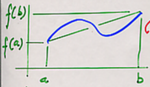
HW: $\frac{e^x - 1}{x} = \frac{1}{x}\left[e^x - 1\right] =: f(x)$

1) Expand e^x in Taylor series w/ remainder

$$R(x) = \frac{(x-0)^{n+1}}{(n+1)!} \exp[\xi(x)]$$

2) Find Taylor series exp. and remainder of $f(x)$. (4) p.6-3.

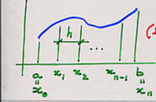
Trap. rule:



Simple rule:

(1) $I_1 = \frac{b-a}{2}\left[f(a) + f(b)\right]$

↑ 1 interval



Comp. rule:

(2) $I_n = h\left[\frac{1}{2} f_0 + f_1 + \cdots + f_{n-1} + \frac{1}{2} f_n\right]$

↑ n interv.

A. p.253 (5.1.5)

(1) $f_i := f(x_i)\ ,\ i = 0, \dots, n$

A. p. 256

Simpson's rule: use 2nd-order poly. (parabolas) to approx. f.

Simple: $I_2 = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$

2 intervals $[x_0, x_1]$ $[x_1, x_2]$ $h := \frac{b-a}{2}$ (2)

Composite: $I_n = \frac{h}{3}\left[f_0 + 4f_1 + 2f(x_2) + 4f_3 + 2f_4 + \cdots + 2f_{n-2} + 4f_{n-1} + f_n\right]$ (3)

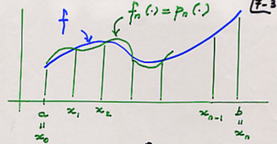
$n = 2k\ ,\ k = 1, 2, \dots$

Newton-Cotes formula:

History Simpson Cotes } Suli & Meyers (2003)

1) Approx. $f(\cdot)$ using Lagrange interp. funcs. $\Rightarrow f_n(\cdot)$

2) Int. $f_n(\cdot) \Rightarrow I_n = \int f_n(x)dx$



$$f_n(x) = p_n(x) = \sum_{i=0}^{n} l_{i,n}(x) f(x_i) \quad (1)$$

$$l_{i,n}(x) = l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \quad (2)$$

$l \equiv$ Lagrange

$$I(f) \cong I_n = \int_a^b p_n(x)$$

$$= \sum_{i=0}^{n} \underbrace{\left(\int_a^b l_i(x)\,dx \right)}_{W_i \text{ (weight)}} f(x_i) \quad (3)$$