	$\bar{x}+y+\bar{z} = 0$
1	1	0	the case when x=1 **and** y=1 **and** z=0	⟺	$\bar{x}+\bar{y}+z = 0$
1	1	1	the case when x=1 **and** y=1 **and** z=1	⟺	$\bar{x}+\bar{y}+\bar{z} = 0$

inputs

All possible
combination of inputs

$\bar{x}+y+\bar{z} = 0$ ⟺



⟺ $x=1$, $y=0$, $z=1$

For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and MAXterms (2)

index	x	y	z		
0	0	0	0	the case when the MAXterm	$M_0 = x + y + z = 0$
1	0	0	1	the case when the MAXterm	$M_1 = x + y + \bar{z} = 0$
2	0	1	0	the case when the MAXterm	$M_2 = x + \bar{y} + z = 0$
3	0	1	1	the case when the MAXterm	$M_3 = x + \bar{y} + \bar{z} = 0$
4	1	0	0	the case when the MAXterm	$M_4 = \bar{x} + y + z = 0$
5	1	0	1	the case when the MAXterm	$M_5 = \bar{x} + y + \bar{z} = 0$
6	1	1	0	the case when the MAXterm	$M_6 = \bar{x} + \bar{y} + z = 0$
7	1	1	1	the case when the MAXterm	$M_7 = \bar{x} + \bar{y} + \bar{z} = 0$

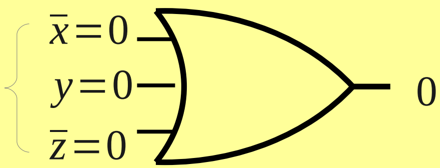
index

inputs

All possible
combination of inputs

Maxterm M_5 ➡ index 5

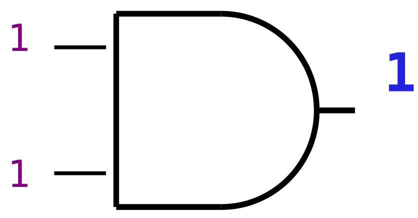
binary 101



$\bar{x} + y + \bar{z} = 0$

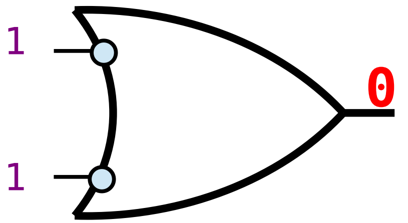
1 → bar

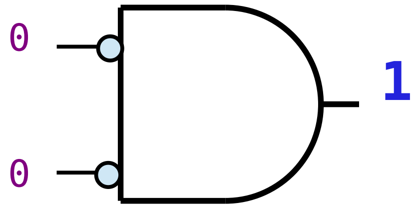
Maxterm and minterm Conditions



x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

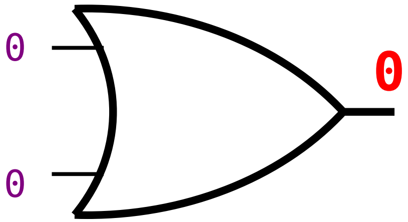
$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
1	1	1
1	0	1
0	1	1
0	0	0





$\bar{x}$	$\bar{y}$	$\bar{x}\bar{y}$
1	1	1
1	0	0
0	1	0
0	0	0

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1



Boolean Function with minterms (1)

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index — inputs (x, y, z) — output (F)

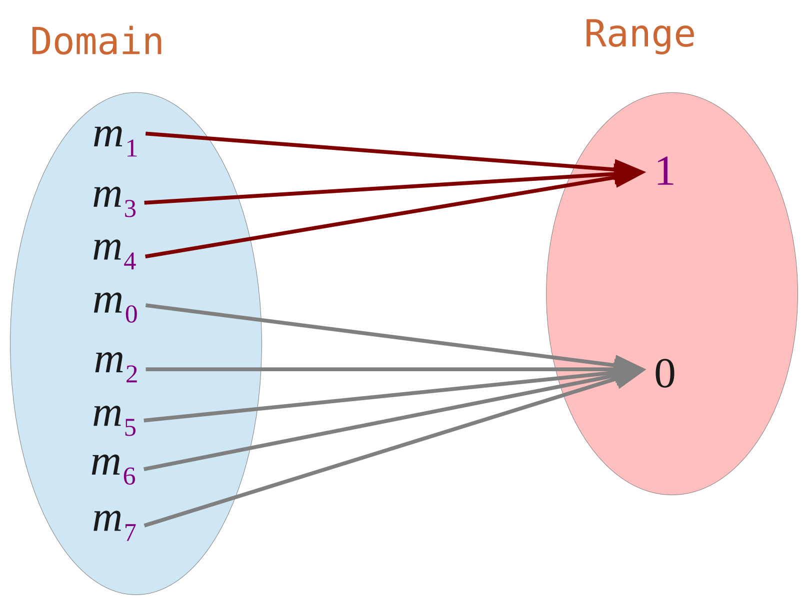
All possible combination of inputs

The output F becomes 1,
for one of the <u>three</u> following cases

(the case when x=0 **and** y=0 **and** z=1) ⟺ $m_1 = \bar{x}\,\bar{y}\,z = 1$

or (the case when x=0 **and** y=1 **and** z=1) ⟺ $m_3 = \bar{x}\,y\,z = 1$

or (the case when x=1 **and** y=0 **and** z=0) ⟺ $m_4 = x\,\bar{y}\,\bar{z} = 1$



Boolean Function with minterms (2)

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

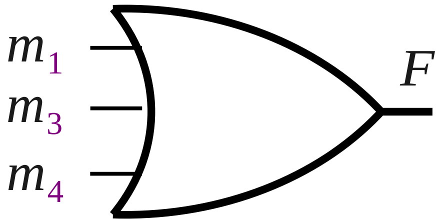
index inputs output

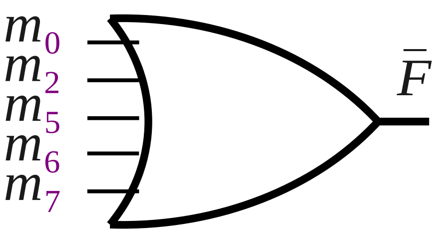
All possible
combination of inputs

The output F becomes 1,
either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$

$\Leftrightarrow F = m_1 + m_3 + m_4$





For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with minterms (3)

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index — inputs (x y z) — output (F)

All possible combination of inputs

The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$$

$$\Leftrightarrow \quad F = m_1 + m_3 + m_4$$

The output F becomes 0,

either $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$$m_0 + m_2 + m_5 + m_6 + m_7 = 1 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad \overline{F} = m_0 + m_2 + m_5 + m_6 + m_7$$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with Maxterms (1)

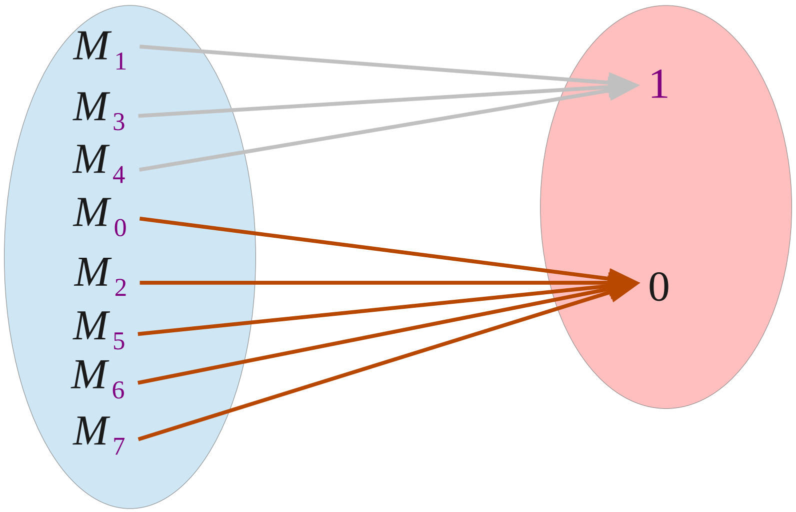
The output F becomes 0,
for one of the five following cases

index	x	y	z	F	
➡ 0	0	0	0	0	(the case when x=0 **and** y=0 **and** z=0) ⟺ $x + y + z = 0$
1	0	0	1	1	
➡ 2	0	1	0	0	**or** (the case when x=0 **and** y=1 **and** z=0) ⟺ $x + \overline{y} + z = 0$
3	0	1	1	1	
4	1	0	0	1	
➡ 5	1	0	1	0	**or** (the case when x=1 **and** y=0 **and** z=1) ⟺ $\overline{x} + y + \overline{z} = 0$
➡ 6	1	1	0	0	**or** (the case when x=1 **and** y=1 **and** z=0) ⟺ $\overline{x} + \overline{y} + z = 0$
➡ 7	1	1	1	0	**or** (the case when x=1 **and** y=1 **and** z=1) ⟺ $\overline{x} + \overline{y} + \overline{z} = 0$

index   inputs output

All possible
combination of inputs

Domain



Range

Boolean Function with Maxterms (2)

	index	x	y	z	F
➡	0	0	0	0	0
	1	0	0	1	1
➡	2	0	1	0	0
	3	0	1	1	1
	4	1	0	0	1
➡	5	1	0	1	0
➡	6	1	1	0	0
➡	7	1	1	1	0

index

inputs output

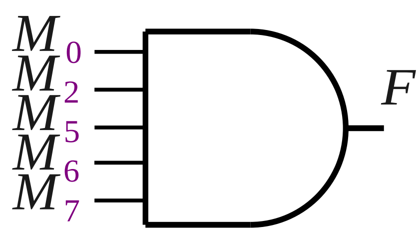
All possible
combination of inputs

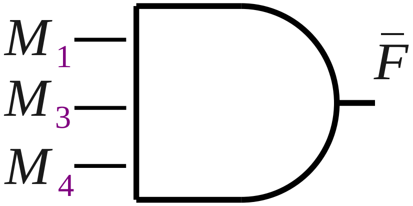
The output F becomes 0,

either $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$





For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with Maxterms (2)

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index — inputs — output

All possible combination of inputs

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \rightleftarrows \quad F = 0$$

$$\Longleftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,

either $M_1=0$ or $M_3=0$ or $M_4=0$

$$M_1 \cdot M_3 \cdot M_4 = 0 \quad \rightleftarrows \quad F = 1$$

$$\Longleftrightarrow \quad \overline{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

Complimentary Relations

index	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible combination of inputs

$$m_i = \overline{M_i}$$

$$M_i = \overline{m_i}$$

$$F(x,y,z) = m_1 + m_3 + m_4$$

The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

For the output of an **or** gate to be 1, at least one must be 1

$$\bar{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$$

$$\Leftrightarrow \quad F(x,y,z) = \overline{m_0 + m_2 + m_5 + m_6 + m_7}$$

$$= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_5} \cdot \overline{m_6} \cdot \overline{m_7}$$

$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,

either $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

For the output of an **and** gate to be 0, at least one input must be 0

Boolean Function Summary

	x	y	z	F
0	0	0	0	
1	0	0	1	1
2	0	1	0	
3	0	1	1	1
4	1	0	0	1
5	1	0	1	
6	1	1	0	
7	1	1	1	

The output F becomes 1,

for the cases

1) when $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$F(x,y,z) = m_1 + m_3 + m_4$ ➡ $F = 1$

2) when $M_1 = 0$ or $M_3 = 0$ or $M_4 = 0$

$\overline{F}(x,y,z) = M_1 \cdot M_3 \cdot M_4$ ➡ $F = 1$ $(\overline{F} = 0)$

	x	y	z	F
0	0	0	0	0
1	0	0	1	
2	0	1	0	0
3	0	1	1	
4	1	0	0	
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 0,

for the cases

1) when $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$\overline{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$ ➡ $F = 0$ $(\overline{F} = 1)$

2) when $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$ ➡ $F = 0$

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$F(x, y, z) = m_1 + m_3 + m_4 \Rightarrow F = 1$$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \Rightarrow F = 0$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\overline{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \Rightarrow F = 0 \quad (\overline{F} = 1)$$

$$\overline{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \Rightarrow F = 1 \quad (\overline{F} = 0)$$

Truth Table

References

[1] http://en.wikipedia.org/