Functor Overview (1A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

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8/2/18

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Typeclasses and Instances

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

such *behavior* is defined by

- **function definition**
- function **type declaration** only

a function definition

```
(==) :: a -> a -> Bool
x == y = not (x /= y)
```

- **a type declaration**

a function type

```
(==) :: a -> a -> Bool
```

- **a type declaration**

A function definition can be **overloaded**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Typeclasses and Type

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

a **type** is an **instance** of a **typeclass** implies

the function types declared by the **typeclass**
are defined (implemented) in the **instance**

so that we can use the functions
that the **typeclass** defines with that **type**

No relation with classes in Java or C++

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

A Concrete Type and a Type Constructor

a : a concrete type

Maybe : not a concrete type
: a **type constructor** that takes one parameter
in order to produces a concrete type.

Maybe a : a concrete type

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe

Maybe : Algebraic Data Type (ADT)

widely used because it effectively extends a type Integer
into a new **context** in which it has an extra value (**Nothing**)
that represents *a lack of value*

can check for that extra value (**Nothing**)
before accessing the possible Integer

good for debugging

many other languages have this sort of "***no-value***" value via NULL references.

the **Maybe** functor handle this ***no-value*** more effectively.

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

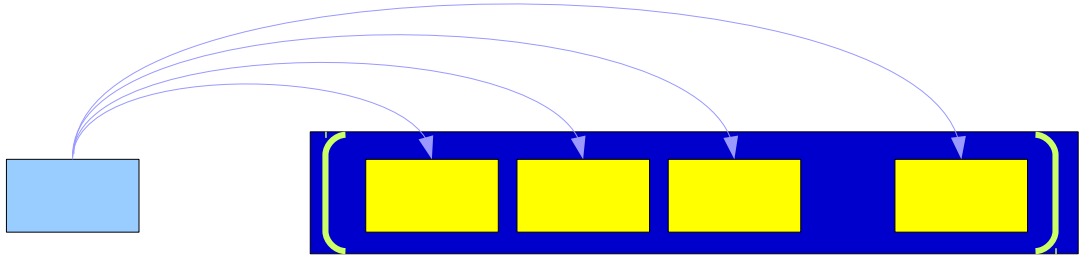
Functor typeclass – “mapped over”

the **Functor typeclass** is basically

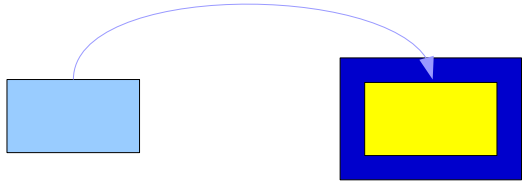
for things that can be *mapped over*

ex) mapping over **list**s

the **list** type is a Functor typeclass



List Functor



Maybe Functor

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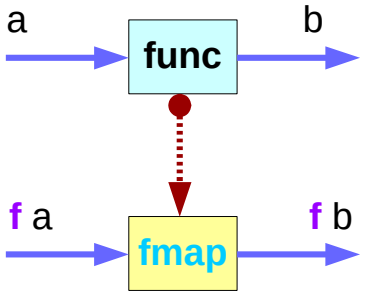
Functor typeclass – instances

class Functor f

instance Functor Maybe
instance Functor [ ]



function fmap
function func
type constructor f

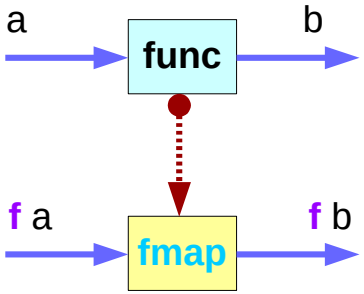
f is a **type constructor** taking one type parameter

Maybe instance of the **Functor typeclass**
[ ] instance of the **Functor typeclass**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Functor typeclass – fmap defined

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```



The **Functor typeclass**

defines the function **fmap**

without a default implementation

function fmap
function func
type constructor f

the **type variable f**

f is not a concrete type (**f** alone cannot hold a **value**)

f is a **type constructor** taking one type parameter

Maybe Int : a concrete type (a concrete type can hold a **value**)

Maybe : a **type constructor** that takes one type as the parameter

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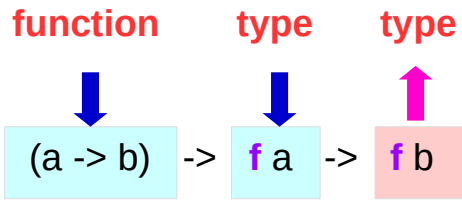
Function map & fmap

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```



func

fmap takes

- a **function** from **one type** to **another** (a -> b)
- a **Functor f** applied with **one type** (**f** a)

fmap returns

- a **Functor f** applied with **another type** (**f** b)

```
map :: (a -> b) -> [a] -> [b]
```

map takes

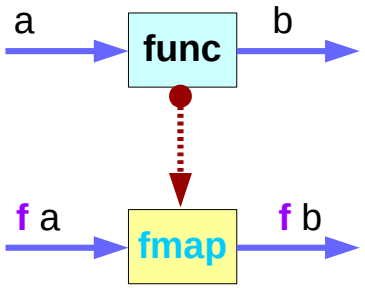
- a function from one type to another (* 2)
- take a list of one type [1, 2, 3]
- returns a list of another type [2, 4, 6]

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

List : an instance of the Functor typeclass

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
    map :: (a -> b) -> [a] -> [b]
```

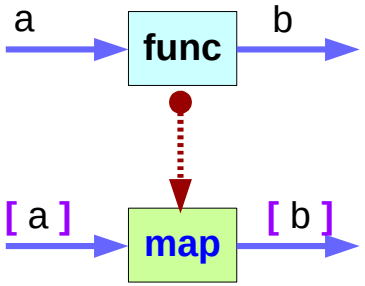


function fmap
function func
type constructor f

map is just a **fmap** that works only on **list**s

a **list** is an **instance** of the **Functor typeclass**.

```
instance Functor [ ] where
    fmap = map
```



f : a type constructor that takes one type
[ ] : a type constructor that takes one type
[a] : a concrete type (**[Int]**, **[String]** or **[[String]]**)

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List Examples

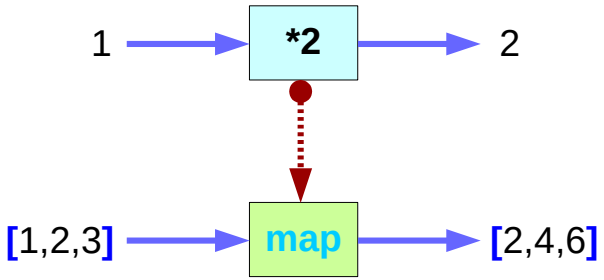
```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
  map :: (a -> b) -> [a] -> [b]
```

```
instance Functor [ ] where
  fmap = map
```

```
map :: (a -> b) -> [a] -> [b]
ghci> fmap (*2) [1..3]
[2,4,6]
ghci> map (*2) [1..3]
[2,4,6]
```

function fmap	map
function func	(*2)
type constructor f	[ ]



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Maybe : an instance of the Functor typeclass

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

f	⟺	Maybe
f a	⟺	Maybe a
f b	⟺	Maybe b

(a -> b)	⟺	func

instance : implementing fmap

func :: (a -> b)

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

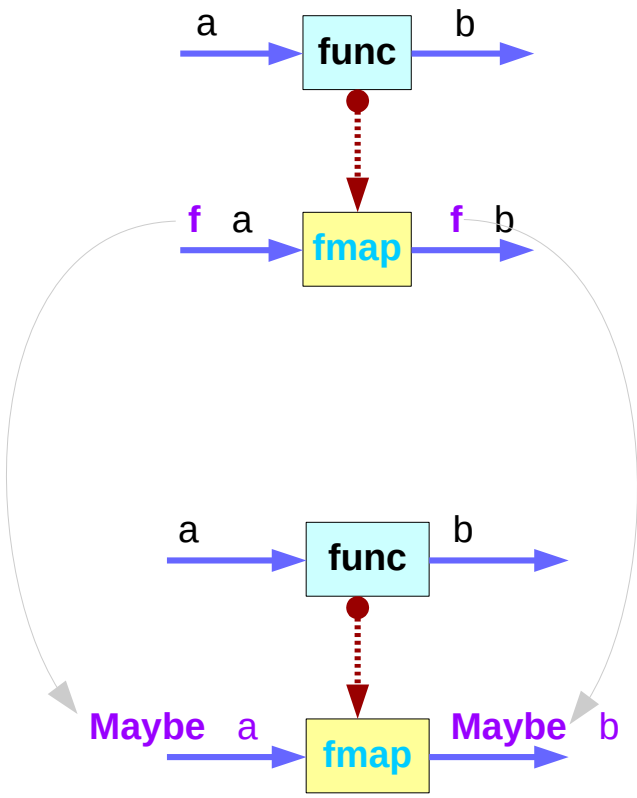
f : a type variable (parameter)

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

f : a type variable

f ⟺ **Maybe**

```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```



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f : a type constructor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

f : a **type constructor** taking one type parameter

type	**f** a	⟺	**Maybe** a	type
type	**f** b	⟺	**Maybe** b	type


```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

f	~~type~~	**Maybe**	~~type~~
f a	type	**Maybe a**	type
f Int	type	**Maybe Int**	type

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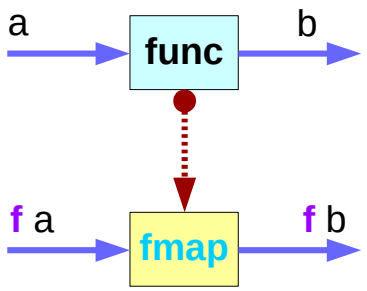
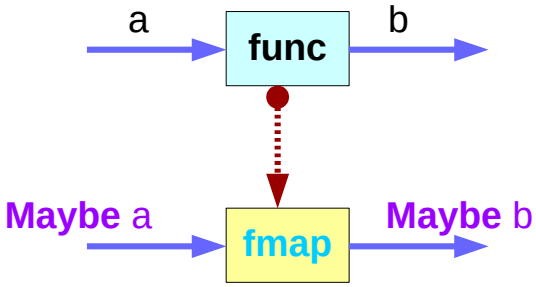
f and Maybe

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap func (Just x) = Just (func x)
  fmap func Nothing = Nothing
```





f : a **type variable**

f : a **type constructor** taking one type parameter

Maybe : an **instance** of **Functor typeclass**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe : an argument to fmap, together with a

```
fmap :: (a -> b) -> f a -> f b
```

f ⟵ Maybe

```
fmap :: (a -> b) -> Maybe a -> Maybe b
fmap func (Just x) = Just (func  x)
fmap func Nothing = Nothing
```

func ⟵ f

```
fmap :: (a -> b) -> Maybe a -> Maybe b
fmap f (Just x) = Just (f  x)
fmap f Nothing = Nothing
```

```
class Functor f
instance Functor Maybe
```

```
func :: a -> b
f    :: a -> b
```

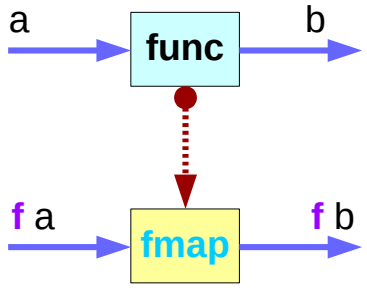
http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe : an argument to fmap, together with a

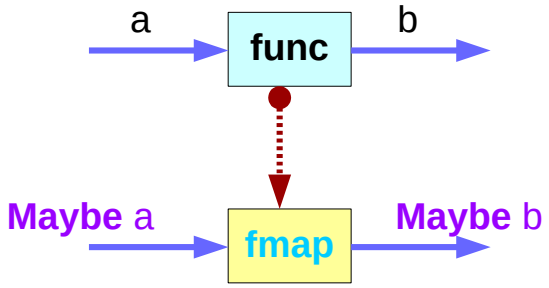
```
class Functor f where
   fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
   fmap func (Just x) = Just (func  x)
   fmap func Nothing = Nothing
```

```
fmap :: (a -> b) -> f a -> f b
```




```
fmap :: (a -> b) -> Maybe a -> Maybe b
```



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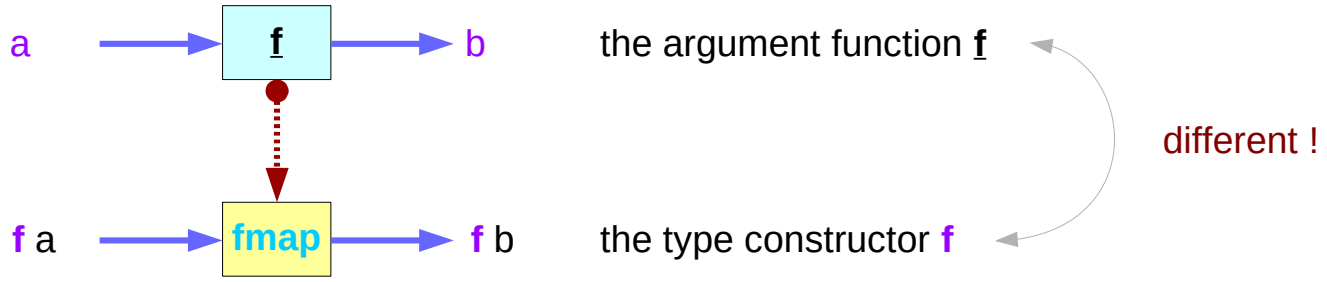
The distinct two f's

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

Functor f

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

f :: (a->b)



An argument f to fmap vs. Functor f

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

f

f : a **type variable**

f : a **type constructor** taking one type parameter

```
instance Functor Maybe where
  fmap func (Just x) = Just (func x)
  fmap func Nothing = Nothing
```

func

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

f

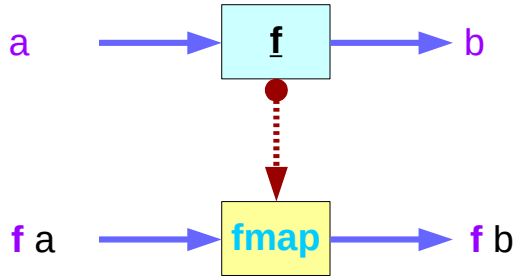
f an argument function to fmap

f is different from the type constructor f

f : a -> b ⟺ func : a -> b

Maybe Functor (Instance)

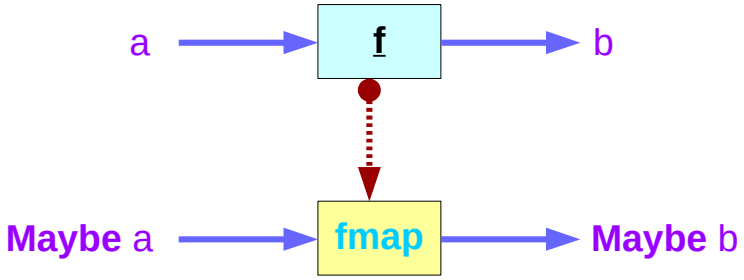
Typeclass

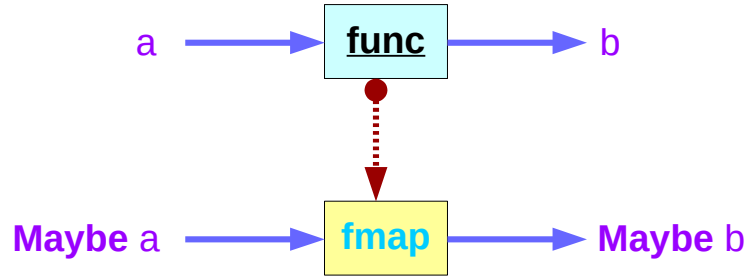



```
class Functor f

instance Functor Maybe
instance Functor [ ]
```

Instance





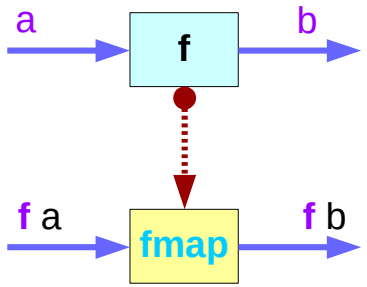
Maybe Functor Examples (1)

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

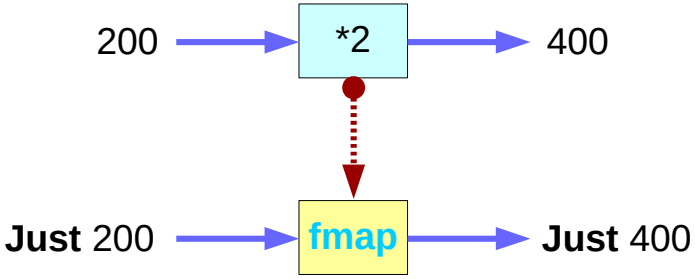
f

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

f




```
ghci> fmap (*2) (Just 200)
Just 400
ghci> fmap (*2) Nothing
Nothing
```



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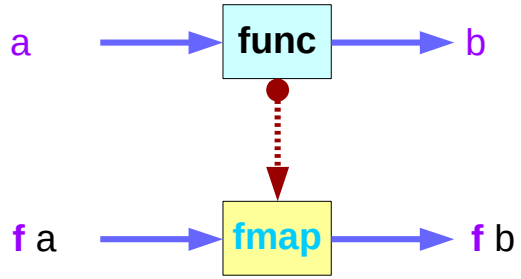
Maybe Functor Examples (2)

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

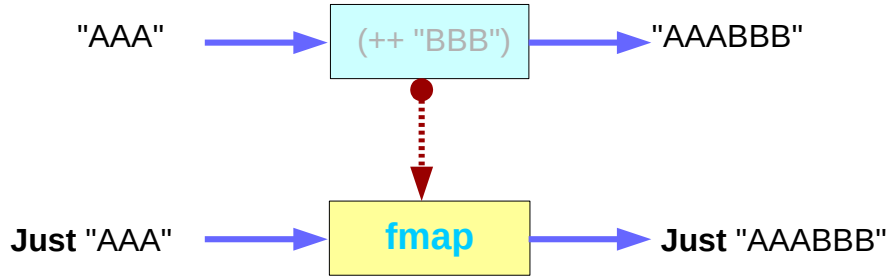
f

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

f




```
ghci> fmap (++ "BBB") (Just "AAA")
Just "AAABBB"
ghci> fmap (++ "BBB") Nothing
Nothing
```



http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe as a functor

Functor typeclass:

- transforming one type to another
- transforming operations of one type to those of another

Int → Maybe Int
(*2) → fmap (*2)

Maybe a is an **instance** of a **functor type class**

Functor instance

Functor provides **fmap** method
maps functions of the base type (such as Integer)
to *functions* of the lifted type (such as **Maybe** Integer).

(*2) **5**
fmap (*2) **Maybe 5**

(*2) :: **Int ->** **Int** *base type function*

fmap (*2) :: Maybe Int -> Maybe Int *lifted type function*

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

A *function* **f** transformed with **fmap**
can work on a **Maybe** value

pattern matching is used

```
case maybeVal of
  Nothing  -> Nothing          -- there is nothing, so just return Nothing
  Just val -> Just (f val)     -- there is a value, so apply the function to it
```

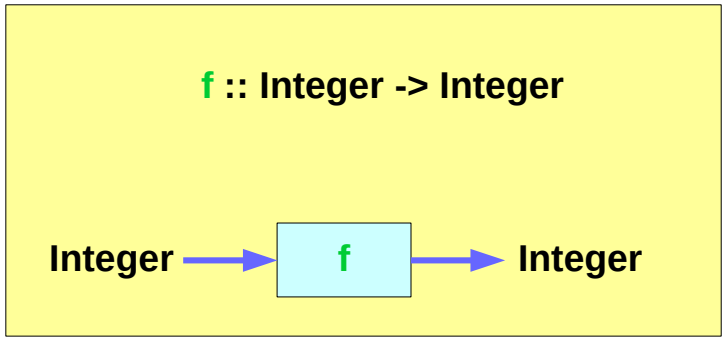
```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

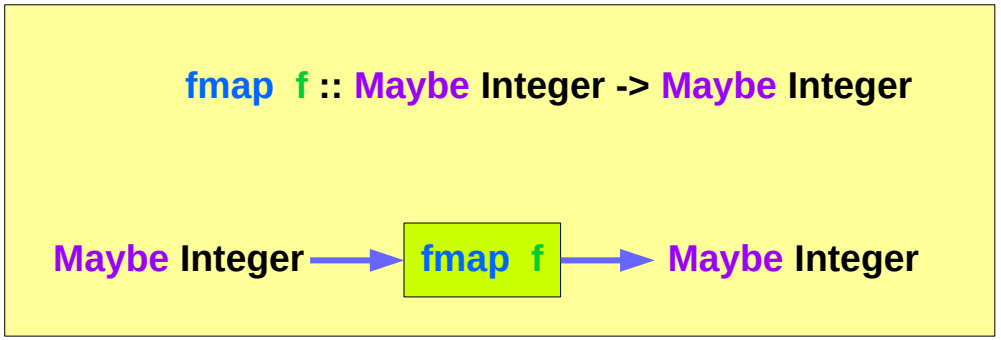
Maybe as a functor

A *function* **f** transformed with **fmap**
cab work on a **Maybe** **value**

base type function



lifted type function



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Maybe as a functor

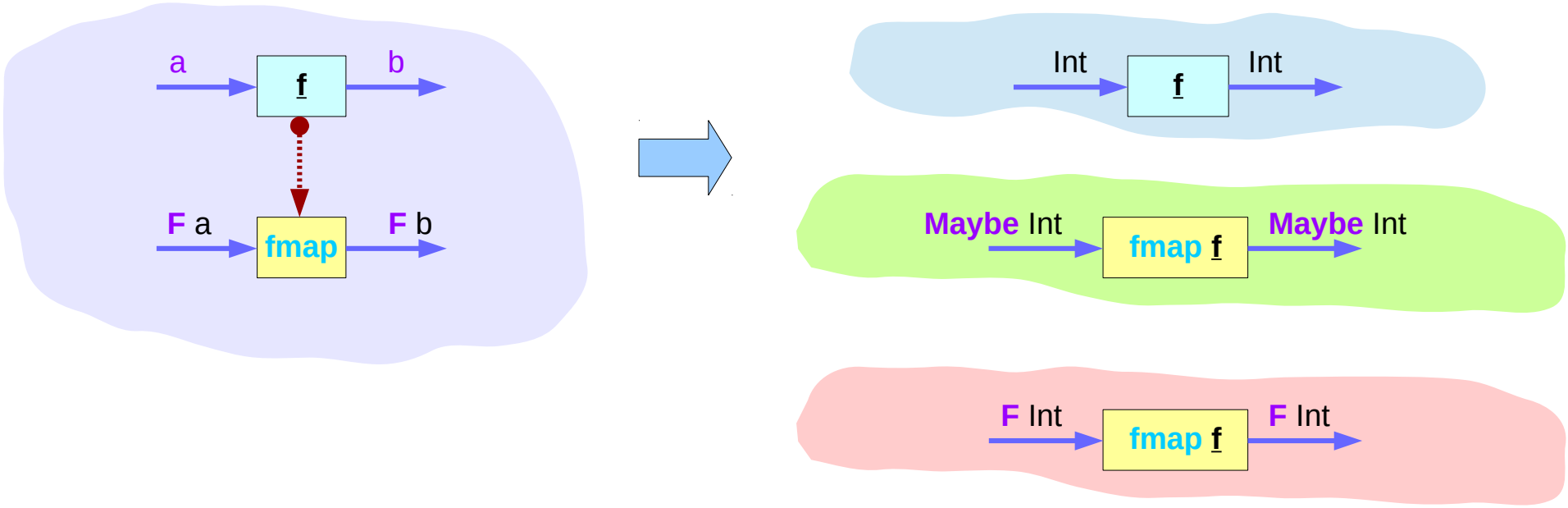
A *function* **f** transformed with **fmap**
to work on a **Maybe** value

```
      f          :: Integer -> Integer

fmap  f          :: Maybe Integer -> Maybe Integer

m_x              :: Maybe Integer

fmap  f  m_x     :: Maybe Integer
```

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Transforming operations

Functor provides **fmap** method
maps *functions* of the base type (such as Integer)
to *functions* of the lifted type (such as **Maybe** Integer).



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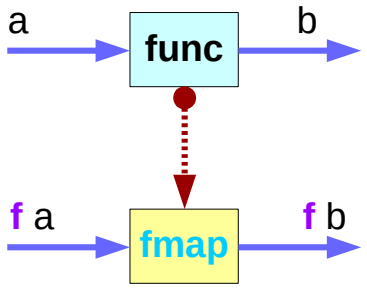
fmap func

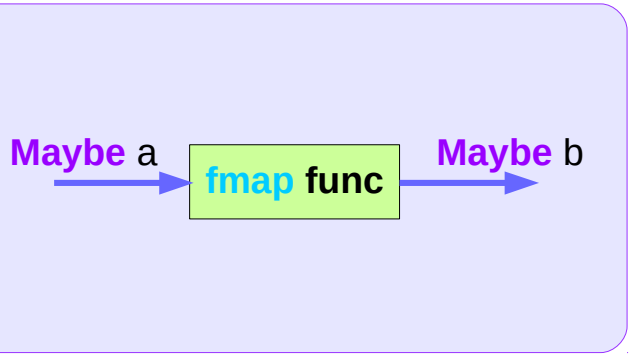
```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

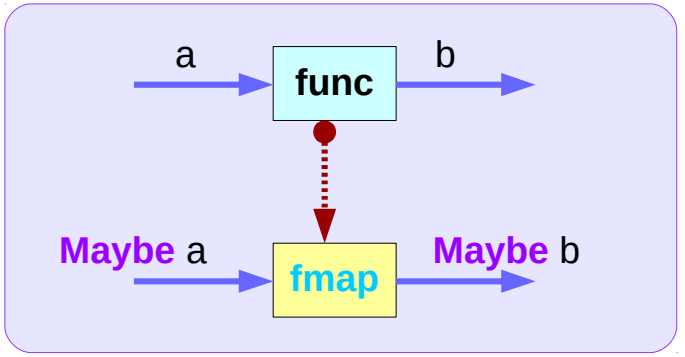
```
instance Functor Maybe where
  fmap func (Just x)  = Just (func  x)
  fmap func Nothing = Nothing
```



fmap func Just x

fmap func Just x





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Apply a function to lifted type values

m_x :: Maybe Integer **(Just 101, Nothing, …)**

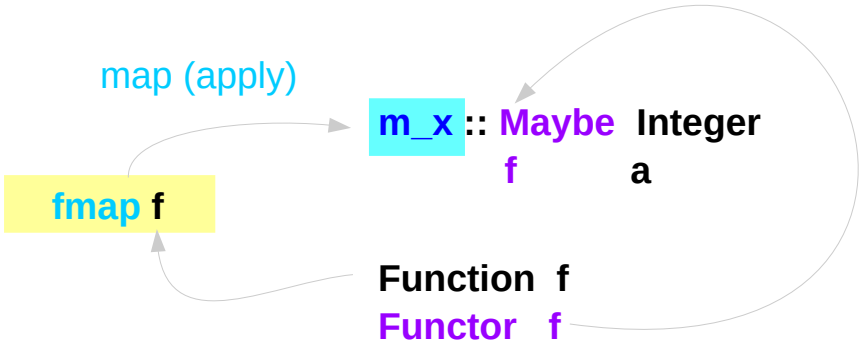
f :: Int -> Int

fmap f m_x

to apply the function **f** directly to the **Maybe Integer** without concerning whether it is **Nothing** or not

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```



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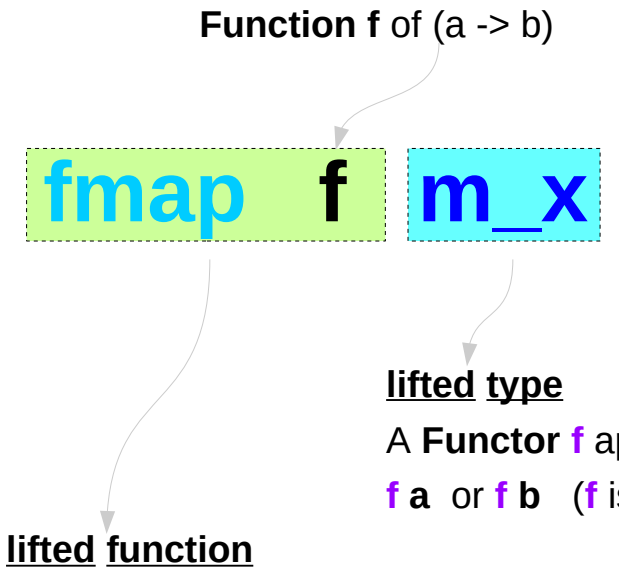
Maybe as a functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```



lifted type

A **Functor f** applied with a type **a** or **b**

f a or **f b** (**f** is not the function **f**)

```
          f       a
m_x :: Maybe  Integer
```

Function f

Functor f

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

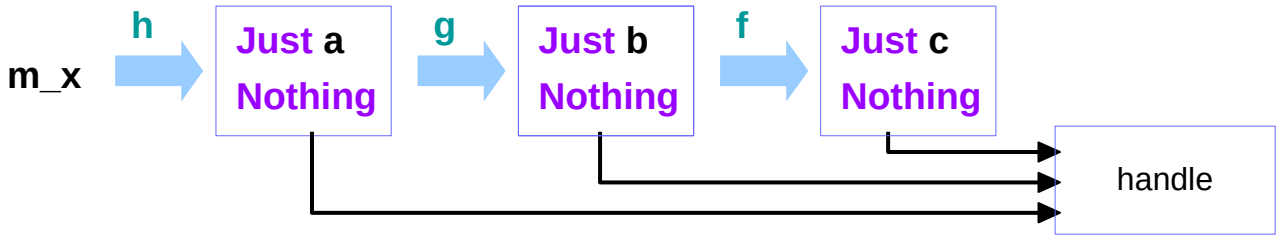
Maybe as a functor

If there a **Maybe Int** value **m_x**
and an **Int -> Int** function **f**,
fmap f m_x can be used
to apply the function **f** directly
to the **Maybe Int** value **m_x**
without worrying if **m_x** actually
has a value or not.

when a whole chain of
lifted **Integer -> Integer** functions
is applied to **Maybe Int** values,
an explicit checking and handling for **Nothing**
can be deferred until the last stage.

A chain of lifted **Integer -> Integer** functions

```
f . g . h x

fmap f . fmap g . fmap h   m_x
```



https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe instances

Maybe is

- an instance of **Eq** and **Ord** (as a base type)
- an instance of **Functor**
- an instance of **Monad**

https://wiki.haskell.org/Maybe

Maybe class

The Maybe type definition

```
data Maybe a = Just a | Nothing
   deriving (Eq, Ord)
```

Maybe is
an instance of **Eq** and **Ord** (as a base type)

https://wiki.haskell.org/Maybe

Maybe Functor

For **Functor**, the **fmap**
moves **f** inside the **Just** constructor
is identity on the **Nothing** constructor.

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

fmap f (Just x) = **Just** (**f** x)
fmap f Nothing = **Nothing**

https://wiki.haskell.org/Maybe

maybe library function

maybe :: b -> (a->b) -> Maybe a -> b

The maybe function takes
    a default value (b),
    a function (a->b), and
    a Maybe value (Maybe a).

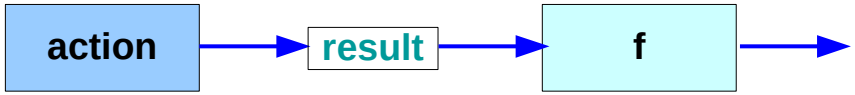
If the Maybe value is **Nothing**,
    the function returns the default value.
Otherwise, it applies the function
    to the value inside the **Just** and returns the result.

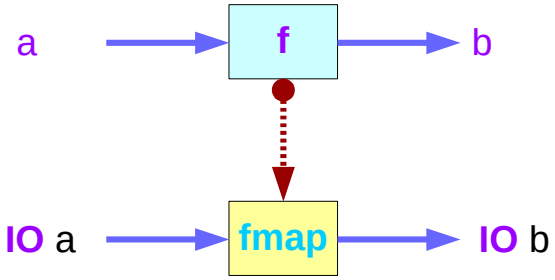
```
>>> maybe False odd (Just 3)
True

>>> maybe False odd Nothing
False
```

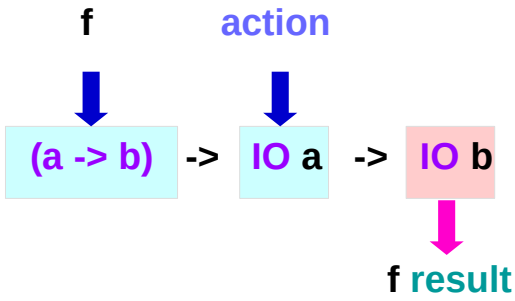
https://hackage.haskell.org/package/base-4.10.0.0/docs/Data-Maybe.html

IO Functor

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```








```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

IO Functor Example

```
main = do line <- getLine
          let line' = reverse line
          putStrLn $ "You said " ++ line' ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line' ++ " backwards!"
```

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```

```
main = do line <- fmap reverse getLine
          putStrLn $ "You said " ++ line ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line ++ " backwards!"
```

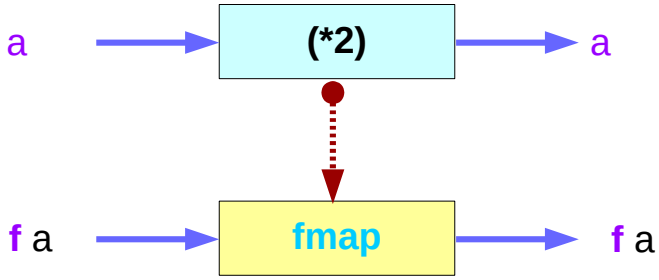
```
fmap f        action  = do
fmap reverse getLine = do
  result <- getLine
  return (reverse result)
```

```
getLine :: IO String
result :: String
reverse :: [a] -> [b]
```

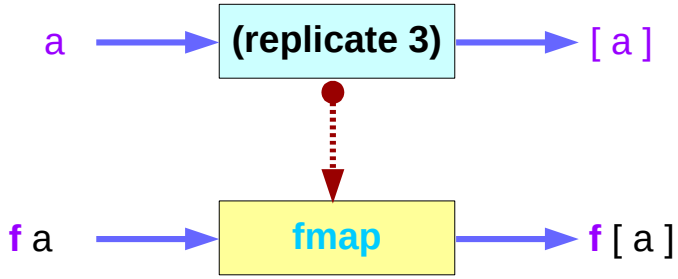
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Typeclass Examples (5)

ghci> :t fmap (*2)
fmap (*2) :: (Num a, Functor f) => f a -> f a



ghci> :t fmap (replicate 3)
fmap (replicate 3) :: (Functor f) => f a -> f [a]



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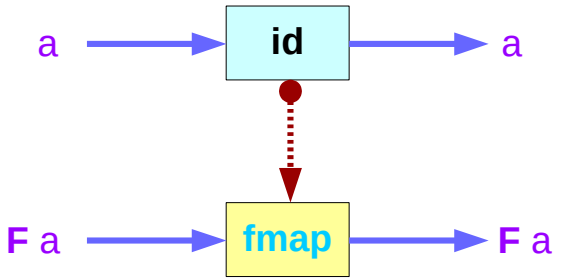
Functor Typeclass Examples (6)

```
ghci> fmap (replicate 3) [1,2,3,4]
[[1,1,1],[2,2,2],[3,3,3],[4,4,4]]

ghci> fmap (replicate 3) (Just 4)
Just [4,4,4]

ghci> fmap (replicate 3) (Right "blah")
Right ["blah","blah","blah"]

ghci> fmap (replicate 3) Nothing
Nothing

ghci> fmap (replicate 3) (Left "foo")
Left "foo"
```

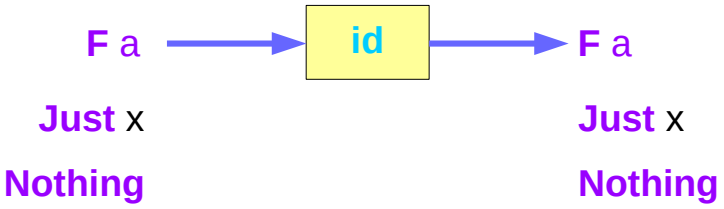
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Laws

```
fmap id = id

id :: a -> a
id    x = x
```




```
instance Functor Maybe where
   fmap func (Just x) = Just (func  x)
   fmap func Nothing = Nothing

instance Functor Maybe where
   fmap f (Just x) = Just (f x)
   fmap f Nothing = Nothing

instance Functor Maybe where
   fmap id (Just x) = Just (id x)
   fmap id Nothing = Nothing
```



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Functor Typeclass

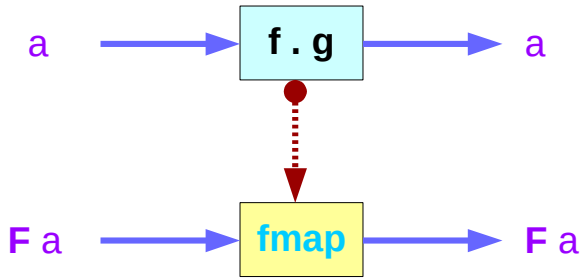
```
ghci> fmap id (Just 3)
Just 3
ghci> id (Just 3)
Just 3
ghci> fmap id [1..5]
[1,2,3,4,5]
ghci> id [1..5]
[1,2,3,4,5]
ghci> fmap id []
[]
ghci> fmap id Nothing
Nothing
```

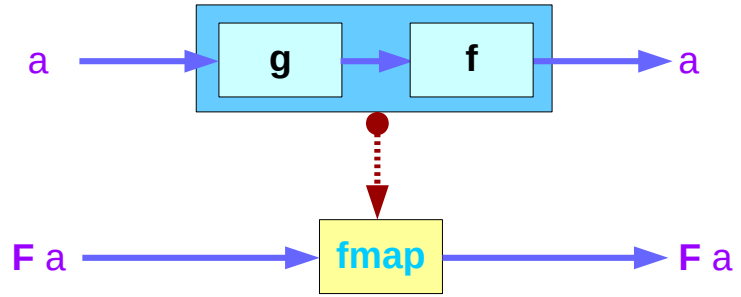
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

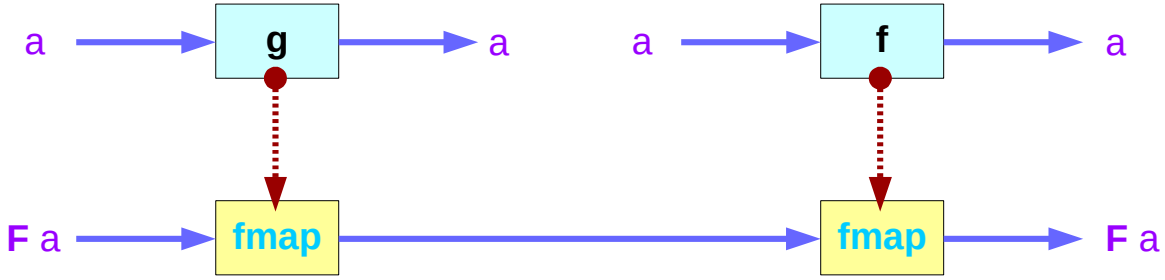
Functor Laws

fmap (f . g) = fmap f . fmap g

fmap (f . g) F = fmap f (fmap g F)







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Functor Laws

```
fmap (f . g) = fmap f . fmap g
fmap (f . g) F = fmap f (fmap g F)

  instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

```
fmap (f . g) Nothing = Nothing
fmap f (fmap g Nothing) = Nothing

fmap (f . g) (Just x) = Just ((f . g) x) = Just (f (g x))
fmap f (fmap g (Just x)) = fmap f (Just (g x)) = Just (f (g x))
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf