K-Map (A3)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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K-Map 3 variables (1)

index				minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$

x y z

	00	01	11	10
0	0	1	3	2
1	4	5	7	6

	y=0		y=1	
	z=0	z=1		z=0
x=0	0	1	3	2
x=1	4	5	7	6

index				minterms	
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}$
1	0	0	1	$\bar{x}\bar{y}z$	
2	0	1	0	$\bar{x}y\bar{z}$	$\bar{x}y$
3	0	1	1	$\bar{x}yz$	
4	1	0	0	$x\bar{y}\bar{z}$	$x\bar{y}$
5	1	0	1	$x\bar{y}z$	
6	1	1	0	$xy\bar{z}$	xy
7	1	1	1	xyz	

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z = \bar{x}\bar{y}(\bar{z} + z) = \bar{x}\bar{y}$$

a group of 2 minterms

$$x \quad y \quad \cancel{z}$$

$$2^1$$

		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0 $\bar{x}\bar{y}$	1	3	2 $\bar{x}y$
x=1	1	4 $x\bar{y}$	5	7	6 xy

K-Map 3 variables (3)

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$	
3	0	1	1	$\bar{x}\,y\,z$	
4	1	0	0	$x\,\bar{y}\,\bar{z}$	$x\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$	$x\,z$
6	1	1	0	$x\,y\,\bar{z}$	
7	1	1	1	$x\,y\,z$	

$$\bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,y\,\bar{z} = \bar{x}\,\bar{z}(\bar{y}+y) = \bar{x}\,\bar{z}$$

a group of 2 minterms

$x \quad \cancel{y} \quad z$

2^1

		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0 $\bar{x}\,\bar{z}$	1 $\bar{x}\,z$	3	2
x=1	1	4 $x\,\bar{z}$	5 $x\,z$	7	6

K-Map 3 variables (4)

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	$\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$	$y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$	$y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$	
5	1	0	1	$x\,\bar{y}\,z$	
6	1	1	0	$x\,y\,\bar{z}$	
7	1	1	1	$x\,y\,z$	

$$\bar{x}\,\bar{y}\,\bar{z} + x\,\bar{y}\,\bar{z} = \bar{y}\,\bar{z}(\bar{x}+x) = \bar{y}\,\bar{z}$$

a group of 2 minterms

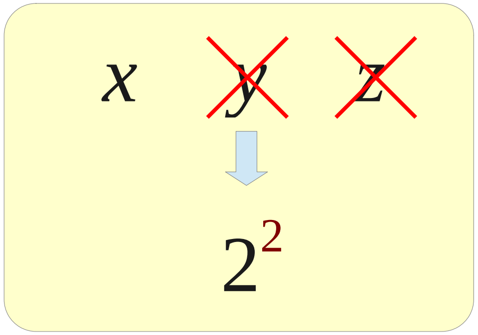
$\cancel{x}$ y z

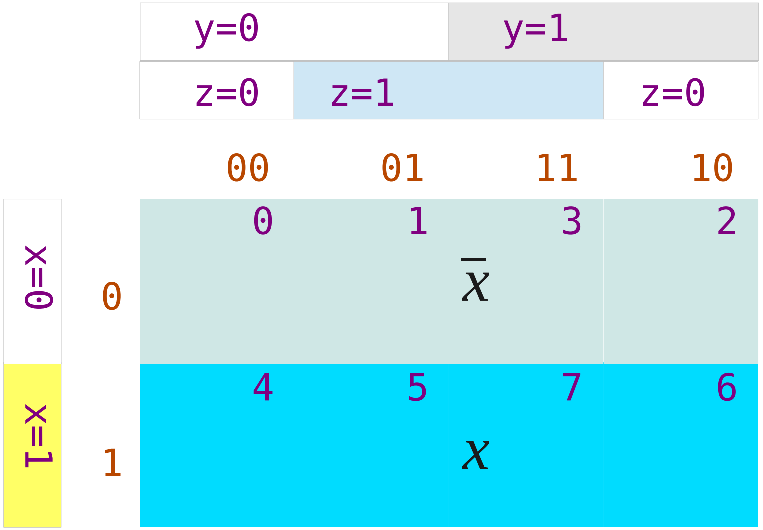
2^1

		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0 $\bar{y}\,\bar{z}$	1 $\bar{y}\,z$	3 $y\,z$	2 $y\,\bar{z}$
x=1	1	4	5	7	6

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	
2	0	1	0	$\bar{x}\,y\,\bar{z}$	
3	0	1	1	$\bar{x}\,y\,z$	
4	1	0	0	$x\,\bar{y}\,\bar{z}$	x
5	1	0	1	$x\,\bar{y}\,z$	
6	1	1	0	$x\,y\,\bar{z}$	
7	1	1	1	$x\,y\,z$	

a group of 4 minterms





K-Map 3 variables (5)

index				minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$

$\bar{y}$

y

a group of 4 minterms

$\cancel{x}$ y $\cancel{z}$

2^2

		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0	1 $\bar{y}$	3	2 y
x=1	1	4	5	7	6

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K-Map 3 variables (5)

index				minterms
0	0	0	0	$\overline{x}\,\overline{y}\,\overline{z}$
1	0	0	1	$\overline{x}\,\overline{y}\,z$
2	0	1	0	$\overline{x}\,y\,\overline{z}$
3	0	1	1	$\overline{x}\,y\,z$
4	1	0	0	$x\,\overline{y}\,\overline{z}$
5	1	0	1	$x\,\overline{y}\,z$
6	1	1	0	$x\,y\,\overline{z}$
7	1	1	1	$x\,y\,z$

$\overline{z}$

z

a group of 4 minterms

$\cancel{x}\quad \cancel{y}\quad z$

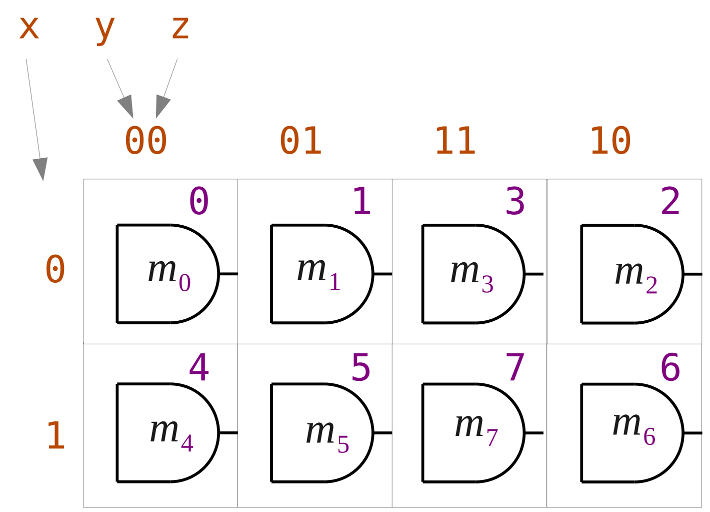
$\Downarrow$

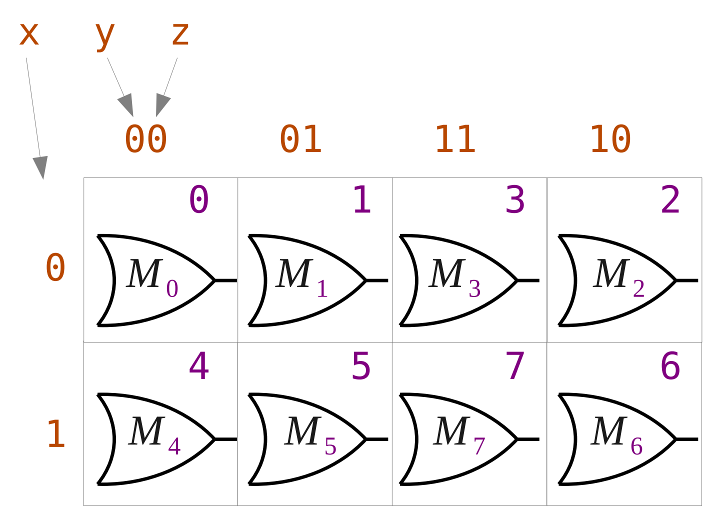
2^2

		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0	1	3	2
x=1	1	4 $\overline{z}$	5 z	7	6

K-Map, minterms, and Maxterms

index				minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$



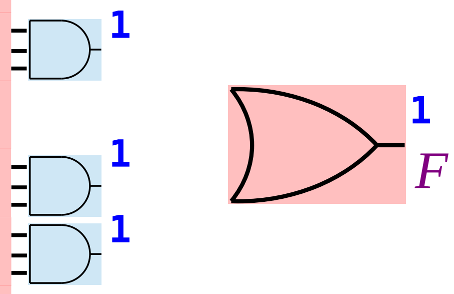


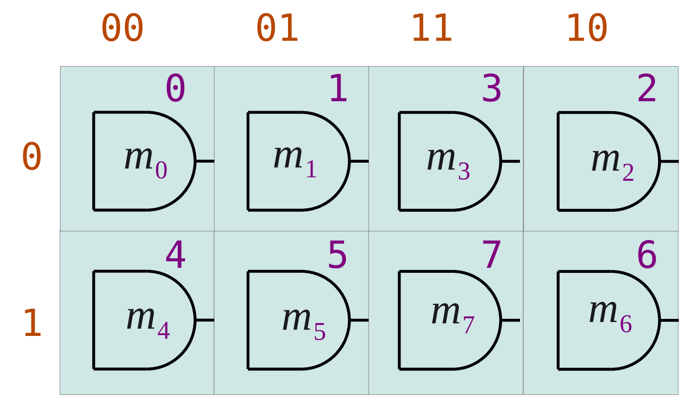
Each rectangle is associated with a minterm or a maxterm which represents a particular input variable conditions.

In this table, output function value is overlaid

Boolean Function with minterms

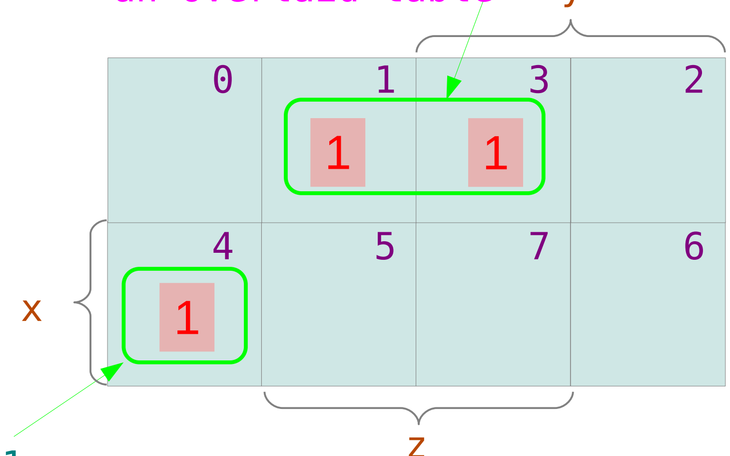
	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0





F=1 when $\bar{x}z$ = 1

an overlaid table



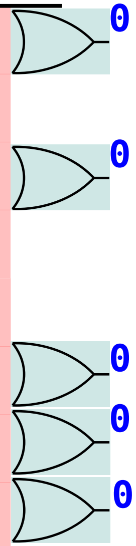
F=1 when $x\bar{y}\bar{z}$ = 1

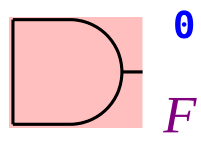
a simplified function

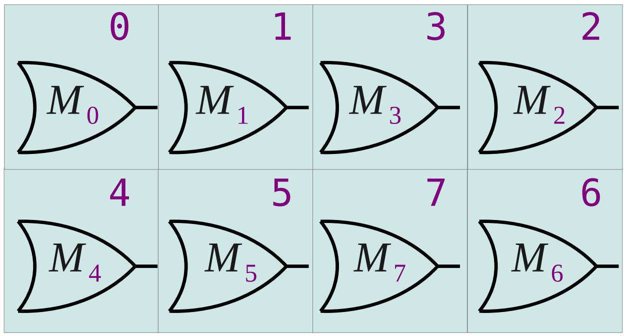
$$F = \bar{x}z + x\bar{y}\bar{z}$$

Boolean Function with Maxterms

	x	y	z	F
⇨ 0	0	0	0	0
1	0	0	1	1
⇨ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
⇨ 5	1	0	1	0
⇨ 6	1	1	0	0
⇨ 7	1	1	1	0

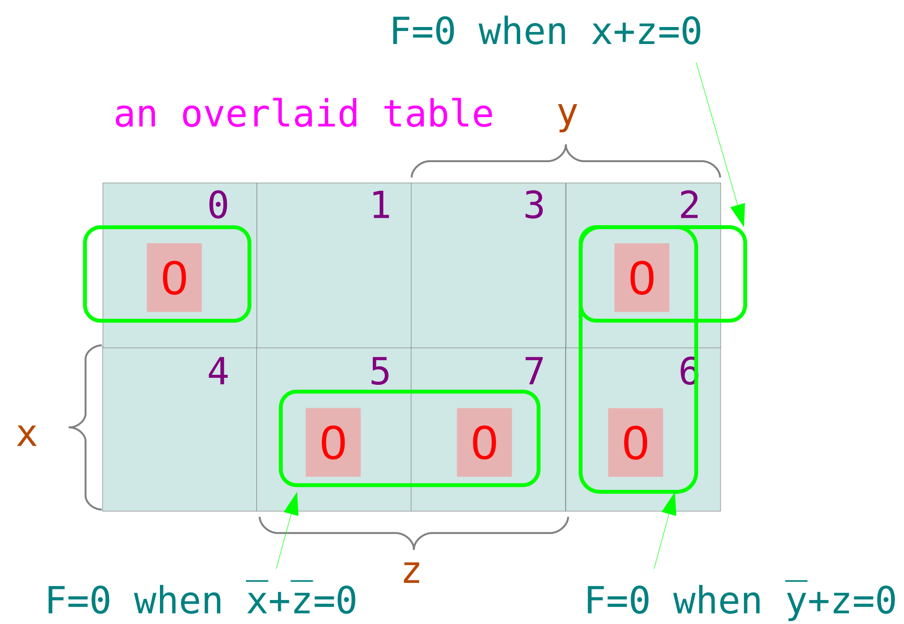






a simplified function

$$F = (\overline{x} + \overline{z})(\overline{y} + z)(x + z)$$



Implicant

F takes the value 1, whenever P equals 1

implies

$$P \Rightarrow F$$

assuming *P* is a product term in a sum of products

$$F(x,y,z,w) = xy + yz + w$$

$$(xy = 1) \Rightarrow (f=1)$$
$$(xyz = 1) \Rightarrow (f=1)$$
$$(xyzw = 1) \Rightarrow (f=1)$$

General,
Reduced

$$xy \Rightarrow F(x,y,z,w) = xy + yz + w$$
$$xyz \Rightarrow F(x,y,z,w) = xy + yz + w$$
$$xyzw \Rightarrow F(x,y,z,w) = xy + yz + w$$

implicants

Implicant

F takes the value 1, whenever P equals 1

$$P \overset{\text{implies}}{\Rightarrow} F$$

assuming P is a product term in a sum of products

$$F(x, y, z, w) = xy + yz + w$$

general,
reduced

$(x=1,\ y=1)$	$xy \Rightarrow$	$F(x, y, z, w) = xy + yz + w$	$(f=1)$
$(x=1,\ y=1,\ z=1)$	$xyz \Rightarrow$	$F(x, y, z, w) = xy + yz + w$	$(f=1)$
$(x=1,\ y=1,\ z=1,\ w=1)$	$xyzw \Rightarrow$	$F(x, y, z, w) = xy + yz + w$	$(f=1)$

input conditions that will make F = 1

Prime Implicant

Prime Implicant: An implicant that is <u>minimal</u>

The removal of any literal from P results in a non-implicant for F

Prime Implicant

general,
reduced

$(x=1,\ y=1)$ $\quad xy \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w$

$(x=1,\ y=1,\ z=1)$ $\quad xyz \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w$

$(x=1,\ y=1,\ z=1,\ w=1)$ $\quad xyzw \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w$

$xyzw \Rightarrow F$ $\qquad xyzw \Rightarrow F$ $\qquad xyzw \Rightarrow F$

$yzw \Rightarrow F$ $\qquad xy\ w \Rightarrow F$ $\qquad xyz \Rightarrow F$

$zw \Rightarrow F$ $\qquad xy \Rightarrow F$ $\qquad yz \Rightarrow F$

$w \Rightarrow F$

A process of
expanding terms

general,
reduced

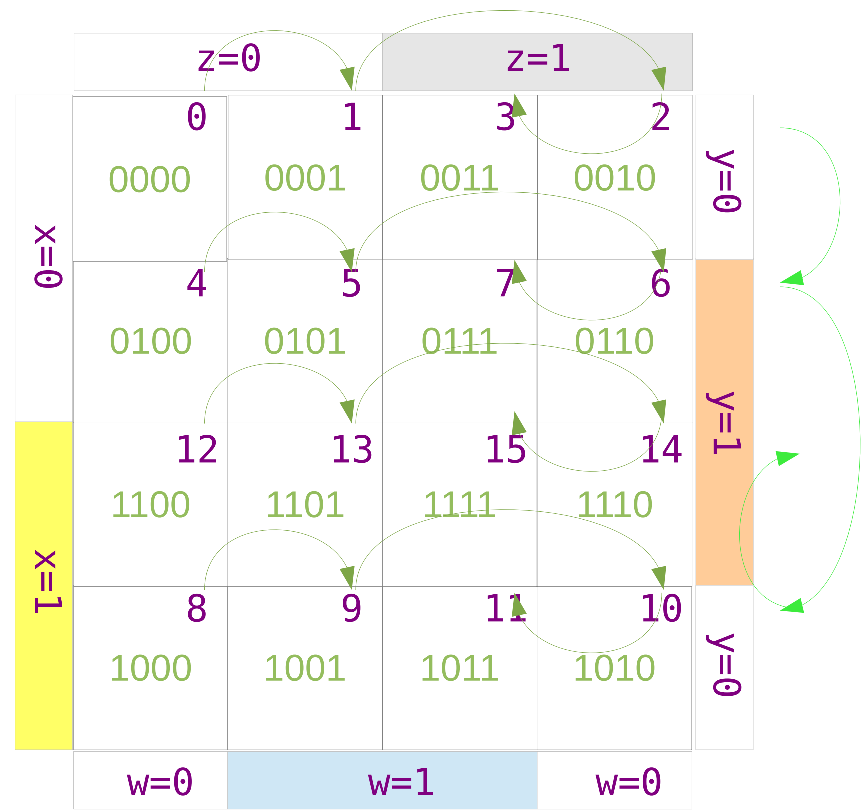
K-Map 4 variables (1)

index					minterms
0	0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}\,\bar{w}$
1	0	0	0	1	$\bar{x}\,\bar{y}\,\bar{z}\,w$
2	0	0	1	0	$\bar{x}\,\bar{y}\,z\,\bar{w}$
3	0	0	1	1	$\bar{x}\,\bar{y}\,z\,w$
4	0	1	0	0	$\bar{x}\,y\,\bar{z}\,\bar{w}$
5	0	1	0	1	$\bar{x}\,y\,\bar{z}\,w$
6	0	1	1	0	$\bar{x}\,y\,z\,\bar{w}$
7	0	1	1	1	$\bar{x}\,y\,z\,w$
8	1	0	0	0	$x\,\bar{y}\,\bar{z}\,\bar{w}$
9	1	0	0	1	$x\,\bar{y}\,\bar{z}\,w$
10	1	0	1	0	$x\,\bar{y}\,z\,\bar{w}$
11	1	0	1	1	$x\,\bar{y}\,z\,w$
12	1	1	0	0	$x\,y\,\bar{z}\,\bar{w}$
13	1	1	0	1	$x\,y\,\bar{z}\,w$
14	1	1	1	0	$x\,y\,z\,\bar{w}$
15	1	1	1	1	$x\,y\,z\,w$

			z=0		z=1	
			w=0	w=1		w=0
			00	01	11	10
x=0	y=0	00	0 0000	1 0001	3 0011	2 0010
	y=1	01	4 0100	5 0101	7 0111	6 0110
x=1		11	12 1100	13 1101	15 1111	14 1110
	y=0	10	8 1000	9 1001	11 1011	10 1010

K-Map 4 variables (2)

index					minterms
0	0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}\,\bar{w}$
1	0	0	0	1	$\bar{x}\,\bar{y}\,\bar{z}\,w$
2	0	0	1	0	$\bar{x}\,\bar{y}\,z\,\bar{w}$
3	0	0	1	1	$\bar{x}\,\bar{y}\,z\,w$
4	0	1	0	0	$\bar{x}\,y\,\bar{z}\,\bar{w}$
5	0	1	0	1	$\bar{x}\,y\,\bar{z}\,w$
6	0	1	1	0	$\bar{x}\,y\,z\,\bar{w}$
7	0	1	1	1	$\bar{x}\,y\,z\,w$
8	1	0	0	0	$x\,\bar{y}\,\bar{z}\,\bar{w}$
9	1	0	0	1	$x\,\bar{y}\,\bar{z}\,w$
10	1	0	1	0	$x\,\bar{y}\,z\,\bar{w}$
11	1	0	1	1	$x\,\bar{y}\,z\,w$
12	1	1	0	0	$x\,y\,\bar{z}\,\bar{w}$
13	1	1	0	1	$x\,y\,\bar{z}\,w$
14	1	1	1	0	$x\,y\,z\,\bar{w}$
15	1	1	1	1	$x\,y\,z\,w$



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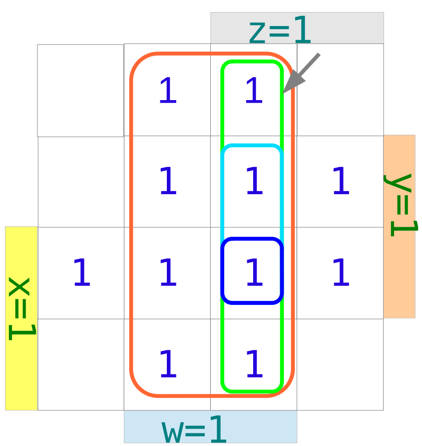
Expanding Terms

$\boxed{xyzw} \Rightarrow F$

$yzw \Rightarrow F$

$zw \Rightarrow F$

$w \Rightarrow F$

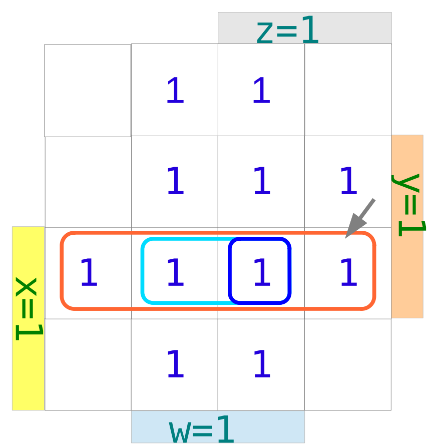


Prime Implicant $w \Rightarrow F$

$\boxed{xyzw} \Rightarrow F$

$xy\ w \Rightarrow F$

$xy \Rightarrow F$

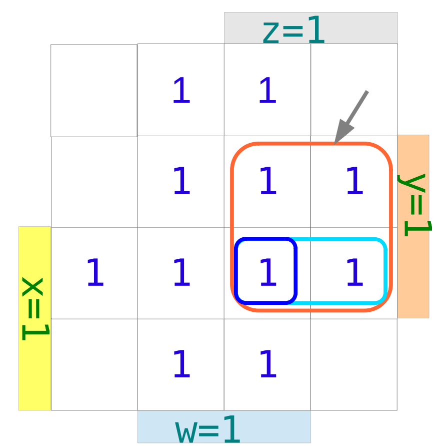


Prime Implicant $xy \Rightarrow F$

$\boxed{xyzw} \Rightarrow F$

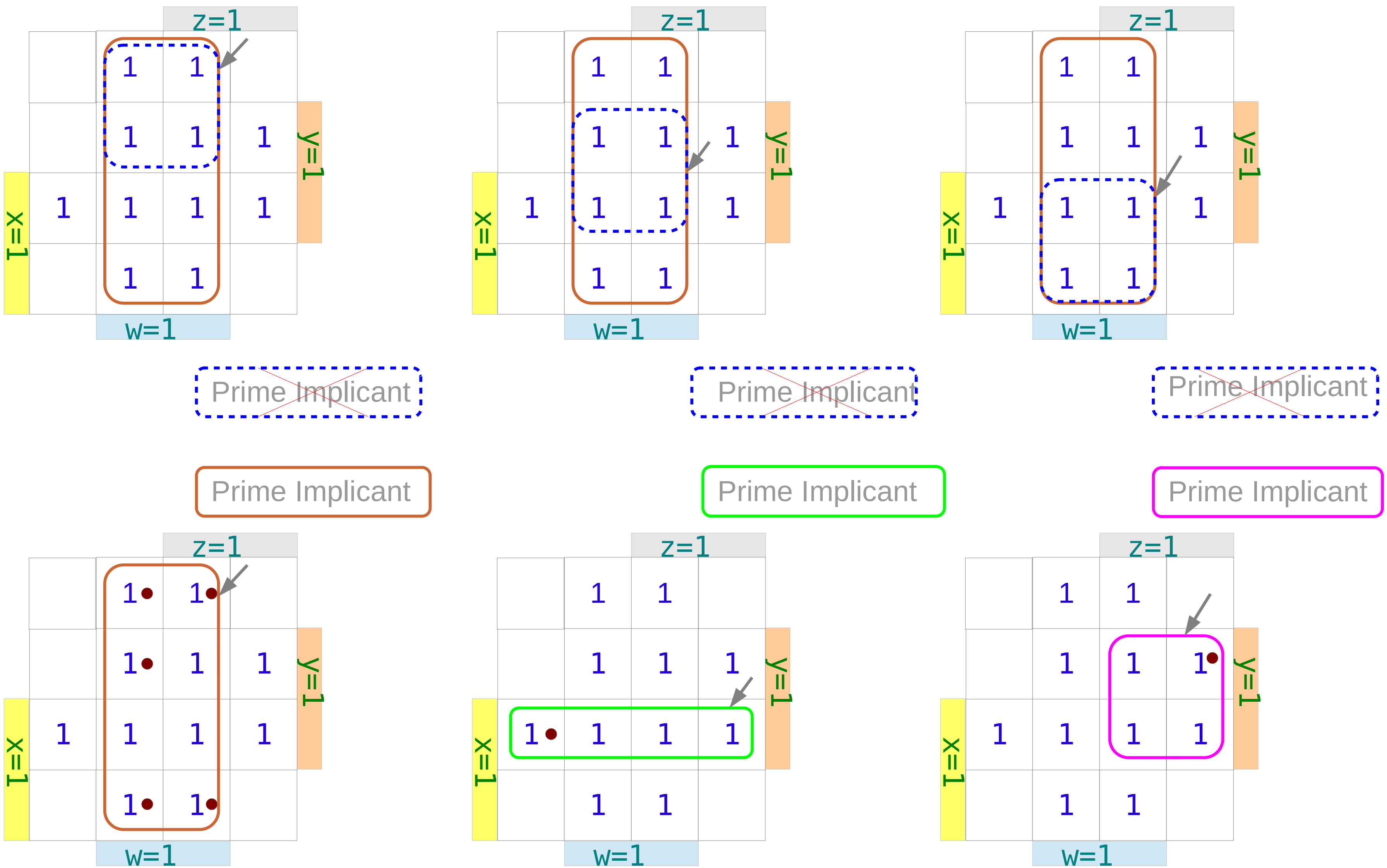
$xyz \Rightarrow F$

$yz \Rightarrow F$



Prime Implicant $yz \Rightarrow F$

Prime Implicant Example



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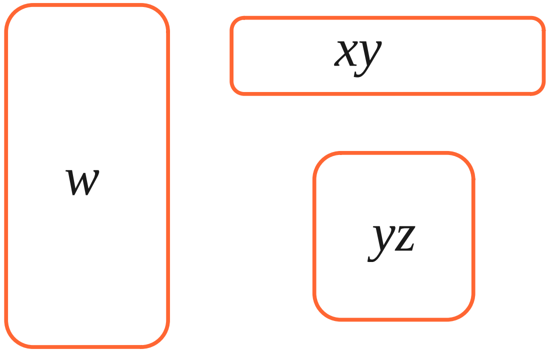
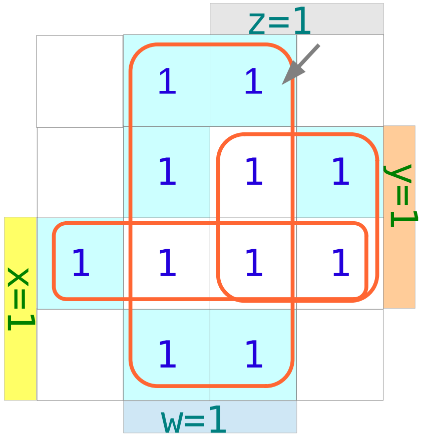
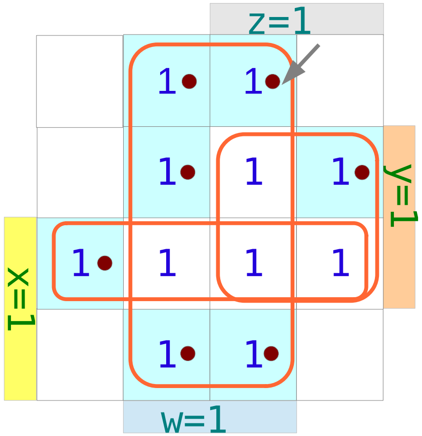
Prime Implicants



Three Prime Implicants





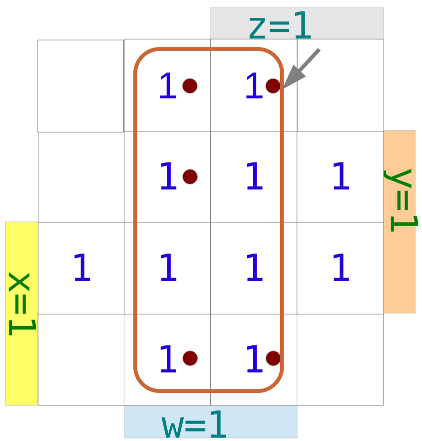
some minterms belong to
only <u>one</u> prime implicant

Those prime implicants containing
any such minterm is called
an **essential prime implicant**

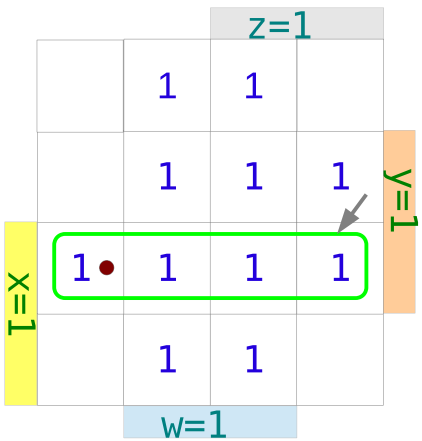
In this example, all three
prime implicants are essential

Essential Prime Implicant

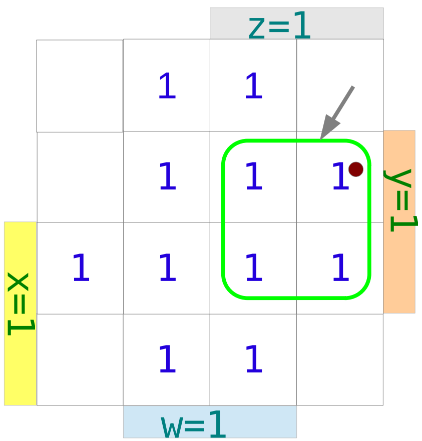
Essential Prime Implicant:
prime implicants
that cover an *output of the function*
that no combination of other implicants is able to cover



Essential Prime Implicant



Essential Prime Implicant



Essential Prime Implicant

2's Complement

Decimal to Binary (1)

Decimal to Binary (2)

Laplace Equation

Decimal

Laplace Equation

Laplace Equation

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”