

Laurent Series and z-Transform

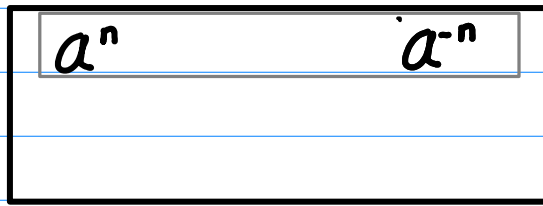
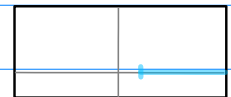
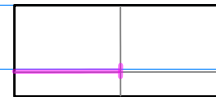
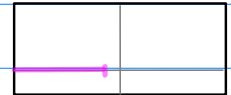
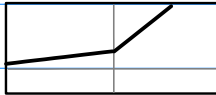
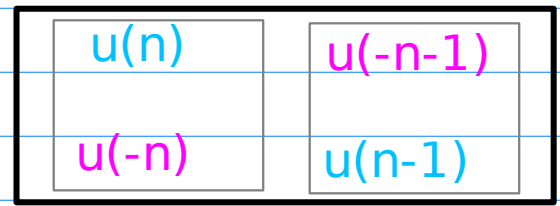
- Geometric Series

Permutations B

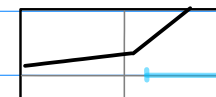
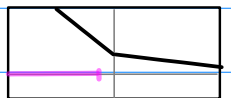
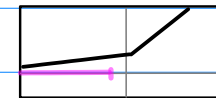
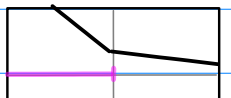
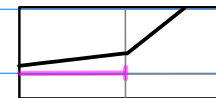
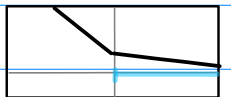
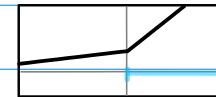
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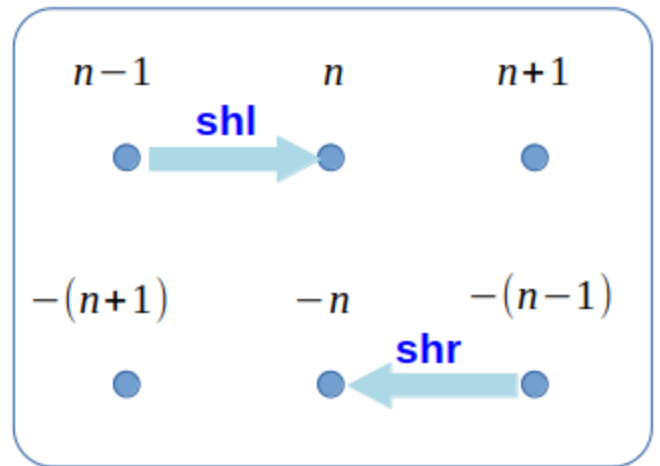
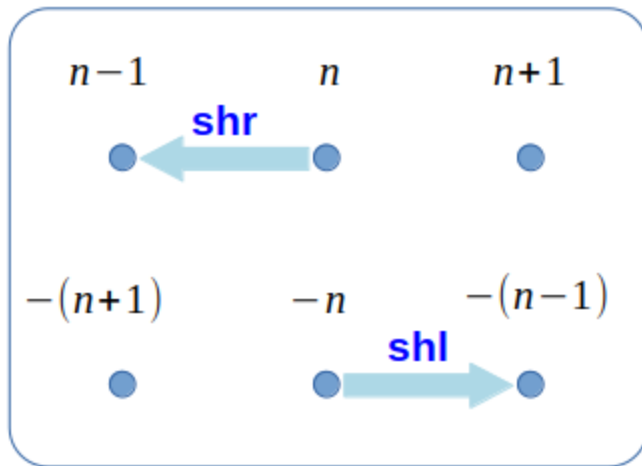
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a^n $\times$ $R(n)$  $\times$ 

- | | | | |
|-----|---------------|------------------|-----|
| (1) | $a^n u(n)$ | $a^{-n} u(n)$ | (2) |
| (3) | $a^n u(-n)$ | $a^{-n} u(-n)$ | (4) |
| (5) | $a^n u(-n-1)$ | $a^{-n} u(-n-1)$ | (6) |
| (7) | $a^n u(n-1)$ | $a^{-n} u(n-1)$ | (8) |





$$n \leftarrow n-1$$

$$n \leftarrow n+1$$

$$f(n-1) \xleftarrow{\text{shr}} f(n)$$

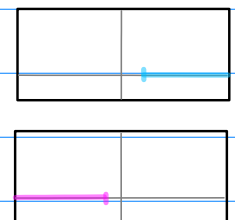
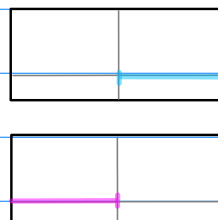
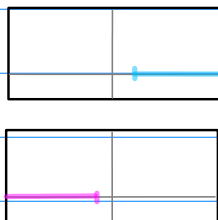
$$f(n) \xleftarrow{\text{shl}} f(n-1)$$

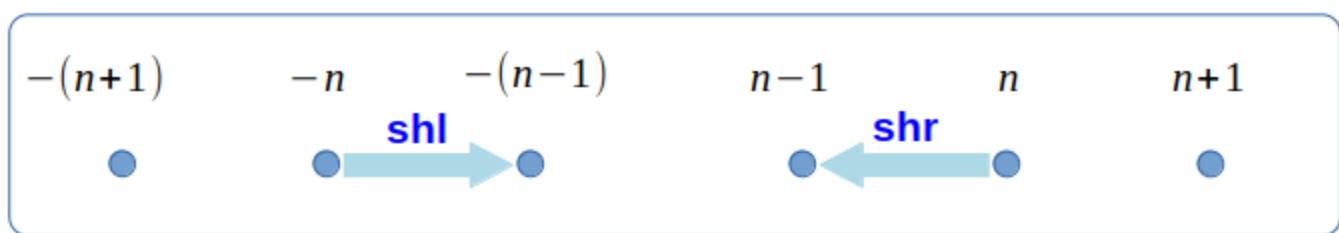
$$f(-n+1) \xleftarrow{\text{shl}} f(-n)$$

$$f(-n) \xleftarrow{\text{shr}} f(-n+1)$$

$$-n \leftarrow -n+1$$

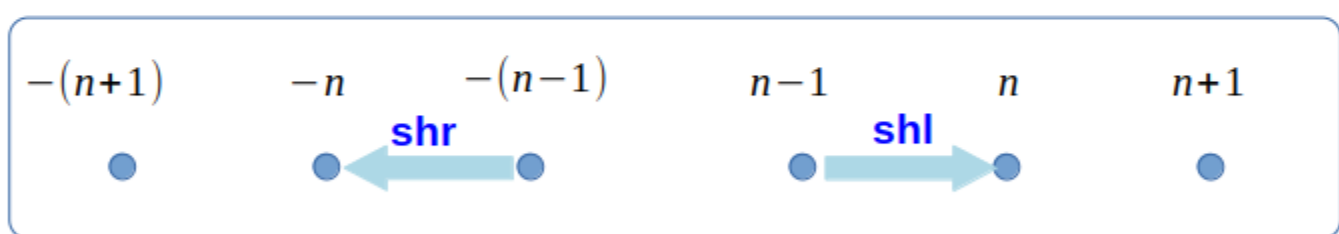
$$-n \leftarrow -n-1$$





$$-n \leftarrow -(n+1) \qquad n \leftarrow n-1$$

$$f(-(n-1)) \xleftarrow{\text{shl}} f(-n) \qquad f(n-1) \xleftarrow{\text{shr}} f(n)$$



$$-n \leftarrow -(n-1) \qquad n \leftarrow n+1$$

$$f(-n) \xleftarrow{\text{shr}} f(-(n-1)) \qquad f(n) \xleftarrow{\text{shl}} f(n-1)$$

$$a^{(n-1)} \quad a^n$$



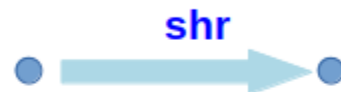
$$a^{-(n-1)} \quad a^{-n}$$



$$a^{(n-1)} \quad a^n$$



$$a^{-(n-1)} \quad a^{-n}$$



$$a^{(n+1)} \quad a^n$$



$$a^{-(n+1)} \quad a^{-n}$$



$$a^{(n+1)} \quad a^n$$



$$a^{-(n+1)} \quad a^{-n}$$



Exponent Shifting

$$a^{(n-1)} \quad a^n$$

A light blue arrow points from the right dot to the left dot, with the label "shr" in blue above it.

$$a^{-(n-1)} \quad a^{-n}$$

A light blue arrow points from the right dot to the left dot, with the label "shr" in blue above it.

$$a^{(n-1)} \quad a^n$$

A light blue arrow points from the left dot to the right dot, with the label "shl" in blue above it.

$$a^{-(n-1)} \quad a^{-n}$$

A light blue arrow points from the left dot to the right dot, with the label "shl" in blue above it.

$$\text{shl}(b^n) = b^{(n+1)}$$

$$b^n \quad b^{n+1}$$

$$a^n \quad a^{(n+1)}$$

$$a^{-n} \quad a^{-(n+1)}$$

$$\text{shr}(b^n) = b^{(n-1)}$$

$$b^n \quad b^{n-1}$$

$$a^{(n+1)} \quad a^n$$

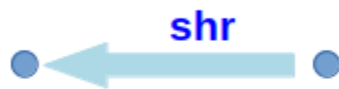
$$a^{-(n+1)} \quad a^{-n}$$

$$\begin{array}{l}
 a^n \\
 a^{-n} \\
 a^{(n+1)} \\
 a^{-(n+1)}
 \end{array}
 \begin{array}{l}
 \searrow \\
 \searrow
 \end{array}
 \begin{array}{l}
 a^n \\
 a^{-n} \\
 a^{(n+1)} \\
 a^{-(n+1)}
 \end{array}$$

$$\begin{array}{l}
 a^n \\
 a^{-n} \\
 a^{(n+1)} \\
 a^{-(n+1)}
 \end{array}
 \begin{array}{l}
 \nearrow \\
 \nearrow
 \end{array}
 \begin{array}{l}
 a^n \\
 a^{-n} \\
 a^{(n+1)} \\
 a^{-(n+1)}
 \end{array}$$

Range Shifting

$$u((n-1)) \quad u(n)$$



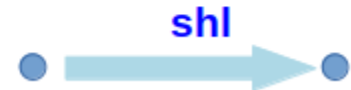
$$u(-(n-1)) \quad u(-n)$$



$$u((n-1)) \quad u(n)$$



$$u(-(n-1)) \quad u(-n)$$

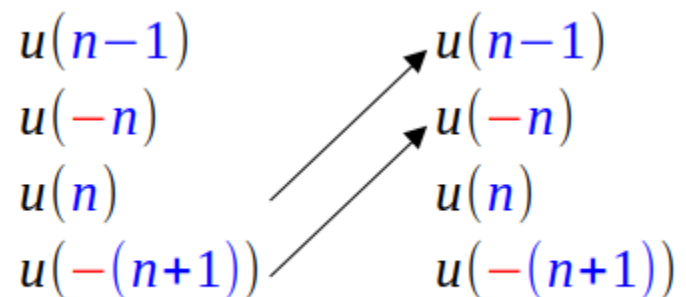
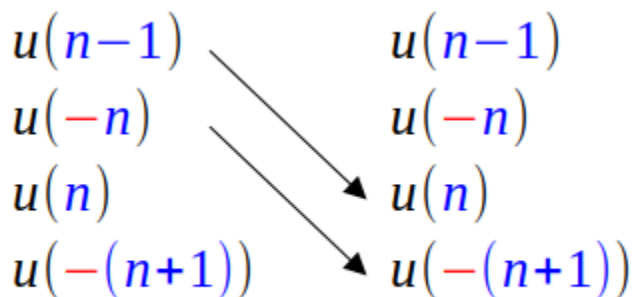


$$\text{shl}(R(n)) = R(n+1)$$

$R(n)$	$R(n+1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-(n+1))$

$$\text{shr}(R(n)) = R(n-1)$$

$R(n)$	$R(n-1)$
$u(n)$	$u(n-1)$
$u(-(n+1))$	$u(-n)$



$$b^n \xrightarrow{\text{sh } l(b^n)} b^{n+1}$$

$$b^n \xrightarrow{\text{sh } r(b^n)} b^{n-1}$$

$$\begin{array}{c} \text{sh } l(b^n) = b^{(n+1)} \\ \hline b^n \qquad \qquad b^{n+1} \\ \hline a^n \qquad \qquad a^{(n+1)} \\ \hline a^{-n} \qquad \qquad a^{-(n+1)} \\ \hline \end{array}$$

$$\begin{array}{c} \text{sh } r(b^n) = b^{(n-1)} \\ \hline b^n \qquad \qquad b^{n-1} \\ \hline a^{(n+1)} \qquad \qquad a^n \\ \hline a^{-(n+1)} \qquad \qquad a^{-n} \\ \hline \end{array}$$

$$R(n) \xrightarrow{\text{sh } l(R(n))} R(n+1)$$

$$R(n) \xrightarrow{\text{sh } r(R(n))} R(n-1)$$

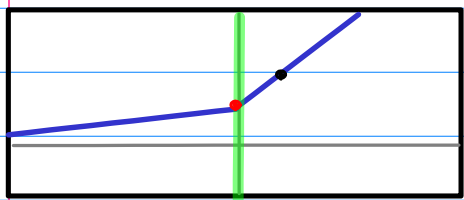
$$\begin{array}{c} \text{sh } l(R(n)) = R(n+1) \\ \hline R(n) \qquad \qquad R(n+1) \\ \hline u(n-1) \qquad \qquad u(n) \\ \hline u(-n) \qquad \qquad u(-(n+1)) \\ \hline \end{array}$$

$$\begin{array}{c} \text{sh } r(R(n)) = R(n-1) \\ \hline R(n) \qquad \qquad R(n-1) \\ \hline u(n) \qquad \qquad u(n-1) \\ \hline u(-(n+1)) \qquad \qquad u(-n) \\ \hline \end{array}$$

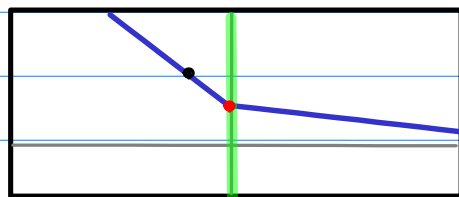
Exponent Shifting

$$shl(b^n) = b^{n+1}$$

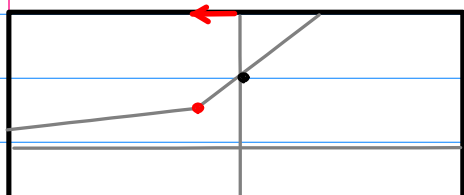
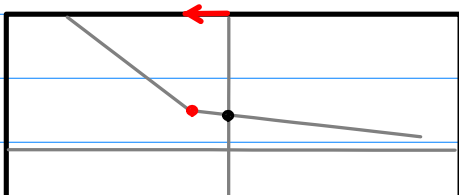
$$shr(b^n) = b^{n-1}$$

 2^n


shift right $n \leftarrow n-1$  shift left $n \leftarrow n+1$ 


 2^{-n}

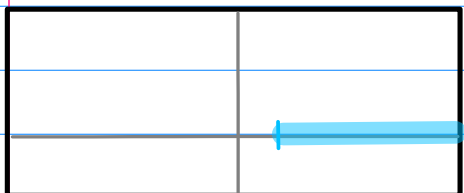
shift right $n \leftarrow n-1$  shift left $n \leftarrow n+1$ 

 2^{n+1}

 $2^{-(n+1)}$


Range Shifting

$$shl(R(n)) = R(n+1)$$

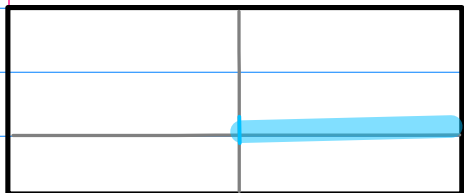
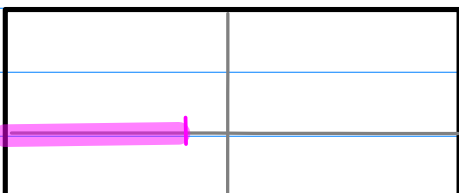
$$shr(R(n)) = R(n-1)$$

 $u(n-1)$


shift right $n \leftarrow n-1$  shift left $n \leftarrow n+1$ 

 $u(-n)$


shift right $n \leftarrow n-1$  shift left $n \leftarrow n+1$ 

 $u(n)$

 $u(-(n+1))$


$$a^n$$

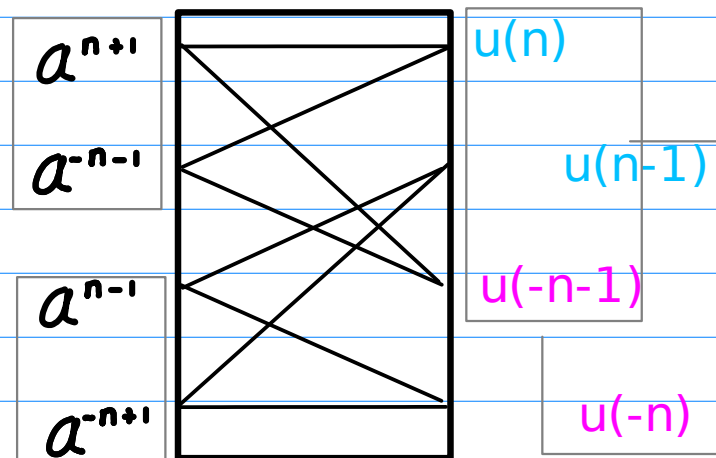
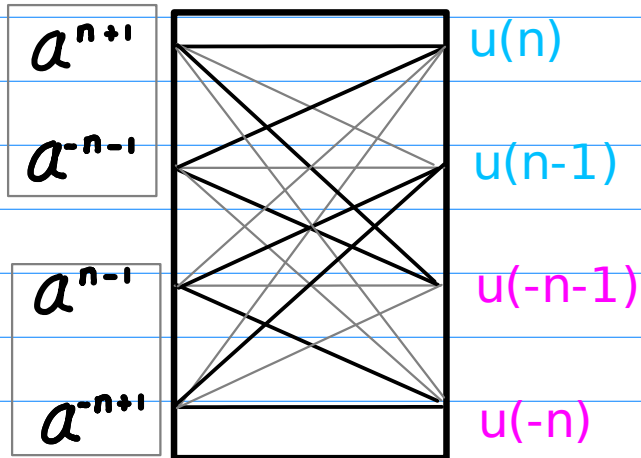
 $\times$

$$R(n)$$

$$\begin{array}{|c|c|} \hline a^{n+1} & a^{-n-1} \\ \hline a^{n-1} & a^{-n+1} \\ \hline \end{array}$$

 $\times$

$$\begin{array}{|c|c|} \hline u(n) & u(-n-1) \\ \hline u(n-1) & u(-n) \\ \hline \end{array}$$



$$\begin{array}{l} n \leftarrow n+2 \text{ or} \\ n \leftarrow n-2 \end{array}$$

$$\begin{array}{|c|c|c|} \hline (1') & a^{n+1}u(n) & a^{-n-1}u(n) & (2') \\ \hline (3') & a^{n-1}u(-n) & a^{-n+1}u(-n) & (4') \\ \hline (5') & a^{n+1}u(-n-1) & a^{-n-1}u(-n-1) & (6') \\ \hline (7') & a^{n-1}u(n-1) & a^{-n+1}u(n-1) & (8') \\ \hline \end{array}$$

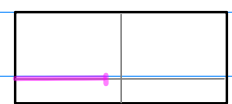
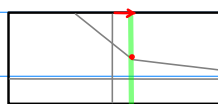
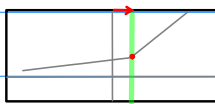
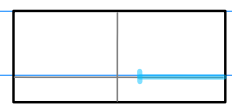
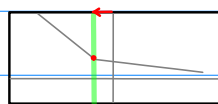
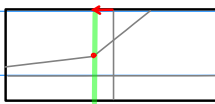
$$\begin{array}{|c|c|c|} \hline (1'') & a^{n-1}u(n) & a^{-n+1}u(n) & (2'') \\ \hline (3'') & a^{n+1}u(-n) & a^{-n-1}u(-n) & (4'') \\ \hline (5'') & a^{n-1}u(-n-1) & a^{-n+1}u(-n-1) & (6'') \\ \hline (7'') & a^{n+1}u(n-1) & a^{-n-1}u(n-1) & (8'') \\ \hline \end{array}$$

a^n $\times$ $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

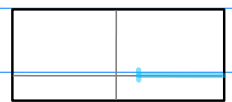
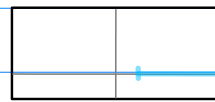
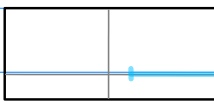
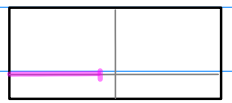
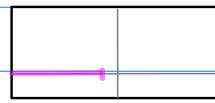
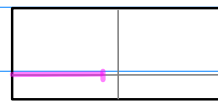
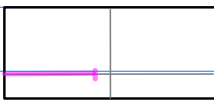
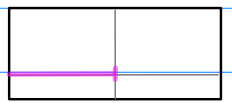
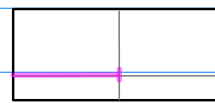
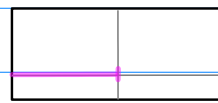
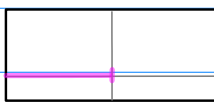
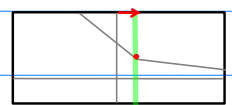
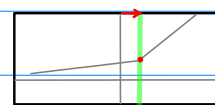
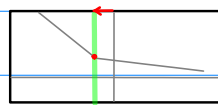
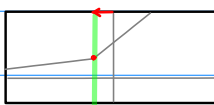
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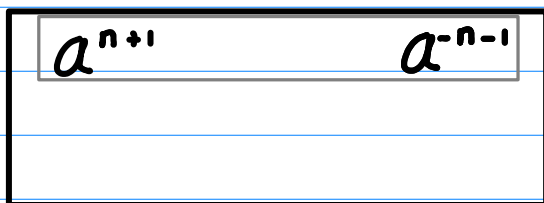
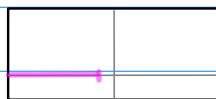
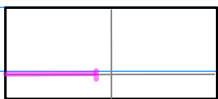
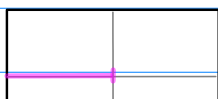
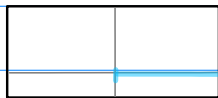
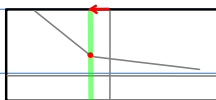
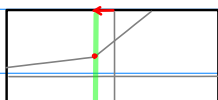
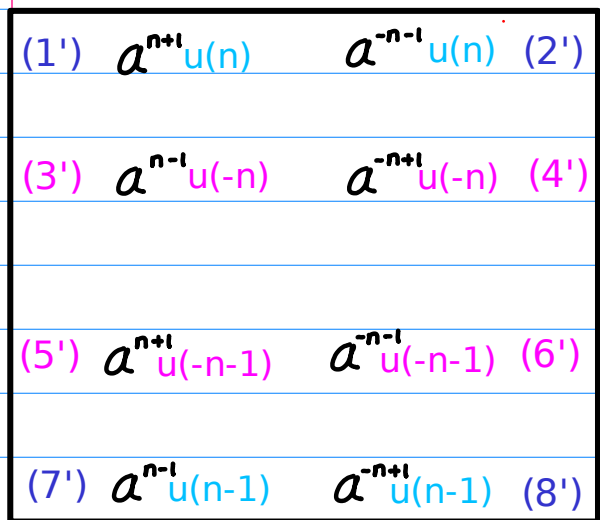
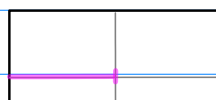
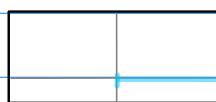
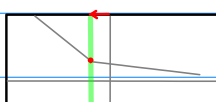
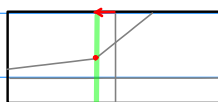
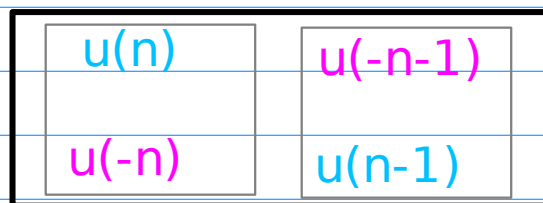
$u(n)$	$u(-n-1)$
$u(-n)$	$u(n-1)$

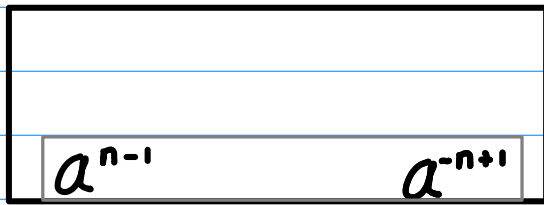
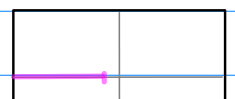
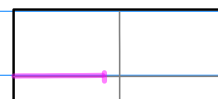
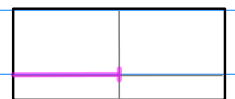
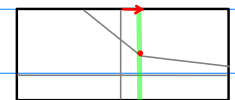
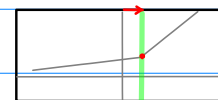
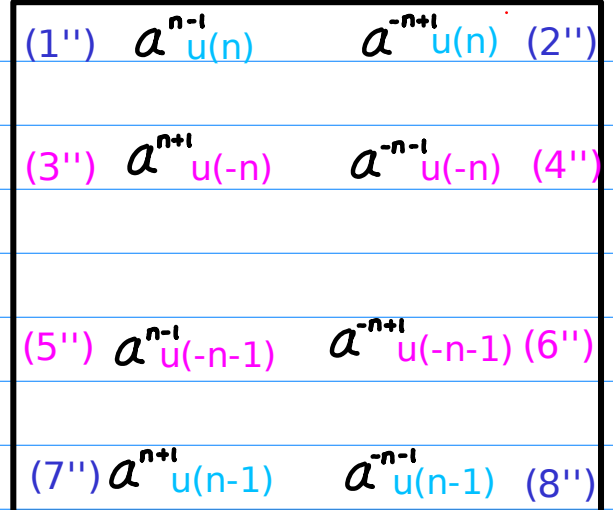
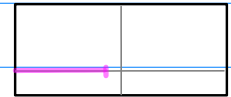
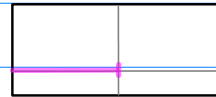
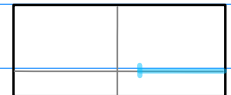
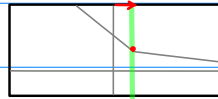
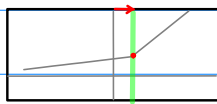
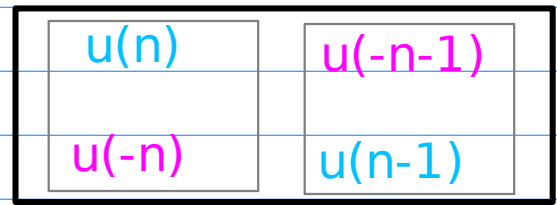


(1')	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	(2')
(3')	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	(4')
(5')	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	(6')
(7')	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	(8')

(1'')	$a^{n-1}u(n)$	$a^{-n+1}u(n)$	(2'')
(3'')	$a^{n+1}u(-n)$	$a^{-n-1}u(-n)$	(4'')
(5'')	$a^{n-1}u(-n-1)$	$a^{-n+1}u(-n-1)$	(6'')
(7'')	$a^{n+1}u(n-1)$	$a^{-n-1}u(n-1)$	(8'')



a^n $\times$ $R(n)$  $\times$ 

a^n $\times$ $R(n)$  $\times$ 

$$\begin{array}{lcl}
 (1) & \boxed{a^n u(n)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{n+l} u(n)} \quad (1') \\
 (2) & \boxed{a^{-n} u(n)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{-n-l} u(n)} \quad (2')
 \end{array}$$

$$\begin{array}{lcl}
 (3) & \boxed{a^n u(-n)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{n-l} u(-n)} \quad (3') \\
 (4) & \boxed{a^{-n} u(-n)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{-n+l} u(-n)} \quad (4')
 \end{array}$$

$$\begin{array}{lcl}
 (5) & \boxed{a^n u(-n-1)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{n+l} u(-n-1)} \quad (5') \\
 (6) & \boxed{a^{-n} u(-n-1)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{-n-l} u(-n-1)} \quad (6')
 \end{array}$$

$$\begin{array}{lcl}
 (7) & \boxed{a^n u(n-1)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{n-l} u(n-1)} \quad (7') \\
 (8) & \boxed{a^{-n} u(n-1)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{-n+l} u(n-1)} \quad (8')
 \end{array}$$

$$\begin{array}{lcl}
 (1) & \boxed{a^n u(n)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{n-l} u(n)} \quad (1'') \\
 (2) & \boxed{a^{-n} u(n)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{-n+l} u(n)} \quad (2'')
 \end{array}$$

$$\begin{array}{lcl}
 (3) & \boxed{a^n u(-n)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{n+l} u(-n)} \quad (3'') \\
 (4) & \boxed{a^{-n} u(-n)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{-n-l} u(-n)} \quad (4'')
 \end{array}$$

$$\begin{array}{lcl}
 (5) & \boxed{a^n u(-n-1)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{n-l} u(-n-1)} \quad (5'') \\
 (6) & \boxed{a^{-n} u(-n-1)} & \xrightarrow{\text{shr}(b^n)} \boxed{a^{-n+l} u(-n-1)} \quad (6'')
 \end{array}$$

$$\begin{array}{lcl}
 (7) & \boxed{a^n u(n-1)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{n+l} u(n-1)} \quad (7'') \\
 (8) & \boxed{a^{-n} u(n-1)} & \xrightarrow{\text{shl}(b^n)} \boxed{a^{-n-l} u(n-1)} \quad (8'')
 \end{array}$$

**Unshifted
Sequence x**

**Shifted
Sequence 1 x'**

**Shifted
Sequence 2 x''**

(1) $a^n u(n)$

$shl(b^n)$ $a^{n+l} u(n)$ (1')

$shr(b^n)$ $a^{n-l} u(n)$ (1'')

(2) $a^{-n} u(n)$

$shl(b^n)$ $a^{-n-l} u(n)$ (2')

$shr(b^n)$ $a^{-n+l} u(n)$ (2'')

(3) $a^n u(-n)$

$shr(b^n)$ $a^{n-l} u(-n)$ (3')

$shl(b^n)$ $a^{n+l} u(-n)$ (3'')

(4) $a^{-n} u(-n)$

$shr(b^n)$ $a^{-n+l} u(-n)$ (4')

$shl(b^n)$ $a^{-n-l} u(-n)$ (4'')

(5) $a^n u(-n-1)$

$shl(b^n)$ $a^{n+l} u(-n-1)$ (5')

$shr(b^n)$ $a^{n-l} u(-n-1)$ (5'')

(6) $a^{-n} u(-n-1)$

$shl(b^n)$ $a^{-n-l} u(-n-1)$ (6')

$shr(b^n)$ $a^{-n+l} u(-n-1)$ (6'')

(7) $a^n u(n-1)$

$shr(b^n)$ $a^{n-l} u(n-1)$ (7')

$shl(b^n)$ $a^{n+l} u(n-1)$ (7'')

(8) $a^{-n} u(n-1)$

$shr(b^n)$ $a^{-n+l} u(n-1)$ (8')

$shl(b^n)$ $a^{-n-l} u(n-1)$ (8'')

$$shl(b^n) \longleftrightarrow shr(b^n)$$

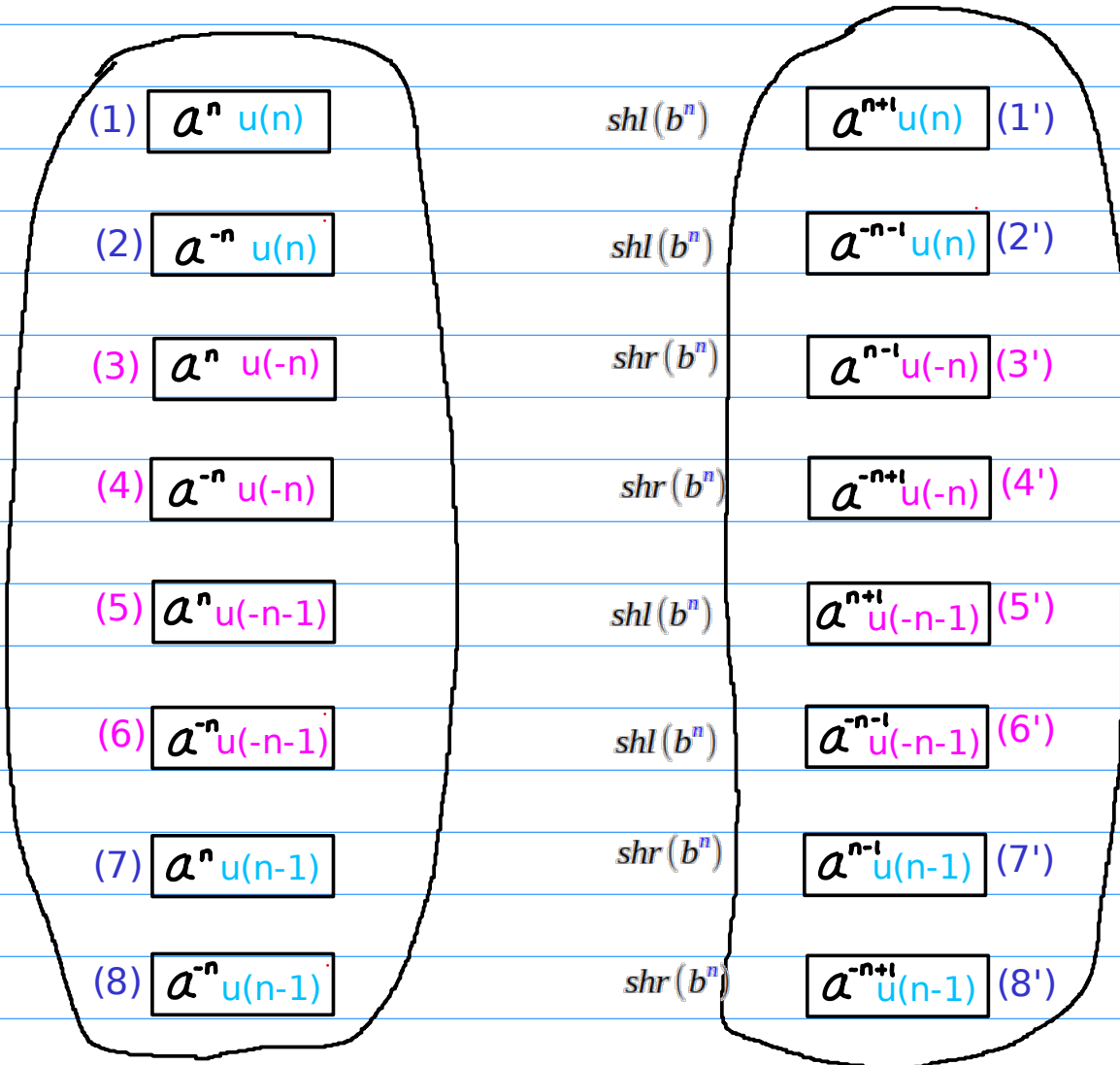
$$shr(b^n) \longleftrightarrow shl(b^n)$$

many possible permutations are possible
but consider these two

**Unshifted
Sequence x**



**Shifted
Sequence 1 x'**



Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

**Unshifted
Sequence x**



**Shifted
Sequence 2 x''**

(1) $a^n u(n)$	$shr(b^n)$	$a^{n-l} u(n)$ (1'')
(2) $a^{-n} u(n)$	$shr(b^n)$	$a^{-n+l} u(n)$ (2'')
(3) $a^n u(-n)$	$shl(b^n)$	$a^{n+l} u(-n)$ (3'')
(4) $a^{-n} u(-n)$	$shl(b^n)$	$a^{-n-l} u(-n)$ (4'')
(5) $a^n u(-n-1)$	$shr(b^n)$	$a^{n-l} u(-n-1)$ (5'')
(6) $a^{-n} u(-n-1)$	$shr(b^n)$	$a^{-n+l} u(-n-1)$ (6'')
(7) $a^n u(n-1)$	$shl(b^n)$	$a^{n+l} u(n-1)$ (7'')
(8) $a^{-n} u(n-1)$	$shl(b^n)$	$a^{-n-l} u(n-1)$ (8'')

Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

$$(x') \longleftrightarrow (x'')$$

$$(1') \sim (8') \qquad (1'') \sim (8'')$$

Exponent Conversion

$$shl(b^n) \longleftrightarrow shr(b^n)$$

$$shr(b^n) \longleftrightarrow shl(b^n)$$

(1') (n+1)	—————	(1'') (n-1)
(2') -(n+1)	—————	(2'') -(n-1)
(3') (n-1)	—————	(3'') (n+1)
(4') -(n-1)	—————	(4'') -(n+1)
(5') (n+1)	—————	(5'') (n-1)
(6') -(n+1)	—————	(6'') -(n-1)
(7') (n-1)	—————	(7'') (n+1)
(8') -(n-1)	—————	(8'') -(n+1)

Identical Exponent Elements

$$(1') \text{ } \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \text{ } (1'')$$

$$(7') \text{ } \text{—————} \text{ } (7'')$$

$$(5') \text{ } \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \text{ } (5'')$$

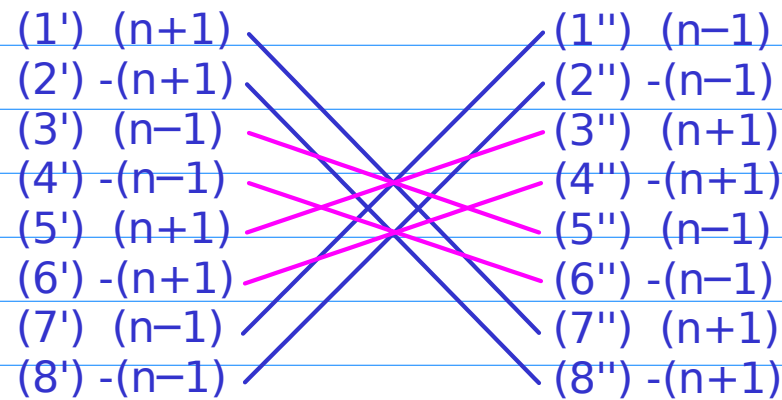
$$(3') \text{ } \text{—————} \text{ } (3'')$$

$$(2') \text{ } \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \text{ } (2'')$$

$$(8') \text{ } \text{—————} \text{ } (8'')$$

$$(6') \text{ } \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \text{ } (6'')$$

$$(4') \text{ } \text{—————} \text{ } (4'')$$



$(1') \ (n+1)$	$\text{shl}(n)$	$(1'') \ (n-1)$	$\text{shr}(n)$
$(7') \ (n-1)$	$\text{shr}(n)$	$(7'') \ (n+1)$	$\text{shl}(n)$

$(2') \ -(n+1)$	$\text{shl}(n)$	$(2'') \ -(n-1)$	$\text{shr}(n)$
$(8') \ -(n-1)$	$\text{shr}(n)$	$(8'') \ -(n+1)$	$\text{shl}(n)$

$(3') \ (n-1)$	$\text{shr}(n)$	$(3'') \ (n+1)$	$\text{shl}(n)$
$(5') \ (n+1)$	$\text{shl}(n)$	$(5'') \ (n-1)$	$\text{shr}(n)$

$(4') \ -(n-1)$	$\text{shr}(n)$	$(4'') \ -(n+1)$	$\text{shl}(n)$
$(6') \ -(n+1)$	$\text{shl}(n)$	$(6'') \ -(n-1)$	$\text{shr}(n)$

(x') $\longleftrightarrow$ (x'')
 $(1') \sim (8')$ $(1'') \sim (8'')$

$(1') \ a^{n+l}u(n)$ $(1'') \ a^{n-l}u(n)$ $a^{-n-l}u(n)$ $(2') \ a^{-n+l}u(n)$ $(2'')$
 $(7') \ a^{n-l}u(n-1)$ $(7'') \ a^{n+l}u(n-1)$ $a^{-n+l}u(n-1)$ $(8') \ a^{-n-l}u(n-1)$ $(8'')$

$(3') \ a^{n-l}u(-n)$ $(3'') \ a^{n+l}u(-n)$ $a^{-n+l}u(-n)$ $(4') \ a^{-n-l}u(-n)$ $(4'')$
 $(5') \ a^{n+l}u(-n-1)$ $(5'') \ a^{n-l}u(-n-1)$ $a^{-n-l}u(-n-1)$ $(6') \ a^{-n+l}u(-n-1)$ $(6'')$

Identical Exponent Elements

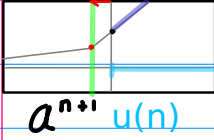
$(1') \ (n+1)$	$(1'') \ (n-1)$
$(2') \ -(n+1)$	$(2'') \ -(n-1)$
$(3') \ (n-1)$	$(3'') \ (n+1)$
$(4') \ -(n-1)$	$(4'') \ -(n+1)$
$(5') \ (n+1)$	$(5'') \ (n-1)$
$(6') \ -(n+1)$	$(6'') \ -(n-1)$
$(7') \ (n-1)$	$(7'') \ (n+1)$
$(8') \ -(n-1)$	$(8'') \ -(n+1)$

Shifted Range Elements

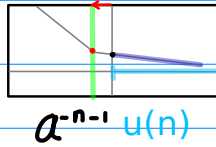
$(1') \ u(n)$	$(1'') \ u(n)$
$(2') \ u(n)$	$(2'') \ u(n)$
$(3') \ u(-n)$	$(3'') \ u(-n)$
$(4') \ u(-n)$	$(4'') \ u(-n)$
$(5') \ u(-n-1)$	$(5'') \ u(-n-1)$
$(6') \ u(-n-1)$	$(6'') \ u(-n-1)$
$(7') \ u(n-1)$	$(7'') \ u(n-1)$
$(8') \ u(n-1)$	$(8'') \ u(n-1)$

(x') $\longleftrightarrow$ (x'')
 $(1') \sim (8')$ $(1'') \sim (8'')$

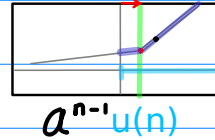
(1') 1000



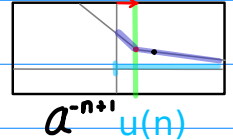
(2') 1001



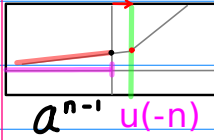
(1'') 1000



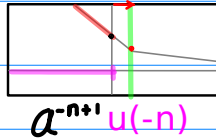
(2'') 1001



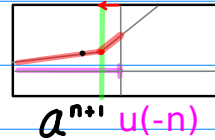
(3') 1010



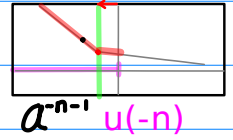
(4') 1011



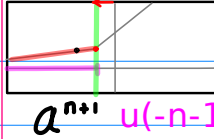
(3'') 1010



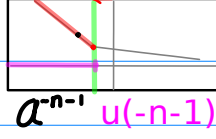
(4'') 1011



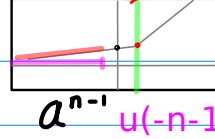
(5') 1100



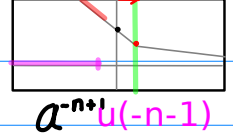
(6') 1101



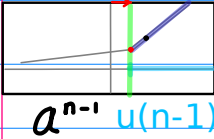
(5'') 1100



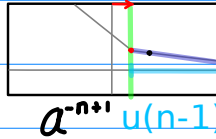
(6'') 1101



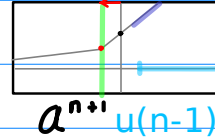
(7') 1110



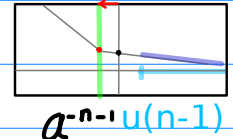
(8') 1111



(7'') 1110



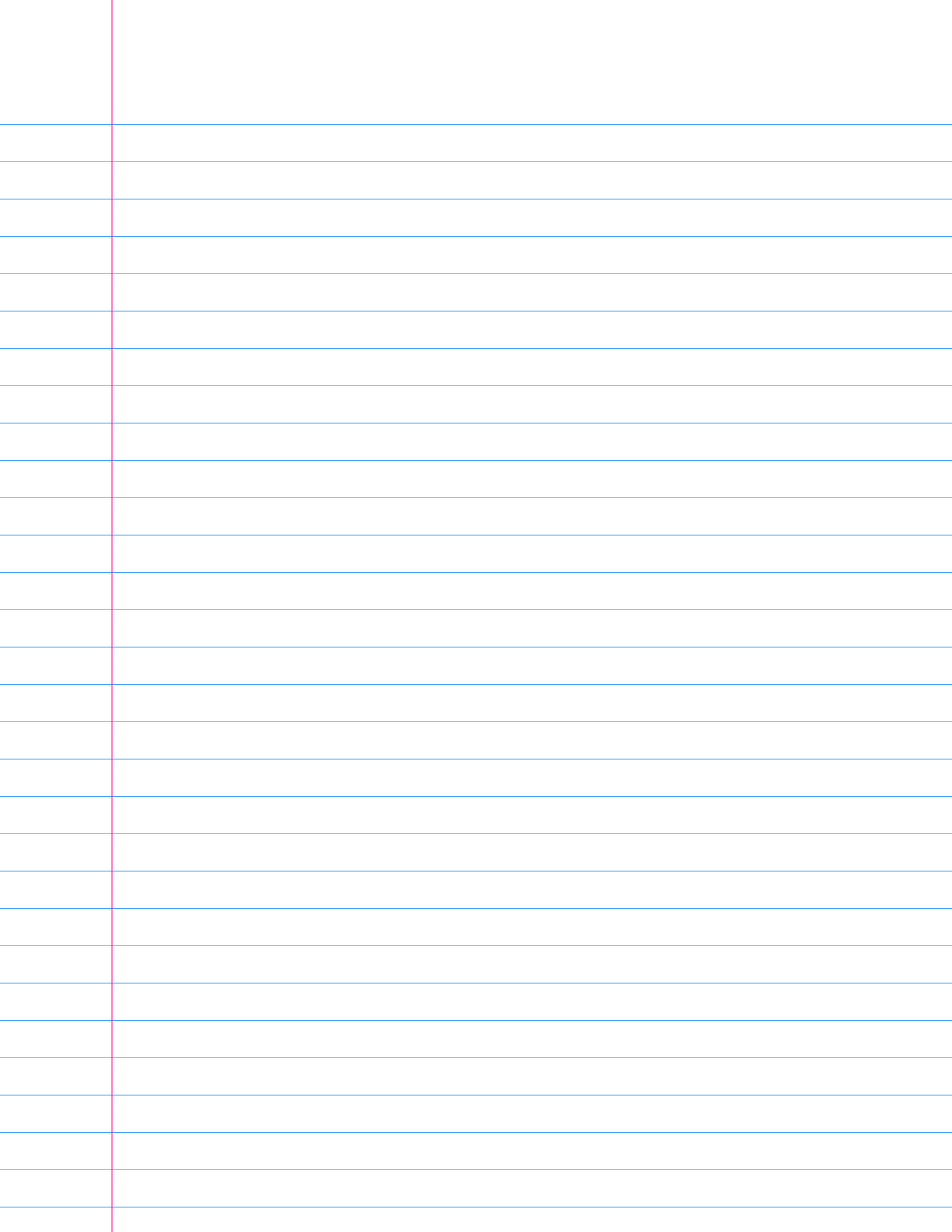
(8'') 1111



Conversion between x' and x''

$shl(b^n) \longleftrightarrow shr(b^n)$

$shr(b^n) \longleftrightarrow shl(b^n)$



(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

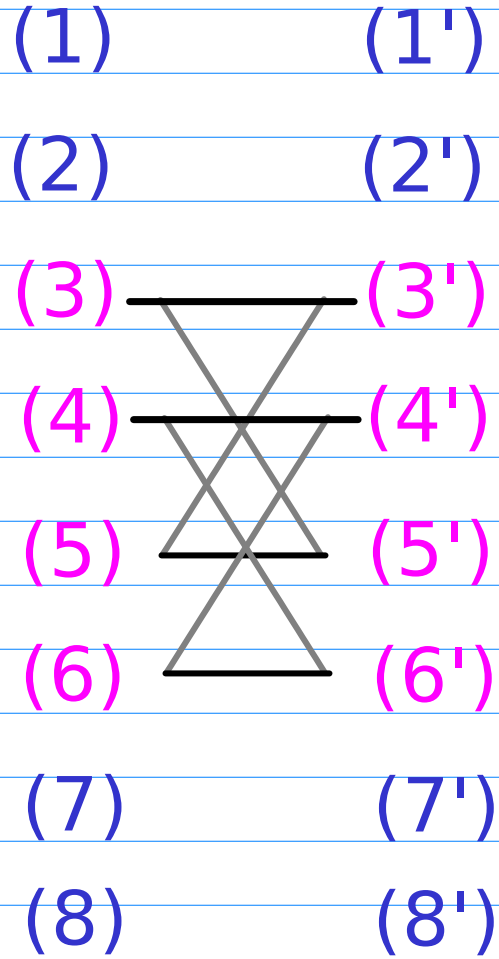
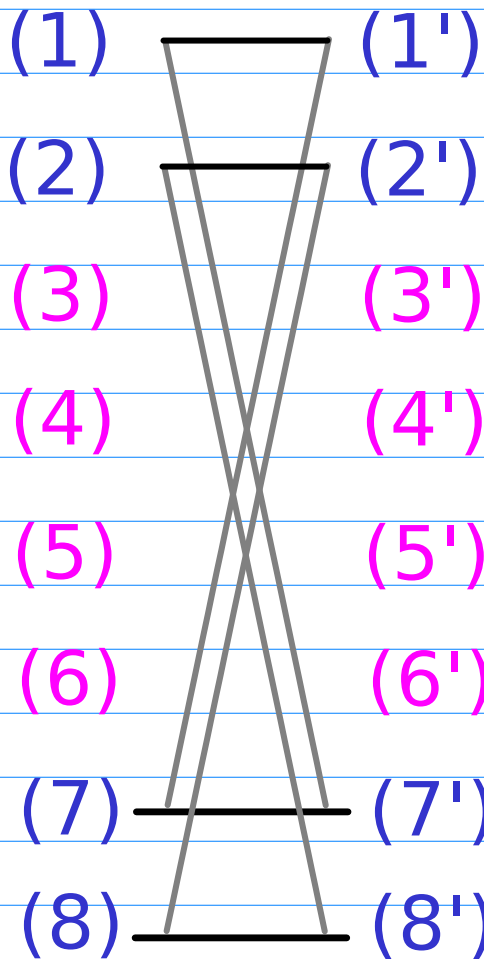
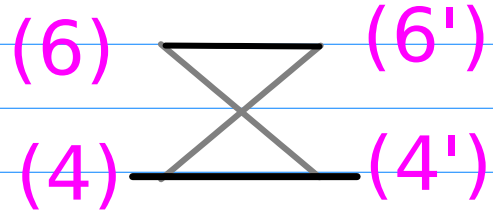
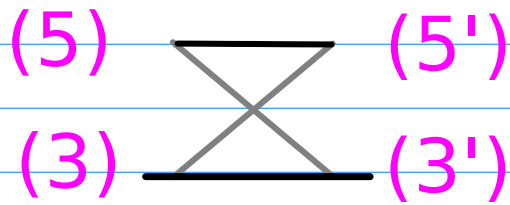
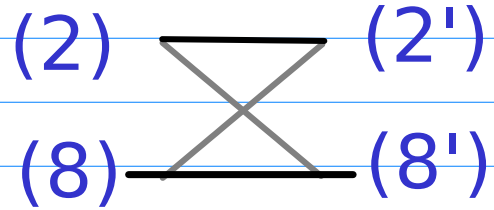
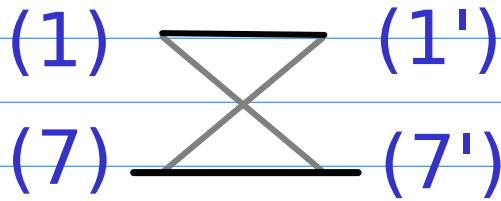
(x)



(x')

(1)~(8)

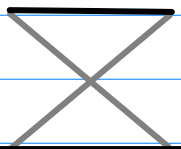
(1')~(8')

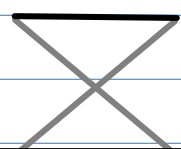


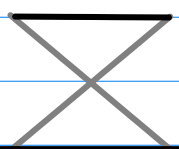
(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

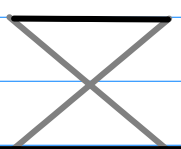
(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

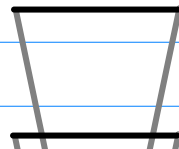
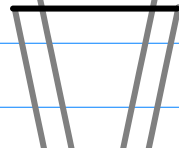
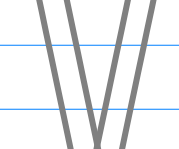
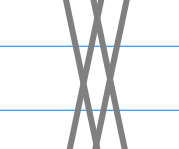
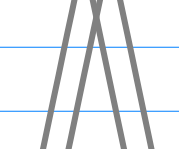
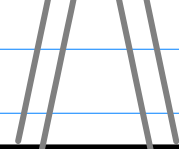
(x)  **(x'')**
(1)~(8) (1')~(8')

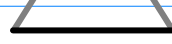
(1)  (1'')
(7)  (7'')

(2)  (2'')
(8)  (8'')

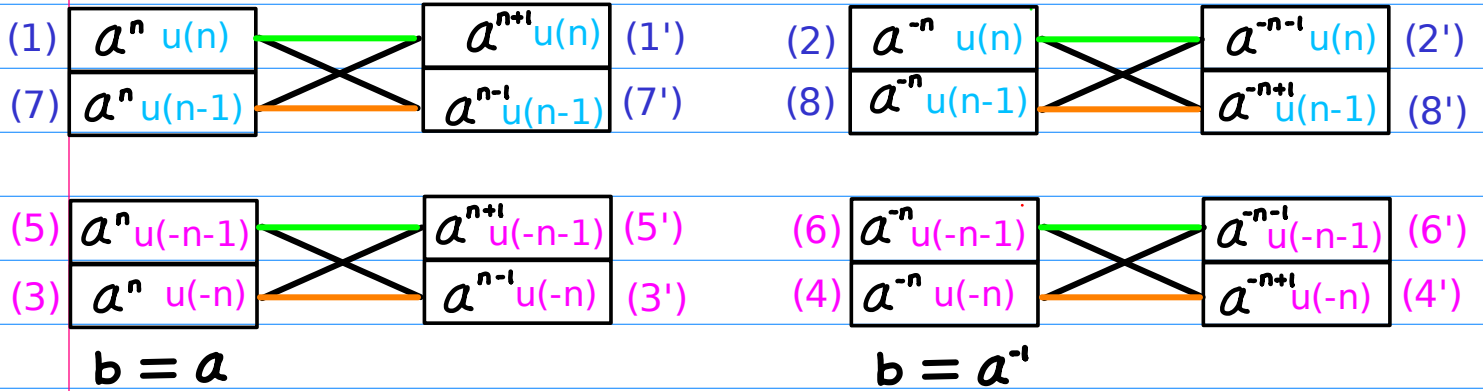
(5)  (5'')
(3)  (3'')

(6)  (6'')
(4)  (4'')

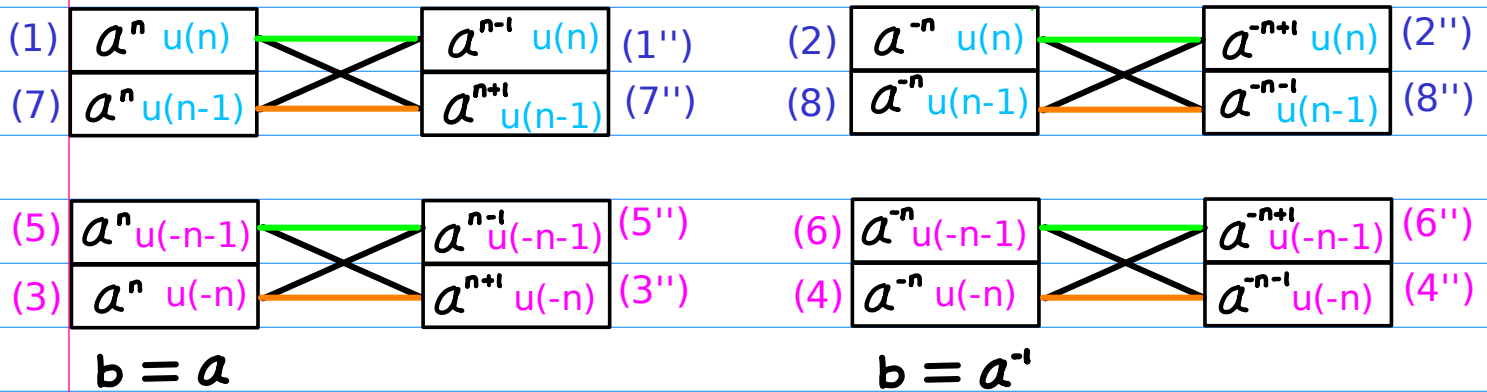
(1)  (1'')
(2)  (2'')
(3)  (3'')
(4)  (4'')
(5)  (5'')
(6)  (6'')
(7)  (7'')
(8)  (8'')

(1) (1'')
(2) (2'')
(3)  (3'')
(4)  (4'')
(5)  (5'')
(6)  (6'')
(7) (7'')
(8) (8'')

A Shifting Shifted Sequence 1



B Shifting Shifted Sequence 2



A Shifting Shifted Sequence 1

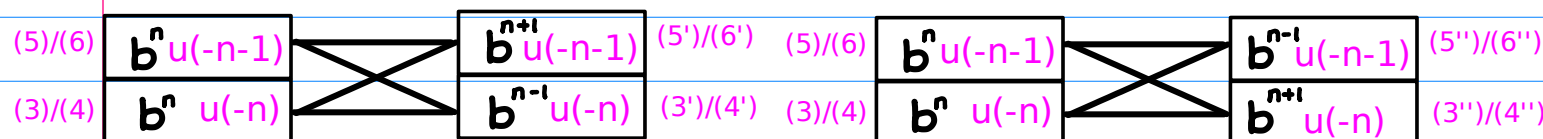
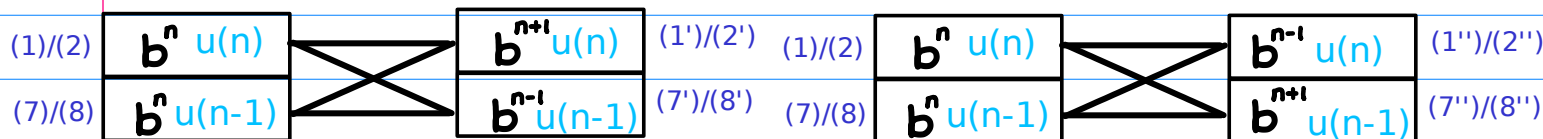
(1) Exponent Shifting

(2) Range Shifting

B Shifting Shifted Sequence 2

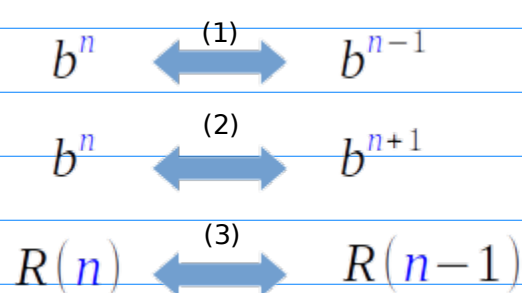
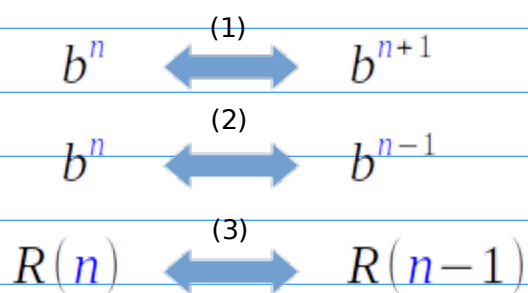
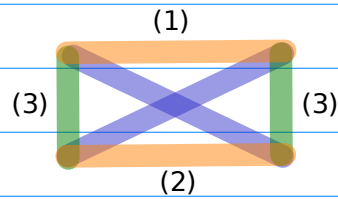
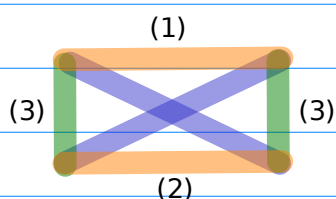
(1) Exponent Shifting

(2) Range Shifting



$$b = a/a^i$$

$$b = a/a^i$$



Exponent Shifting

b^n	$b^{sh(n)}$	
a^n	$a^{(n+1)}$	$shl(b^n)$
a^n	$a^{(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$

Exponent Shifting

b^n	$b^{sh(n)}$	
a^n	$a^{(n-1)}$	$shr(b^n)$
a^n	$a^{(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$

Range Shifting

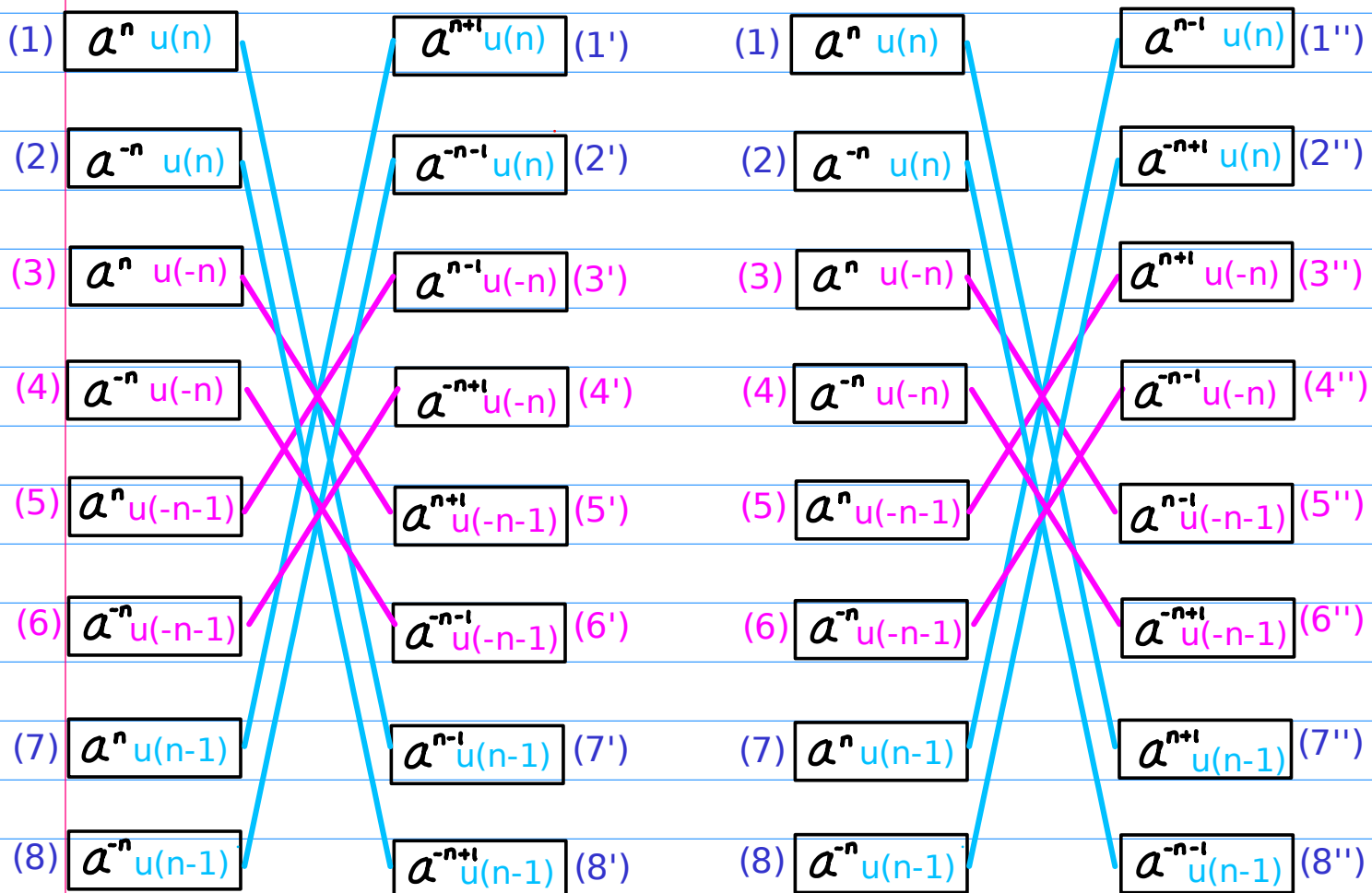
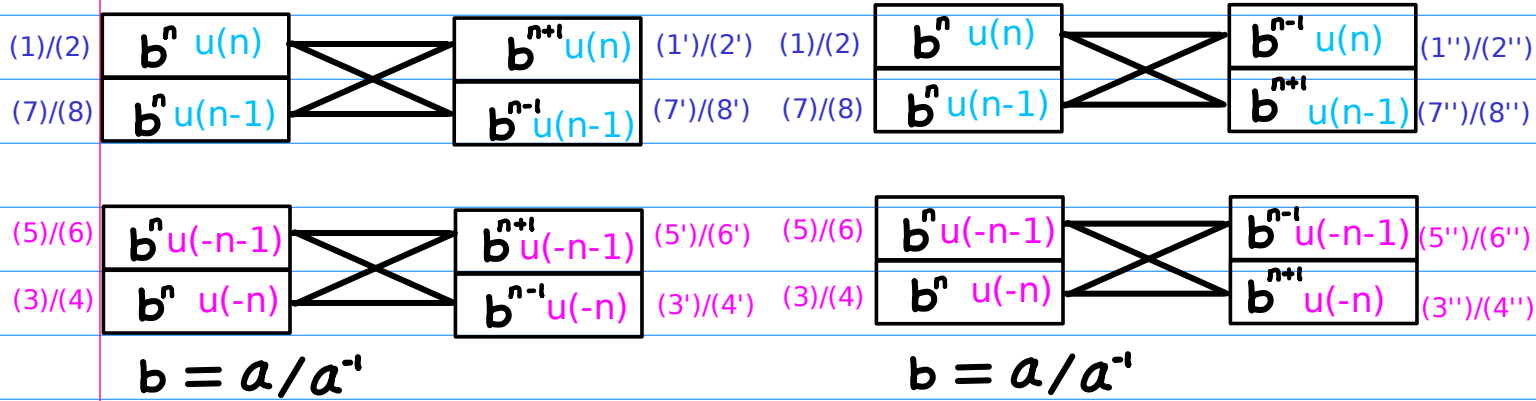
$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

Range Shifting

$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

A Shifting Shifted Sequence 1

B Shifting Shifted Sequence 2



A Shifting Shifted Sequence 1

$$\begin{array}{lcl} (1) & a^n u(n) & a^{n+l} u(n) \quad (1') \\ (7) & a^n u(n-1) & a^{n-l} u(n-1) \quad (7') \end{array}$$

$$\begin{array}{lcl} (2) & a^{-n} u(n) & a^{-n-l} u(n) \quad (2') \\ (8) & a^{-n} u(n-1) & a^{-n+l} u(n-1) \quad (8') \end{array}$$

$$\begin{array}{lcl} (5) & a^n u(-n-1) & a^{n+l} u(-n-1) \quad (5') \\ (3) & a^n u(-n) & a^{n-l} u(-n) \quad (3') \end{array}$$

$$\begin{array}{lcl} (6) & a^{-n} u(-n-1) & a^{-n-l} u(-n-1) \quad (6') \\ (4) & a^{-n} u(-n) & a^{-n+l} u(-n) \quad (4') \end{array}$$

$$b = a$$

$$b = a^{-l}$$

$$(1) \quad a^n u(n) \quad a^{n+l} u(n) \quad (1')$$

$$(2) \quad a^{-n} u(n) \quad a^{-n-l} u(n) \quad (2')$$

$$(3) \quad a^n u(-n) \quad a^{n-l} u(-n) \quad (3')$$

$$(4) \quad a^{-n} u(-n) \quad a^{-n+l} u(-n) \quad (4')$$

$$(5) \quad a^n u(-n-1) \quad a^{n+l} u(-n-1) \quad (5')$$

$$(6) \quad a^{-n} u(-n-1) \quad a^{-n-l} u(-n-1) \quad (6')$$

$$(7) \quad a^n u(n-1) \quad a^{n-l} u(n-1) \quad (7')$$

$$(8) \quad a^{-n} u(n-1) \quad a^{-n+l} u(n-1) \quad (8')$$

$$(1) \quad a^n u(n) \quad a^{n+l} u(n) \quad (1')$$

$$(2) \quad a^{-n} u(n) \quad a^{-n-l} u(n) \quad (2')$$

$$(3) \quad a^n u(-n) \quad a^{n-l} u(-n) \quad (3')$$

$$(4) \quad a^{-n} u(-n) \quad a^{-n+l} u(-n) \quad (4')$$

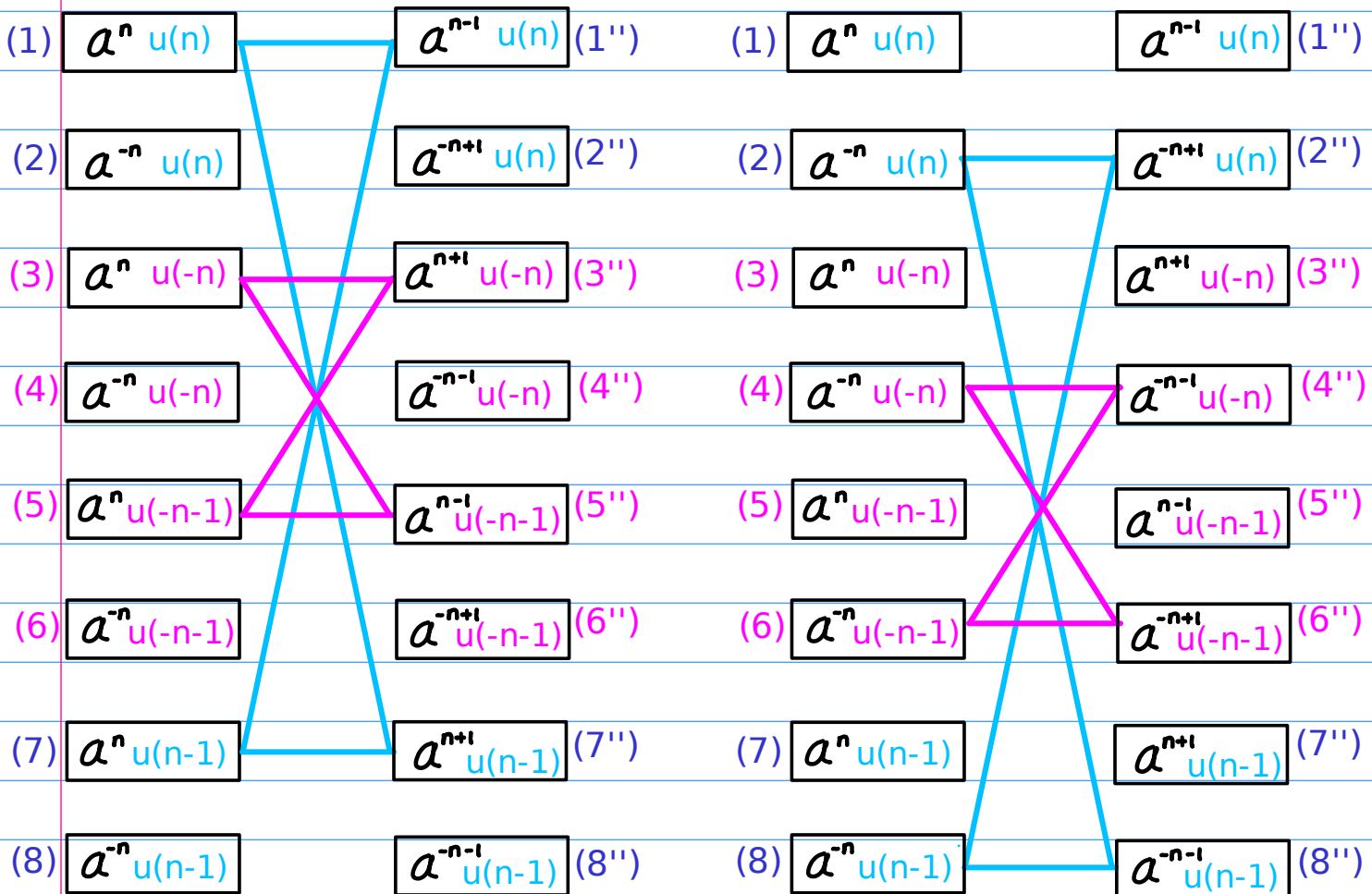
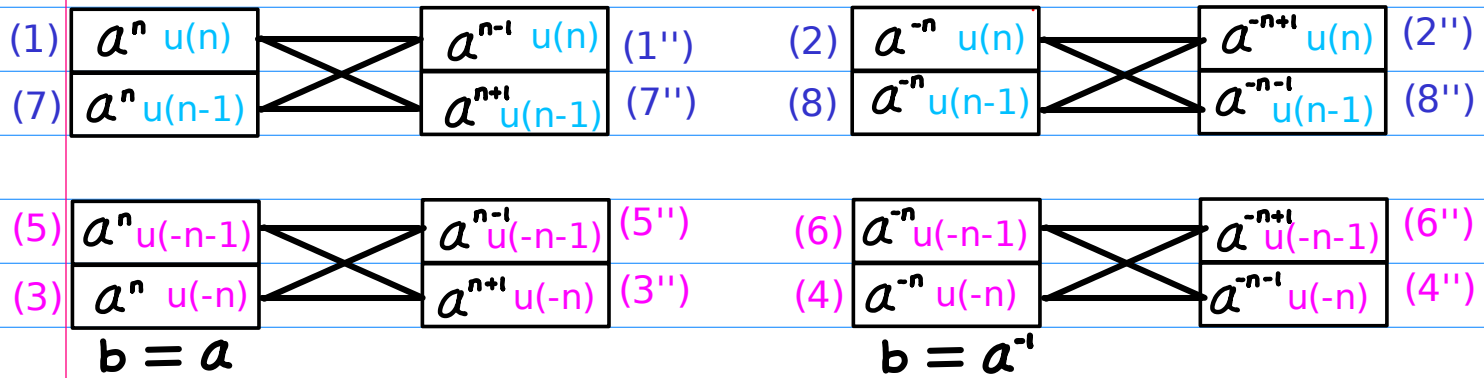
$$(5) \quad a^n u(-n-1) \quad a^{n+l} u(-n-1) \quad (5')$$

$$(6) \quad a^{-n} u(-n-1) \quad a^{-n-l} u(-n-1) \quad (6')$$

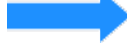
$$(7) \quad a^n u(n-1) \quad a^{n-l} u(n-1) \quad (7')$$

$$(8) \quad a^{-n} u(n-1) \quad a^{-n+l} u(n-1) \quad (8')$$

B Shifting Shifted Sequence 2

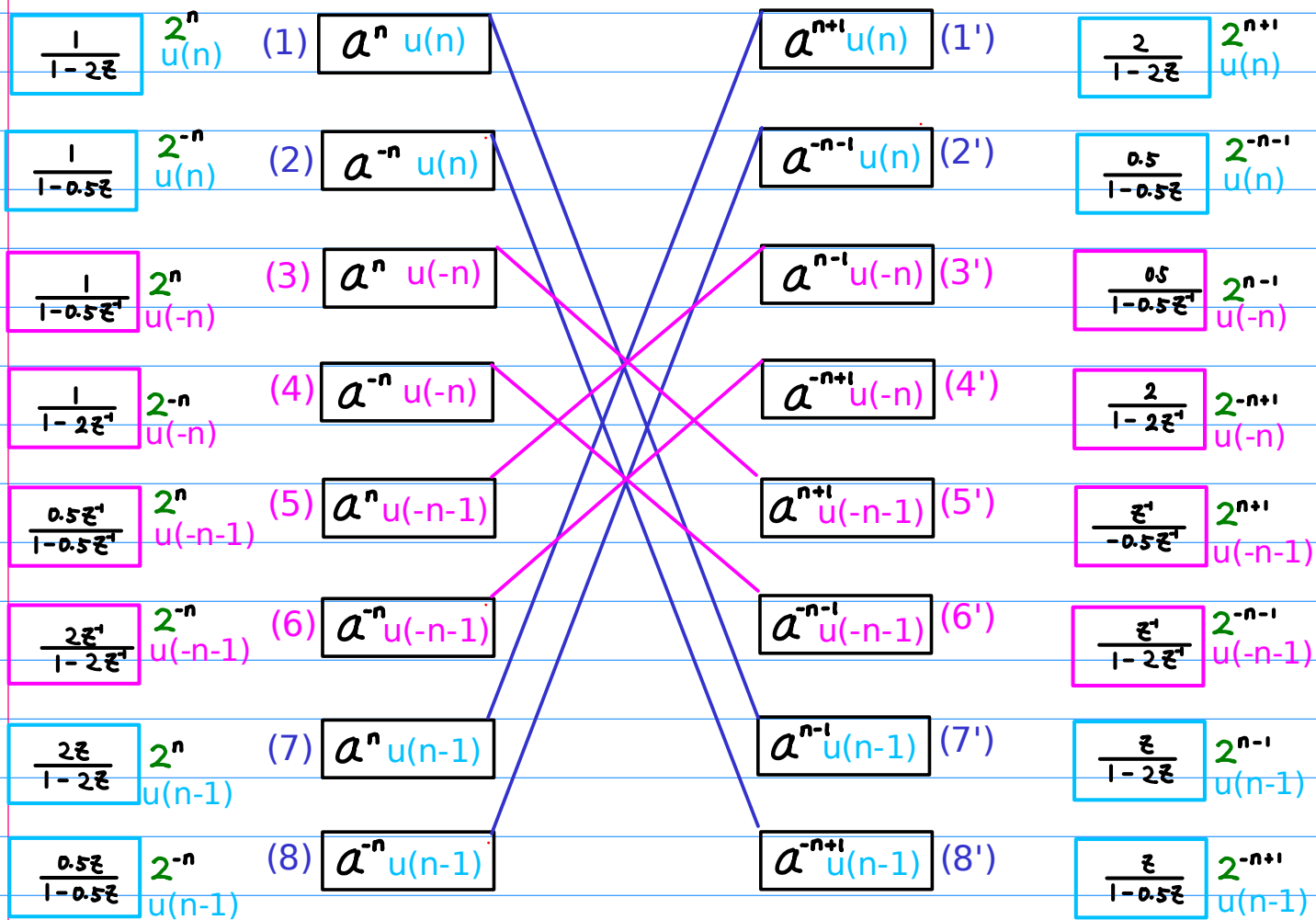


A Shifting Shifted Sequence 1

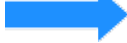
Unshifted Sequence x  Shifted Sequence 1 x'

(1)  (1')
 (7)  (7')
 (5)  (5')
 (3)  (3')

(2)  (2')
 (8)  (8')
 (6)  (6')
 (4)  (4')

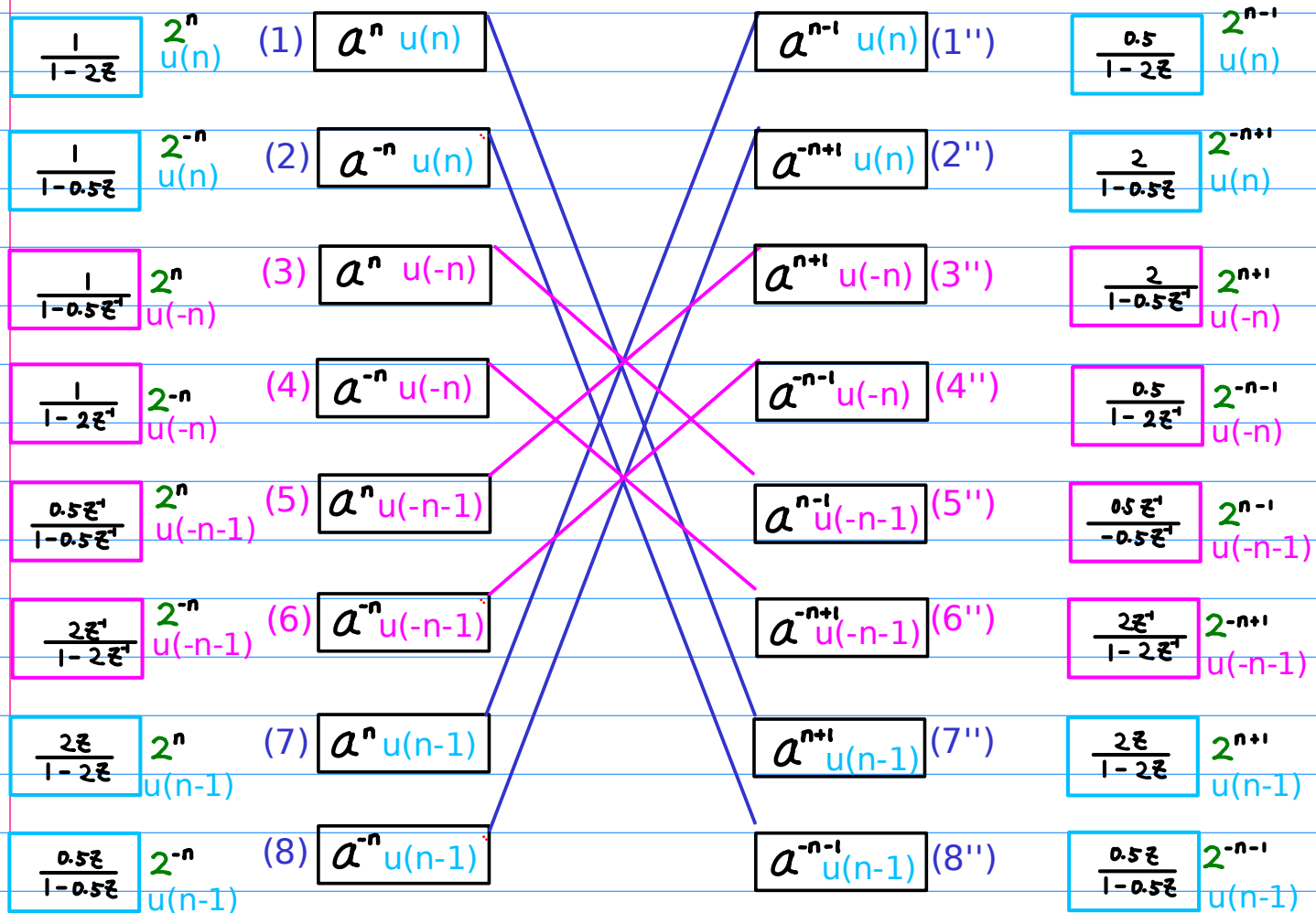


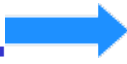
B Shifting Shifted Sequence 2

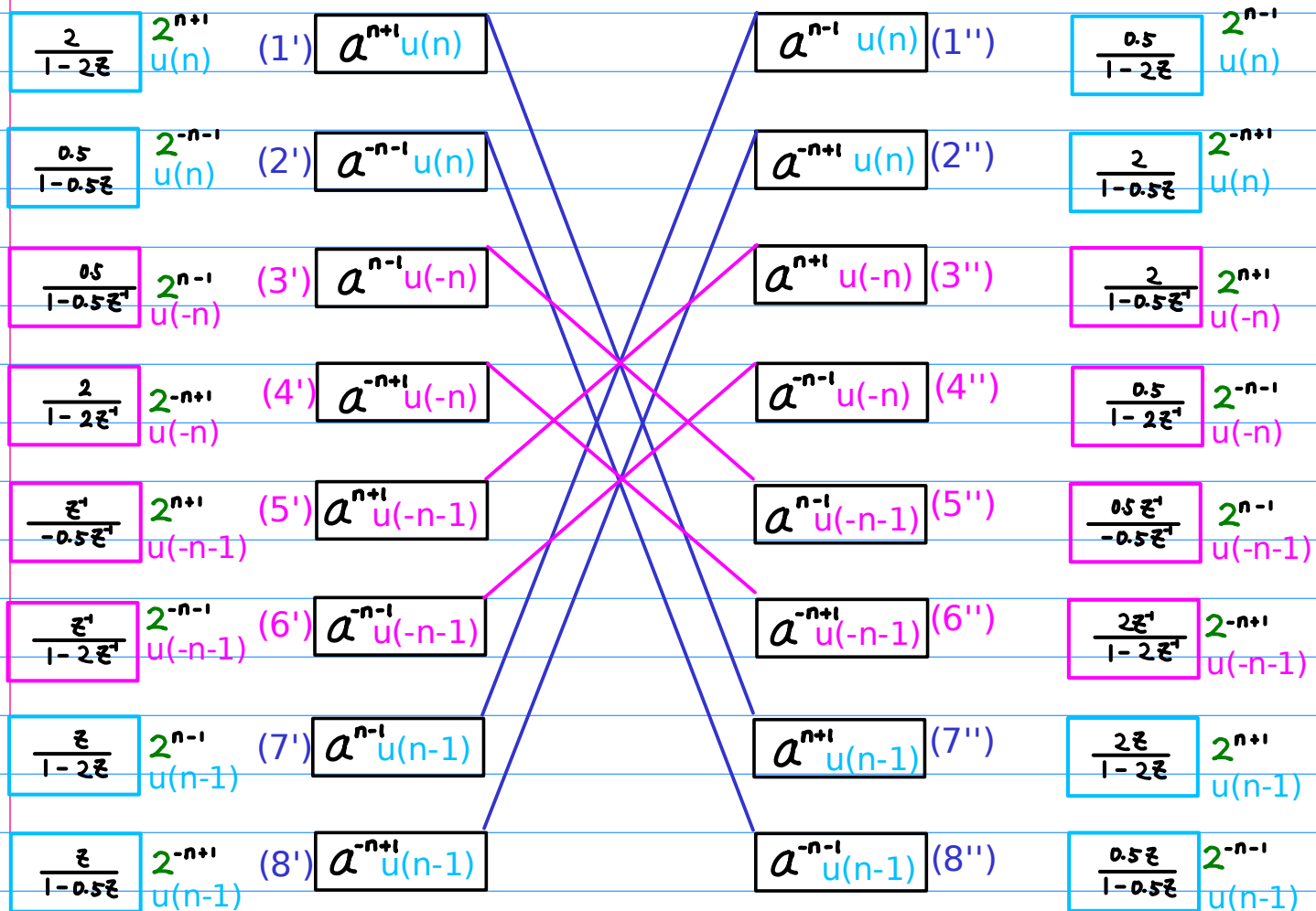
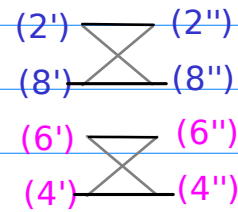
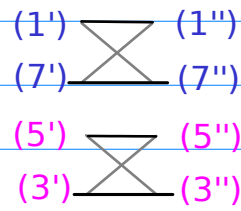
Unshifted Sequence x  Shifted Sequence 2 x''

(1)  (1'')
 (7)  (7'')
 (5)  (5'')
 (3)  (3'')

(2)  (2'')
 (8)  (8'')
 (6)  (6'')
 (4)  (4'')

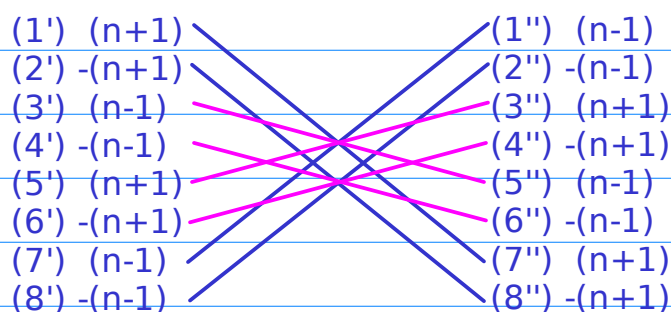


Unshifted Sequence x'  Shifted Sequence 2 x''



$shl(b^n) \longleftrightarrow shr(b^n)$

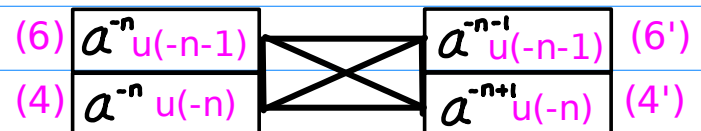
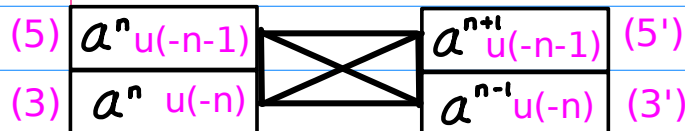
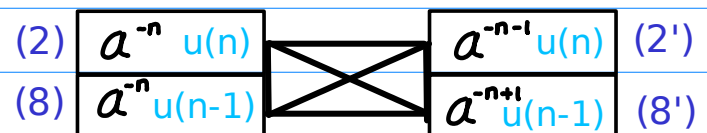
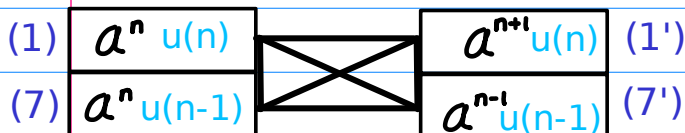
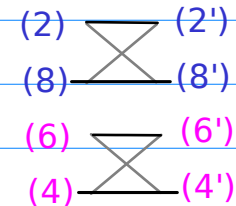
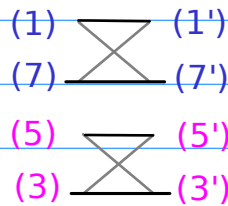
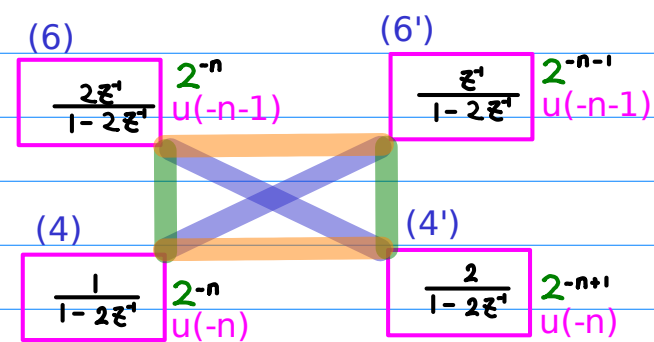
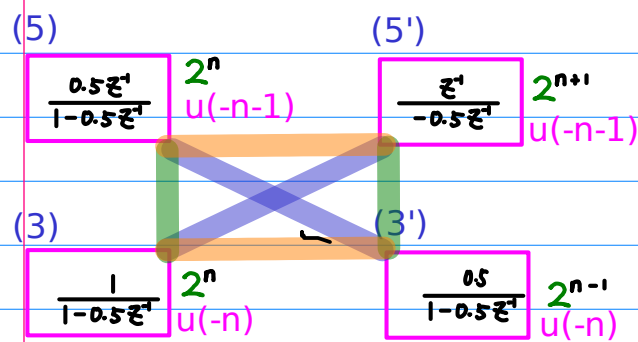
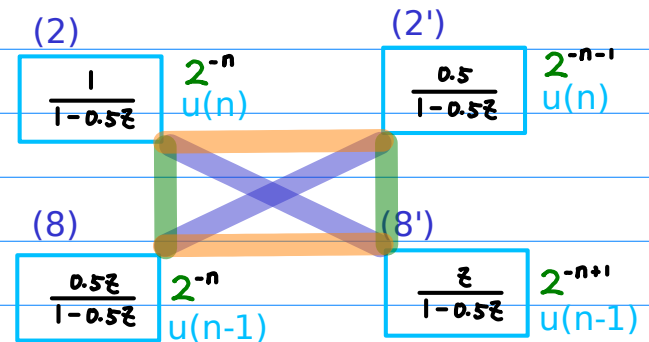
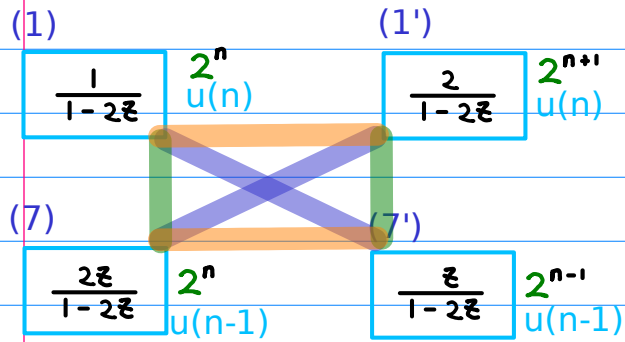
$shr(b^n) \longleftrightarrow shl(b^n)$



Inter-permutation

A Shifting Shifted Sequence 1

(x) $\longrightarrow$ (x')
(1)~(8) (1')~(8')

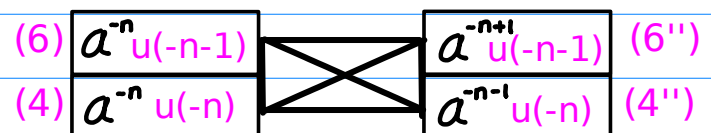
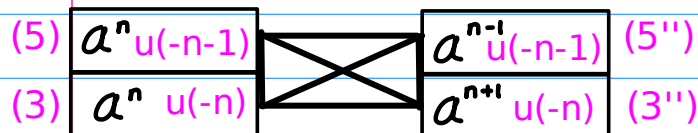
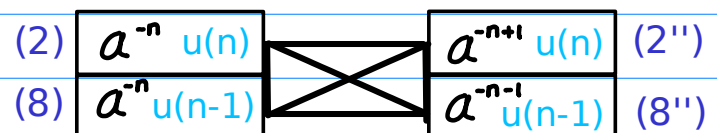
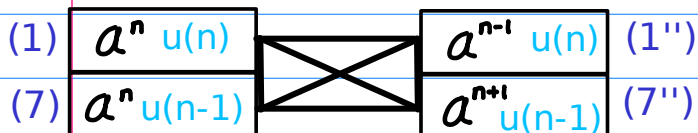
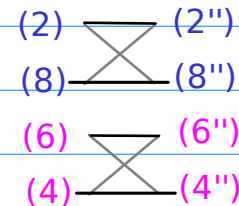
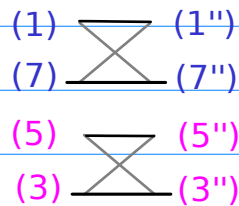
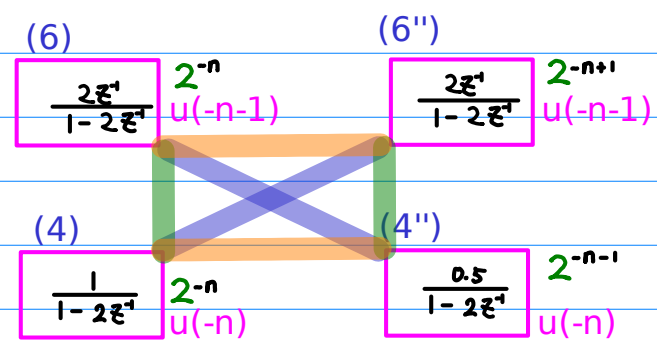
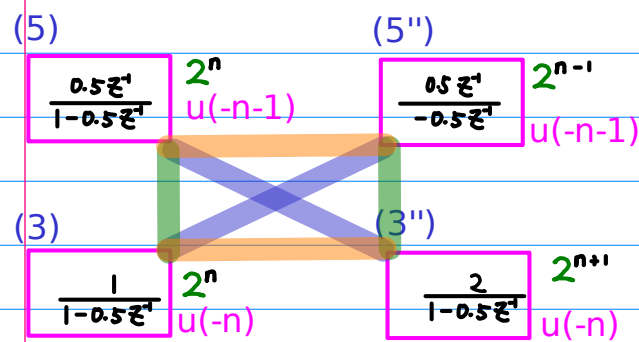
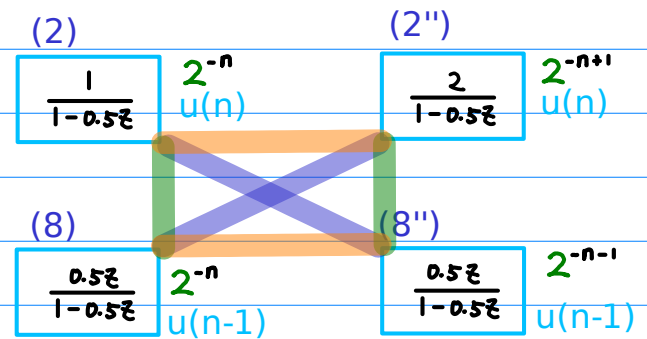
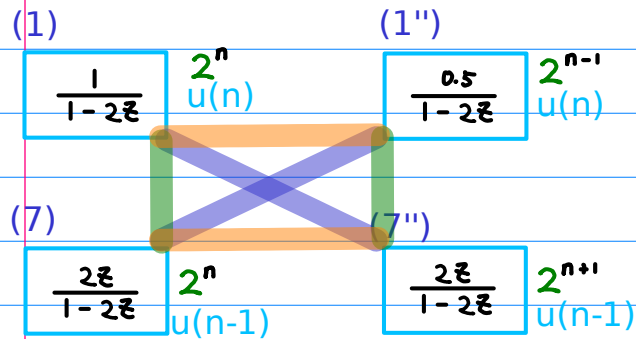


Inter-permutation

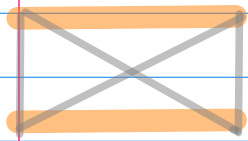
B Shifting Shifted Sequence 2

(x) $\longrightarrow$ (x'')

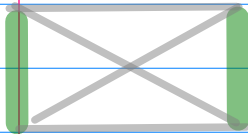
(1)~(8) (1'')~(8'')



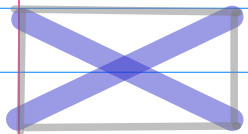
Decomposing Shift Operations



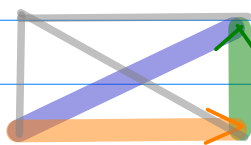
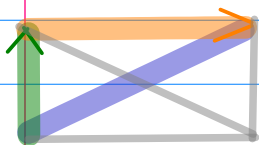
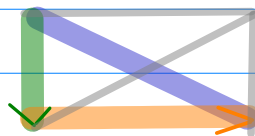
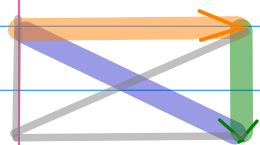
(exponent shift, identity)



(identity, range shift)



(exponent shift, range shift)
 = (exponent shift, identity)
 + (identity, range shift)



A Shifting Shifted Sequence 1
(1) Exponent Shifting
(2) Range Shifting

$$\begin{aligned}(\text{SR}, \text{id}) + (\text{id}, \text{SR}) &= (\text{SR}, \text{SR}) \\ (\text{SL}, \text{id}) + (\text{id}, \text{SL}) &= (\text{SL}, \text{SL})\end{aligned}$$

B Shifting Shifted Sequence 2
(1) Exponent Shifting, Flipping
(2) Range Shifting

$$\begin{aligned}(\text{SR}, \text{id}) + (\text{id}, \text{SL}) &= (\text{SR}, \text{SL}) \\ (\text{SL}, \text{id}) + (\text{id}, \text{SR}) &= (\text{SL}, \text{SR})\end{aligned}$$

A Shifting Shifted Sequence 1

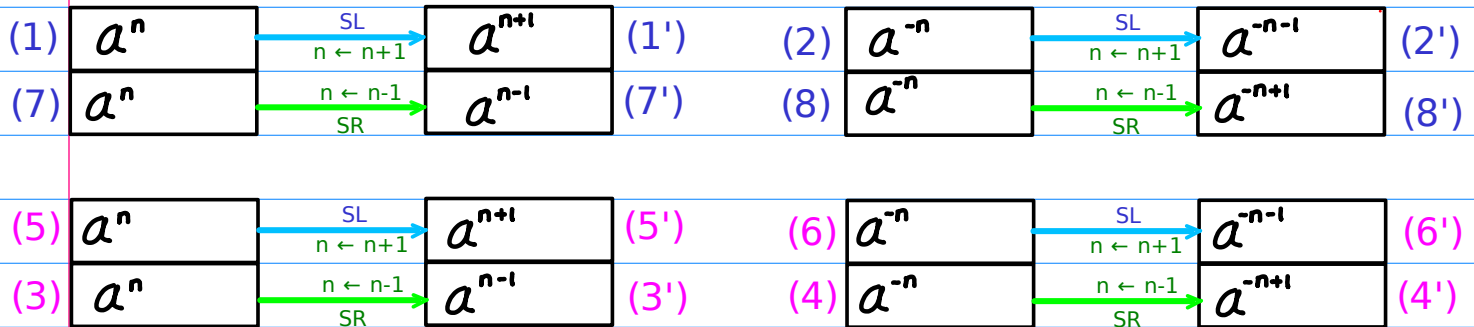
(1) Exponent Shifting

(2) Range Shifting

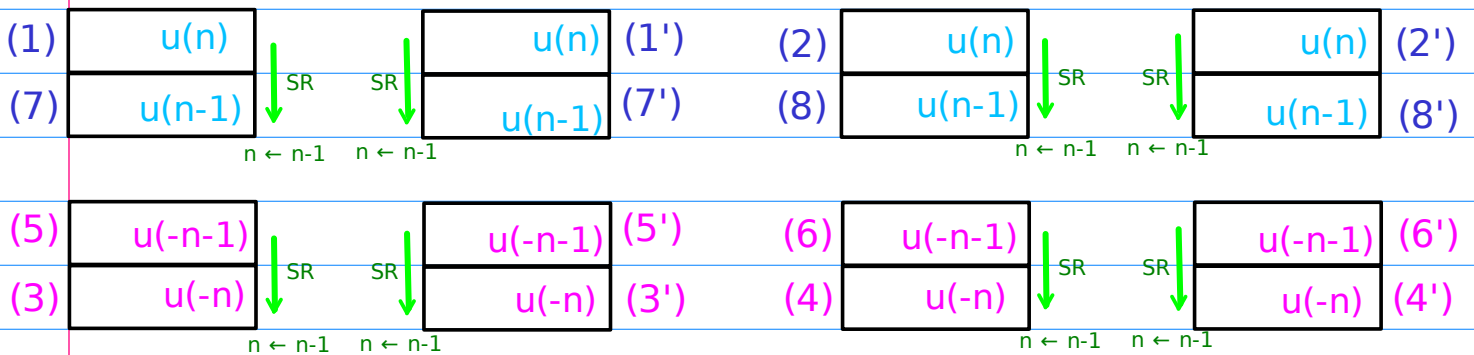
(SR, id) or (SL, id)

(id, SR) or (id, SL)

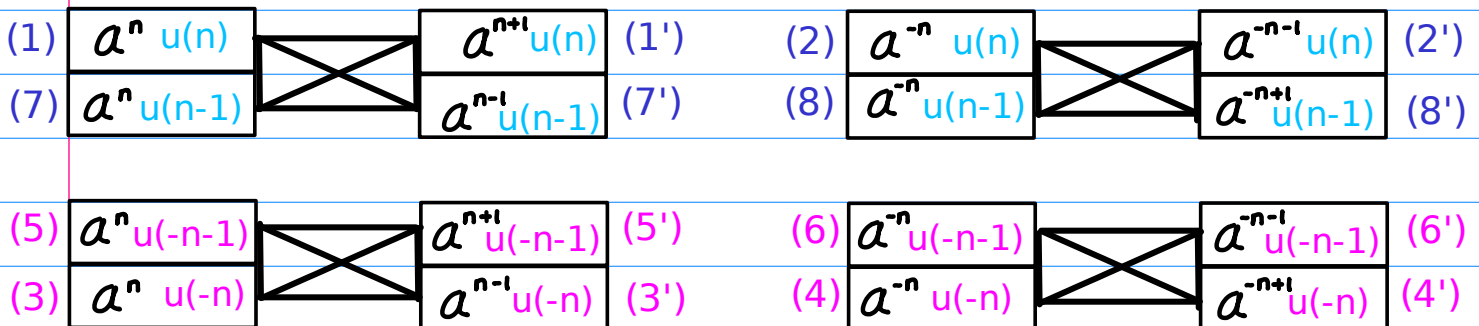
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

$$(EP, RP) = (EP, id) + (id, RP)$$

EP : Exponent Permutations

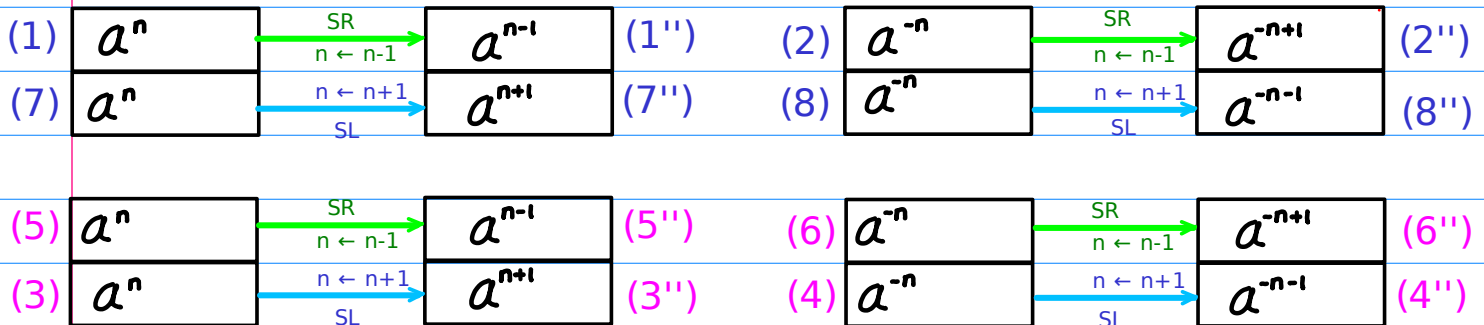
RP : Range Permutations

B Shifting Shifted Sequence 2

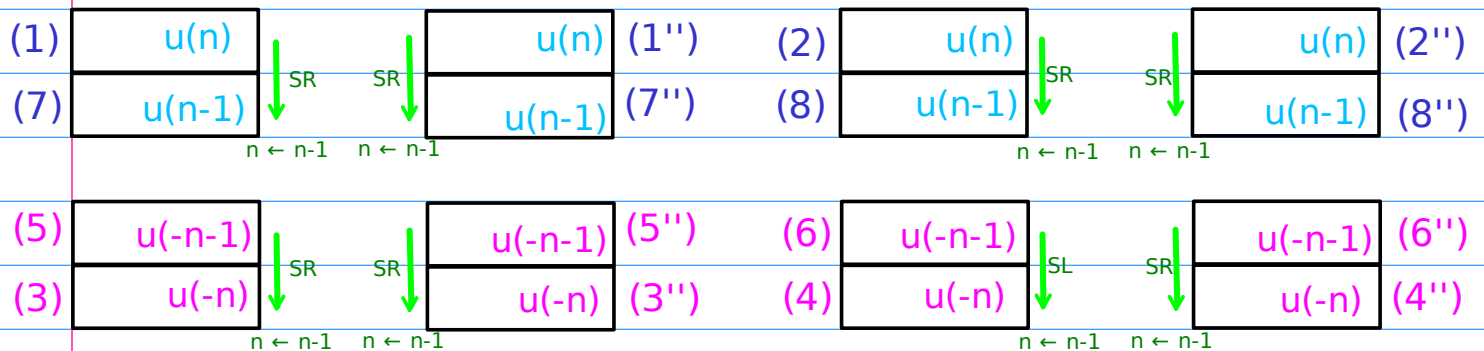
(1) Exponent Shifting, Flipping (SR, id) or (SL, id)

(2) Range Shifting (id, SR) or (id, SL)

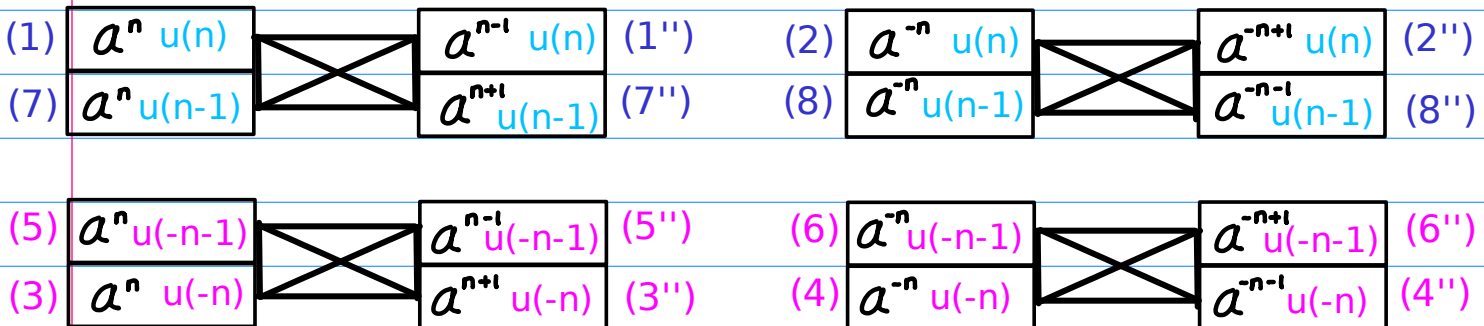
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

$$(\text{EP}, \text{RP}) = (\text{EP}, \text{id}) + (\text{id}, \text{RP})$$

EP : Exponent Permutations

RP : Range Permutations

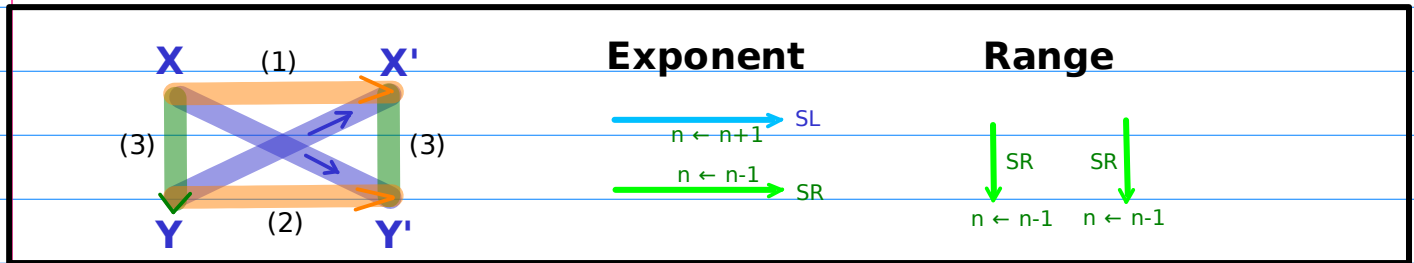
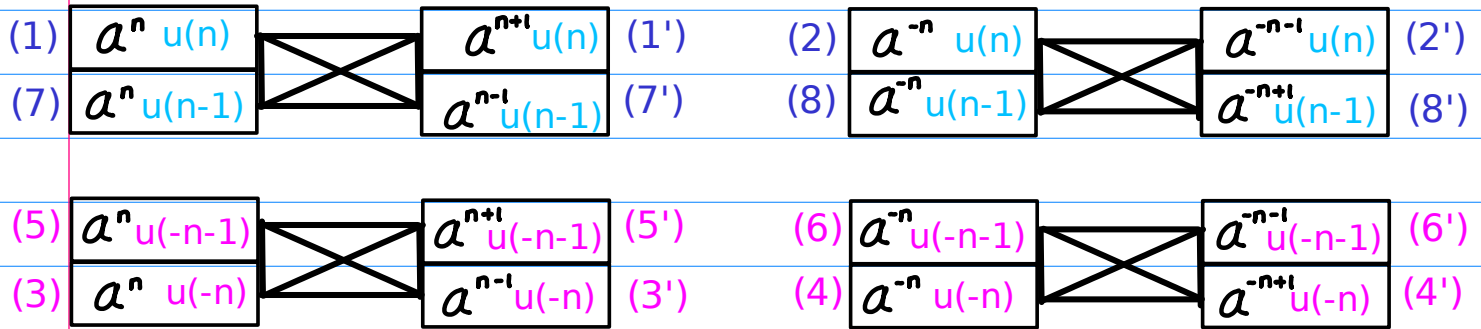
A Shifting Shifted Sequence 1

(1) Exponent Shifting

(2) Range Shifting

$$(SR, id) + (id, SR) = (SR, SR)$$

$$(SL, id) + (id, SL) = (SL, SL)$$



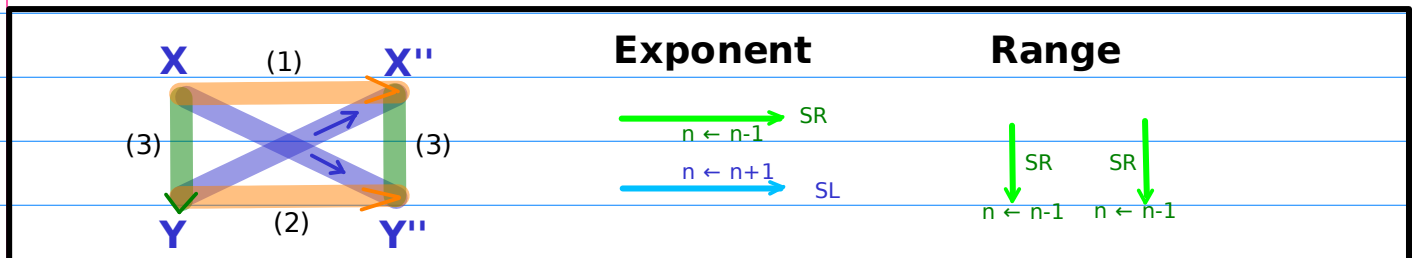
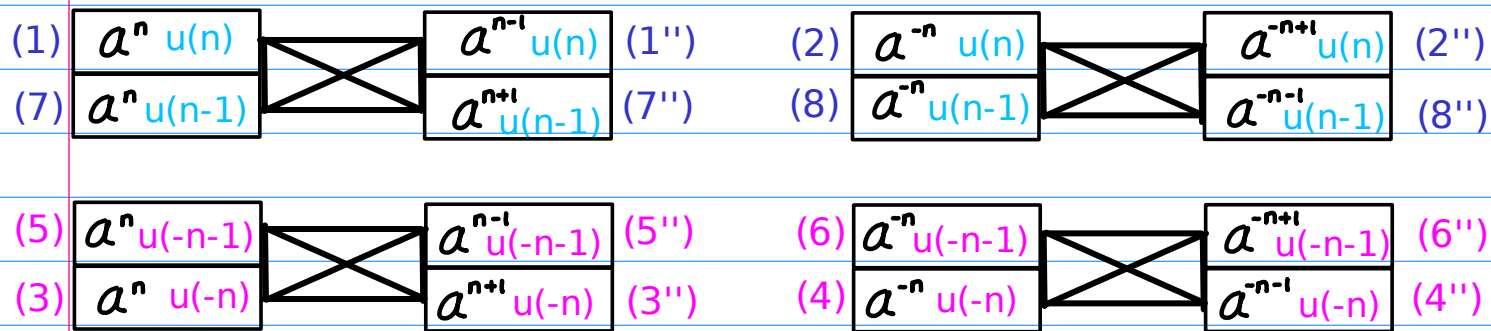
B Shifting Shifted Sequence 2

(1) Exponent Shifting

(2) Range Shifting

$$(SR, id) + (id, SL) = (SR, SL)$$

$$(SL, id) + (id, SR) = (SL, SR)$$



A Shifting Shifted Sequence 1

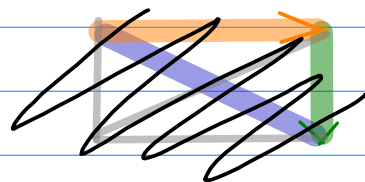
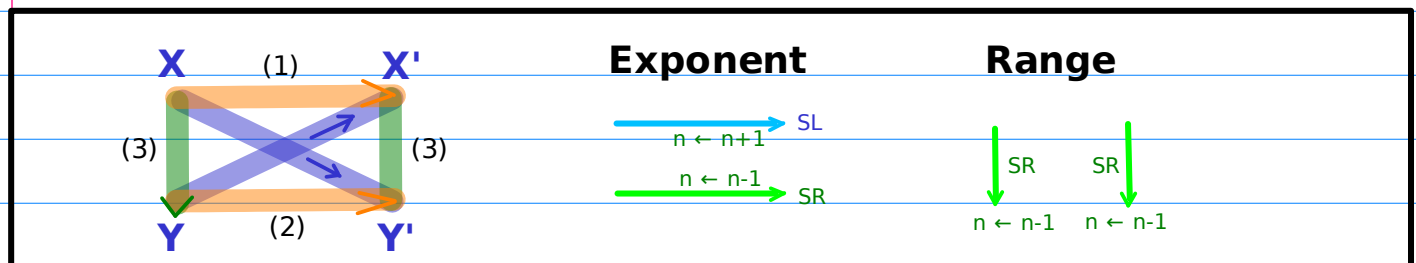
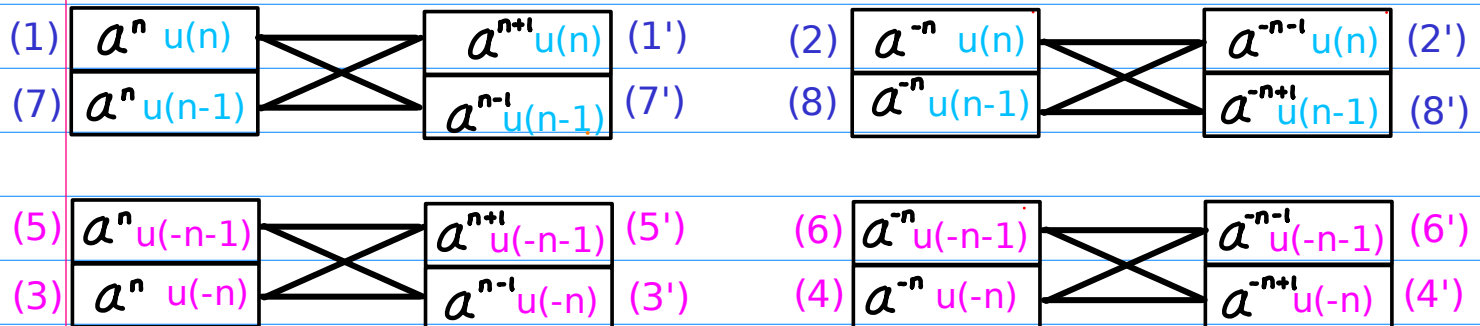
(1) Exponent Shifting

(2) Range Shifting

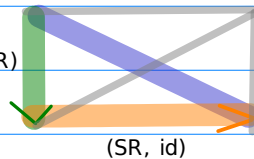
:

$$(SR, id) + (id, SR) = (SR, SR)$$

$$(SL, id) + (id, SL) = (SL, SL)$$



(id, SR)

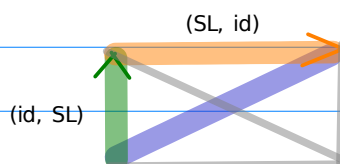


(SR, id)

(SR, id) shift right exponent

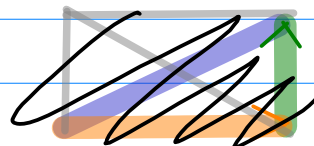
(id, SR) shift right range

(SR, SR)



(SL, id)

(id, SL)



(SL, id) shift left exponent

(id, SL) shift left range

(SL, SL)

$$(SR, id) + (id, SR) = (SR, SR)$$

$$(SL, id) + (id, SL) = (SL, SL)$$

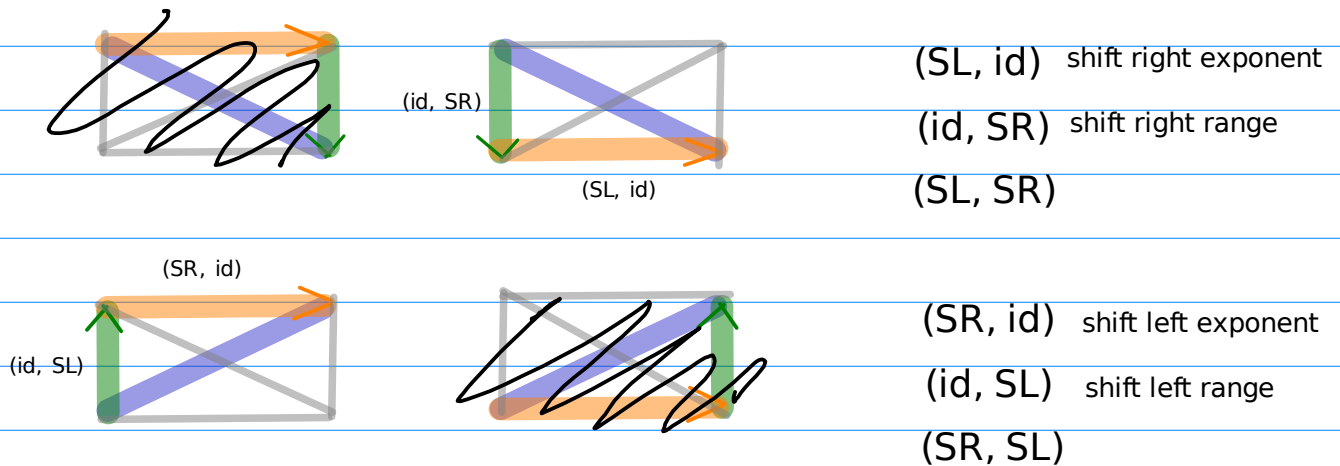
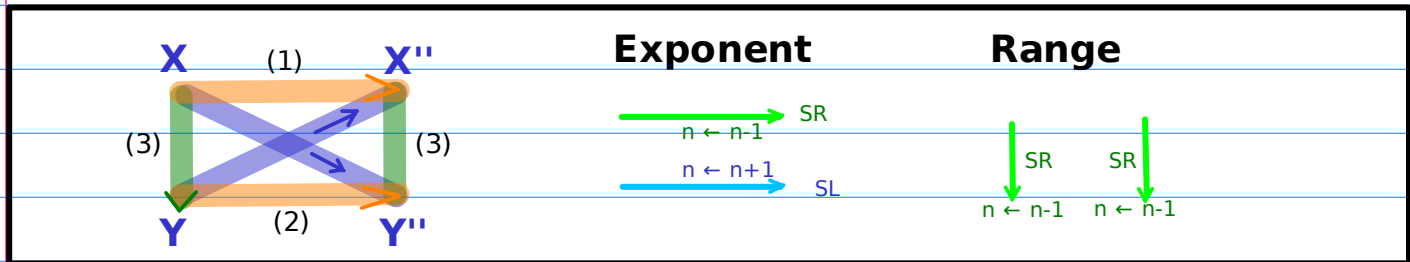
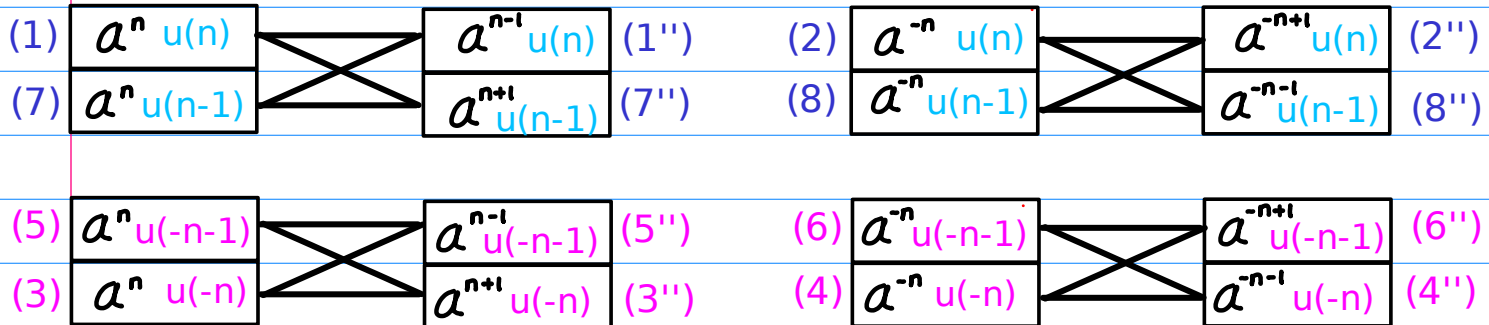
B Shifting Shifted Sequence 2

(1) Exponent Shifting

(2) Range Shifting

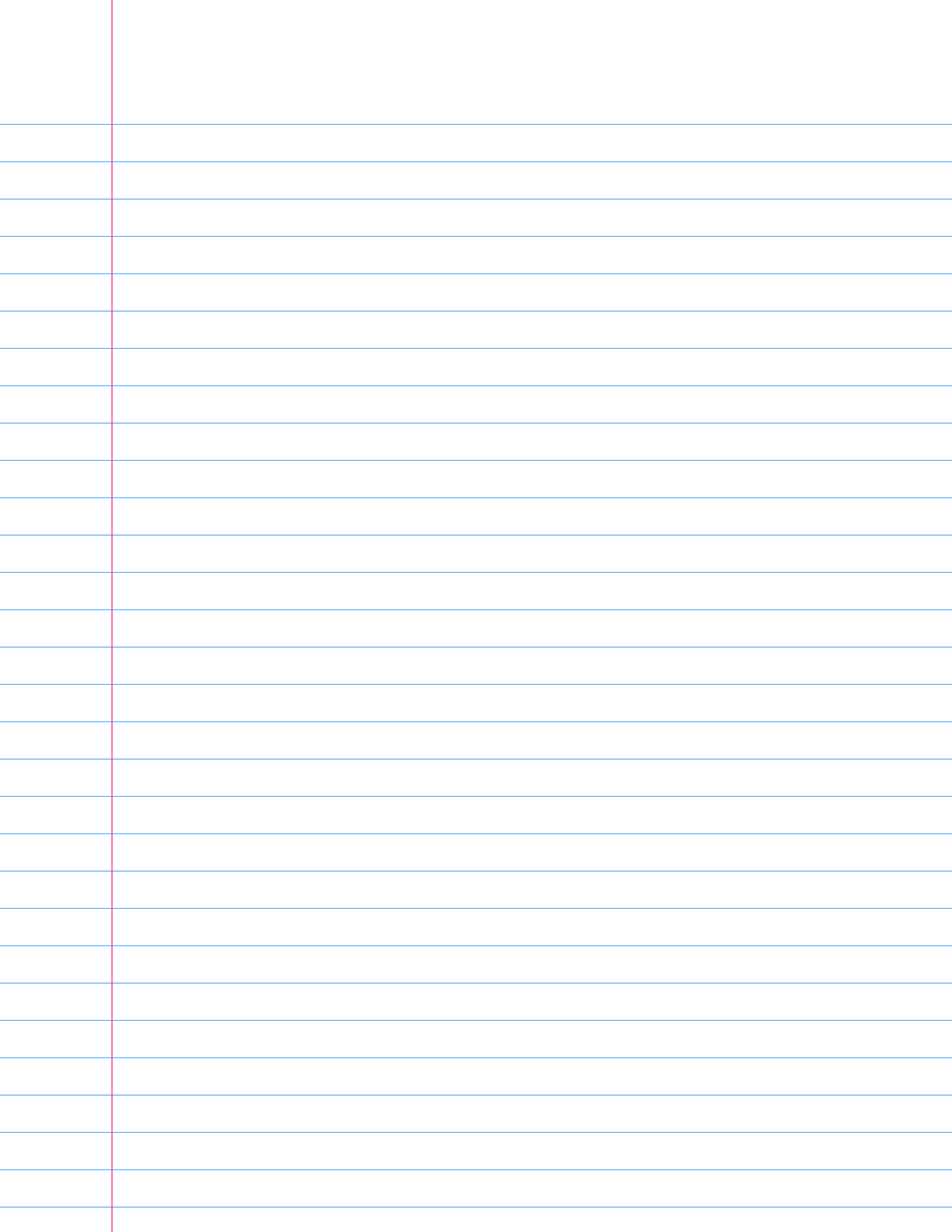
$$(SR, id) + (id, SL) = (SR, SL)$$

$$(SL, id) + (id, SR) = (SL, SR)$$



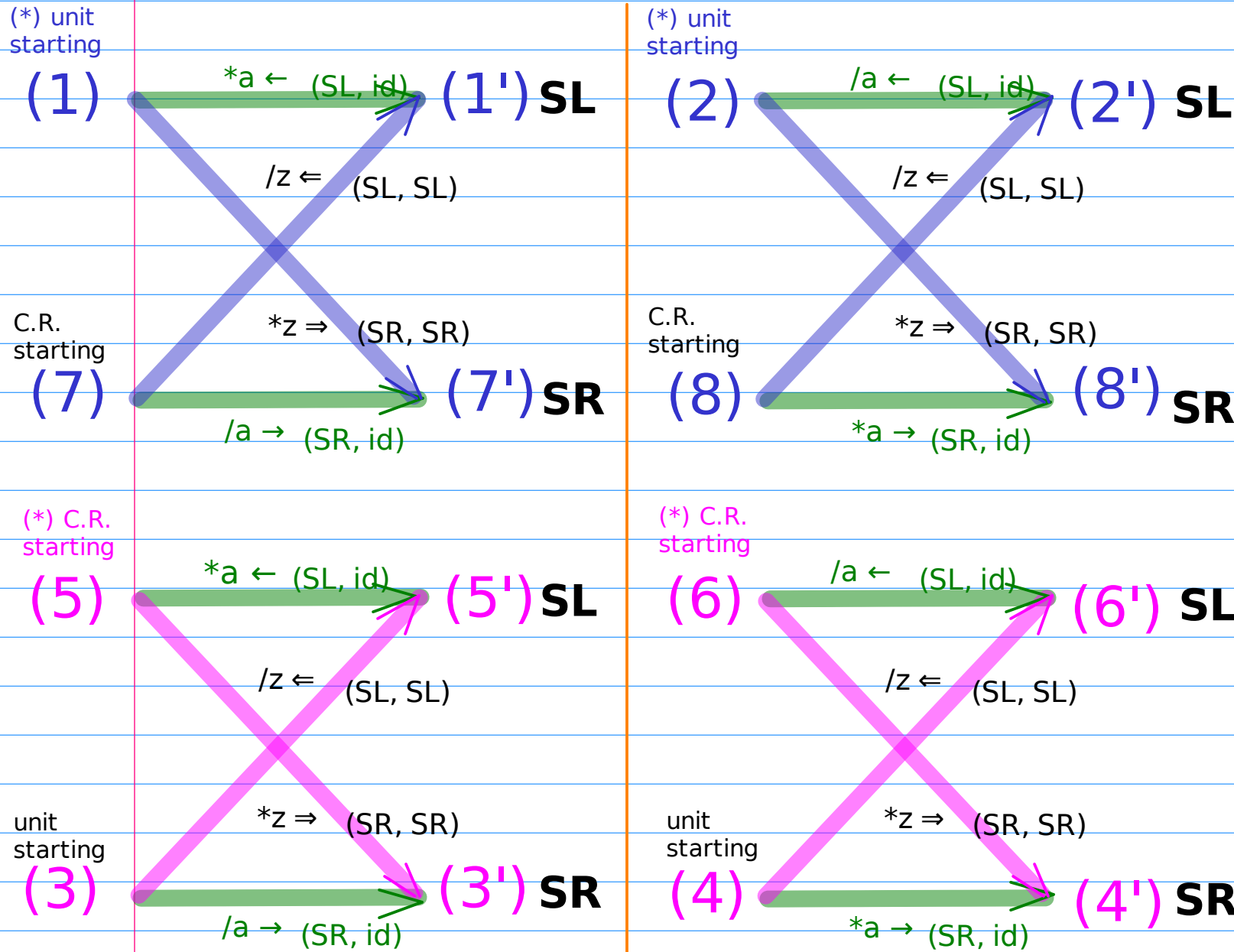
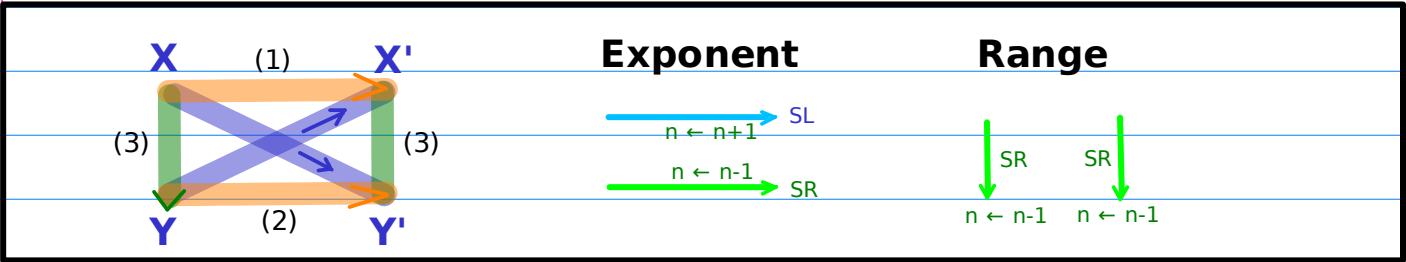
$$(SL, id) + (id, SR) = (SL, SR)$$

$$(SR, id) + (id, SL) = (SR, SL)$$



Butterfly Relations

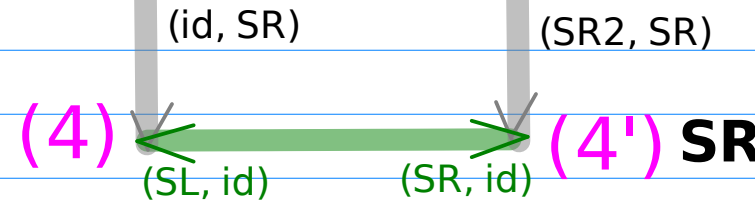
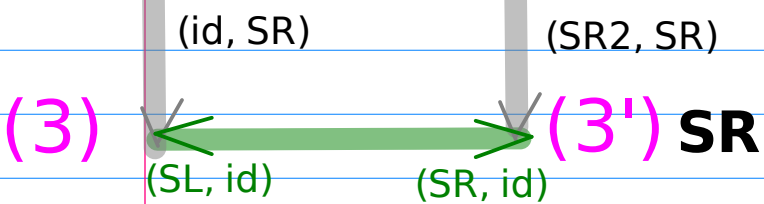
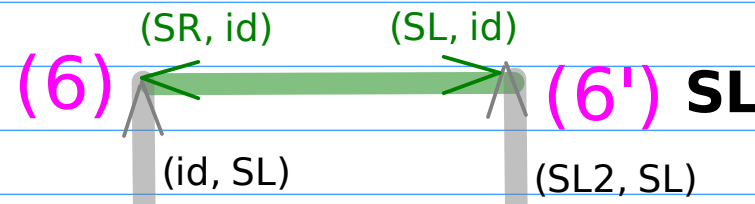
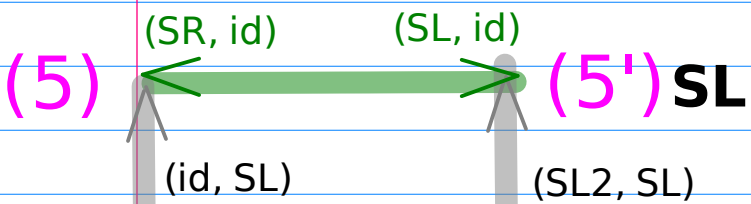
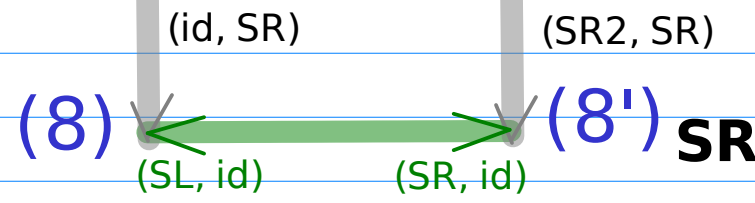
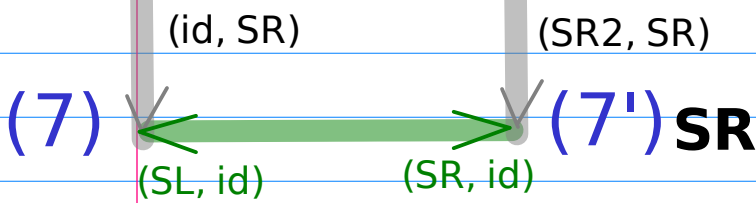
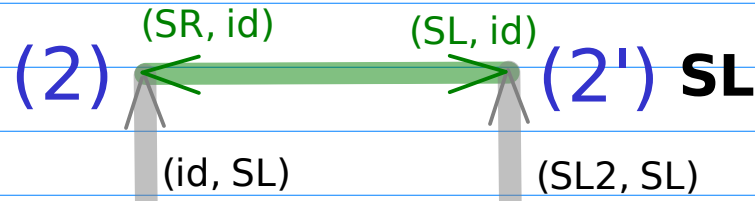
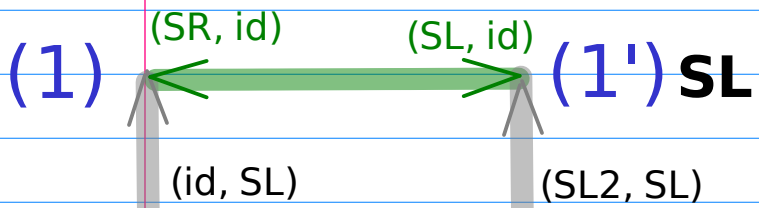
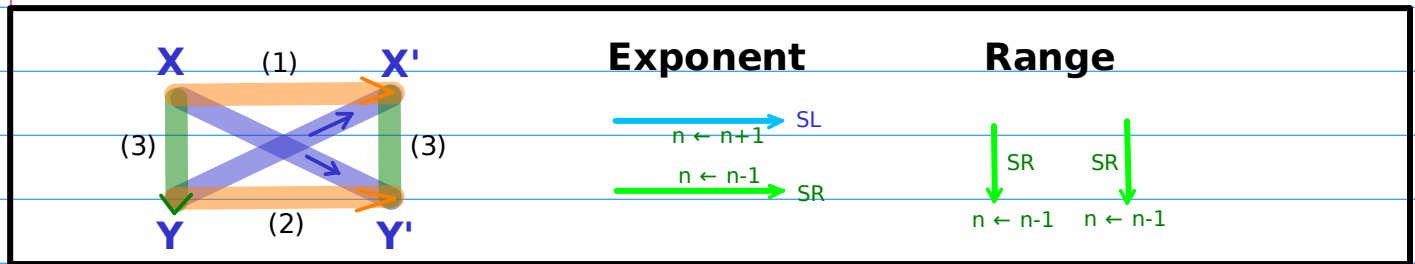
A Shifting Shifted Sequence 1
Unshifted Sequence x $\longrightarrow$ Shifted Sequence 1 x'



Butterfly Relations

A Shifting Shifted Sequence 1

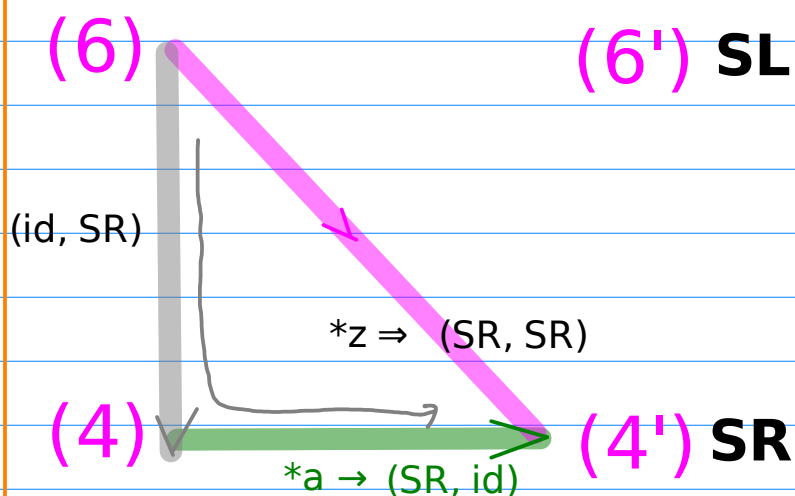
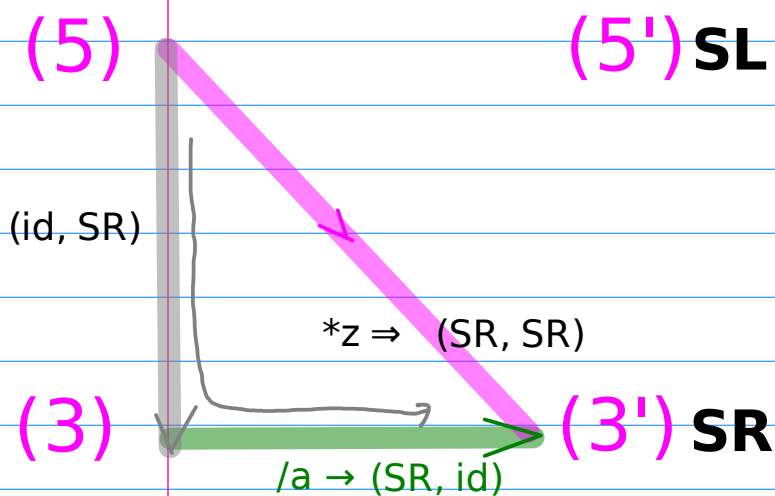
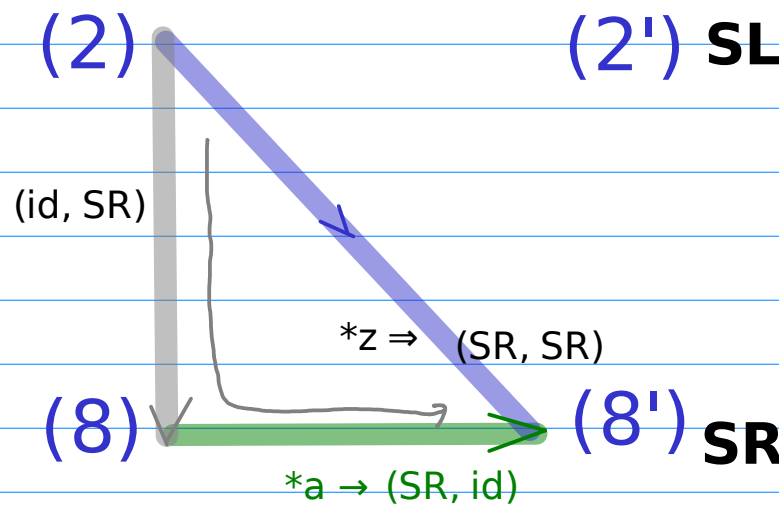
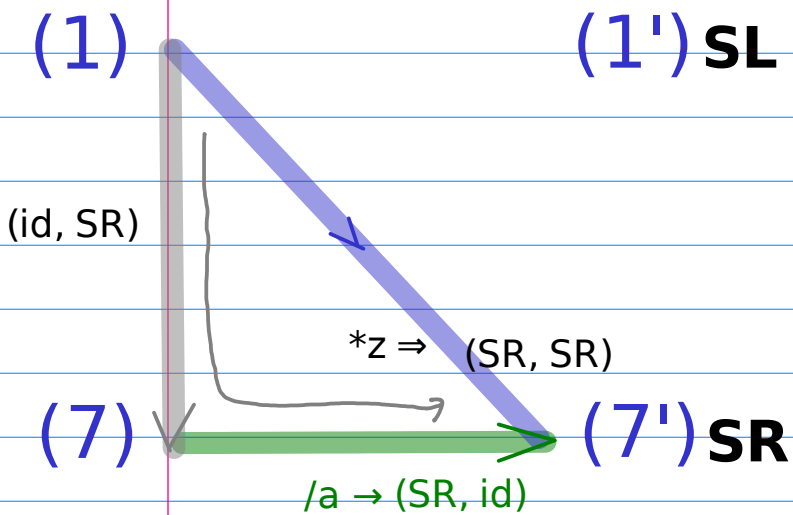
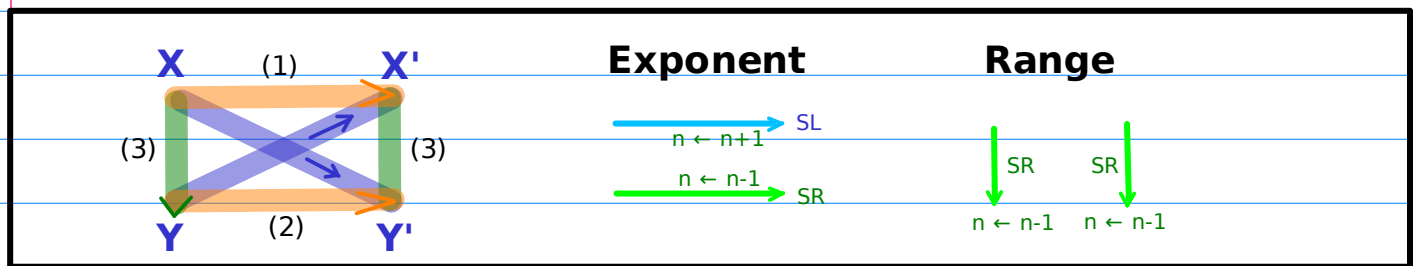
Unshifted Sequence x $\rightarrow$ Shifted Sequence 1 x'



Decomposition of Exp and Rng Shifts (1)

A Shifting Shifted Sequence 1
 Unshifted Sequence x $\longrightarrow$ Shifted Sequence 1 x'

$$(id, SR) + (SR, id) = (SR, SR)$$

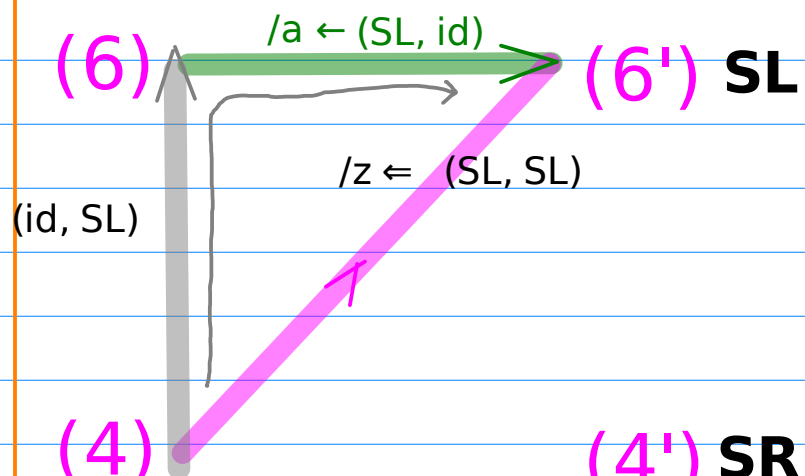
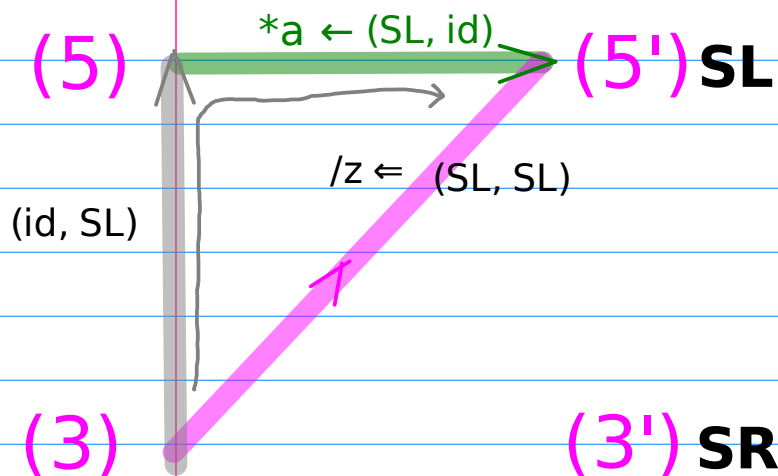
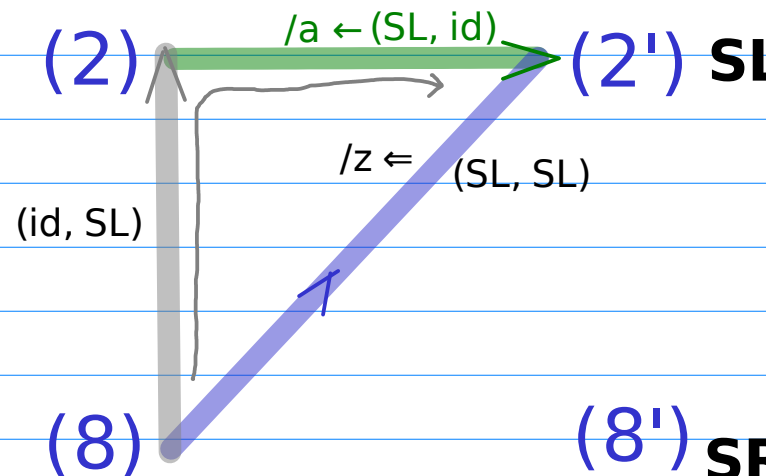
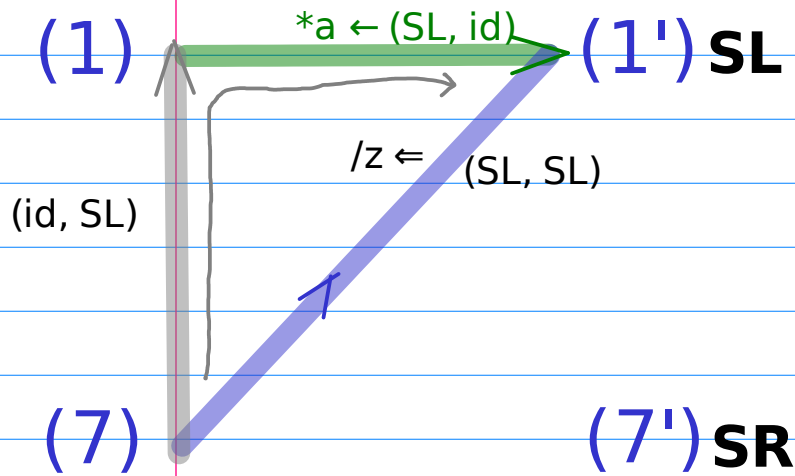
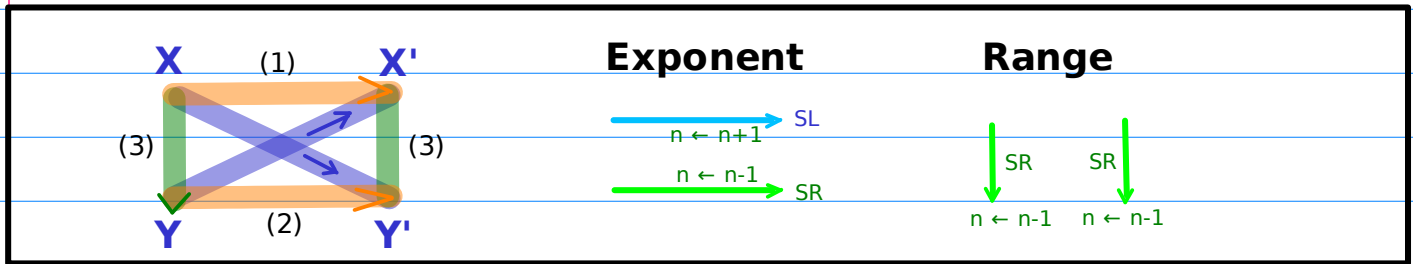


Decomposition of Exp and Rng Shifts (2)

A Shifting Shifted Sequence 1

Unshifted Sequence x $\longrightarrow$ Shifted Sequence 1 x'

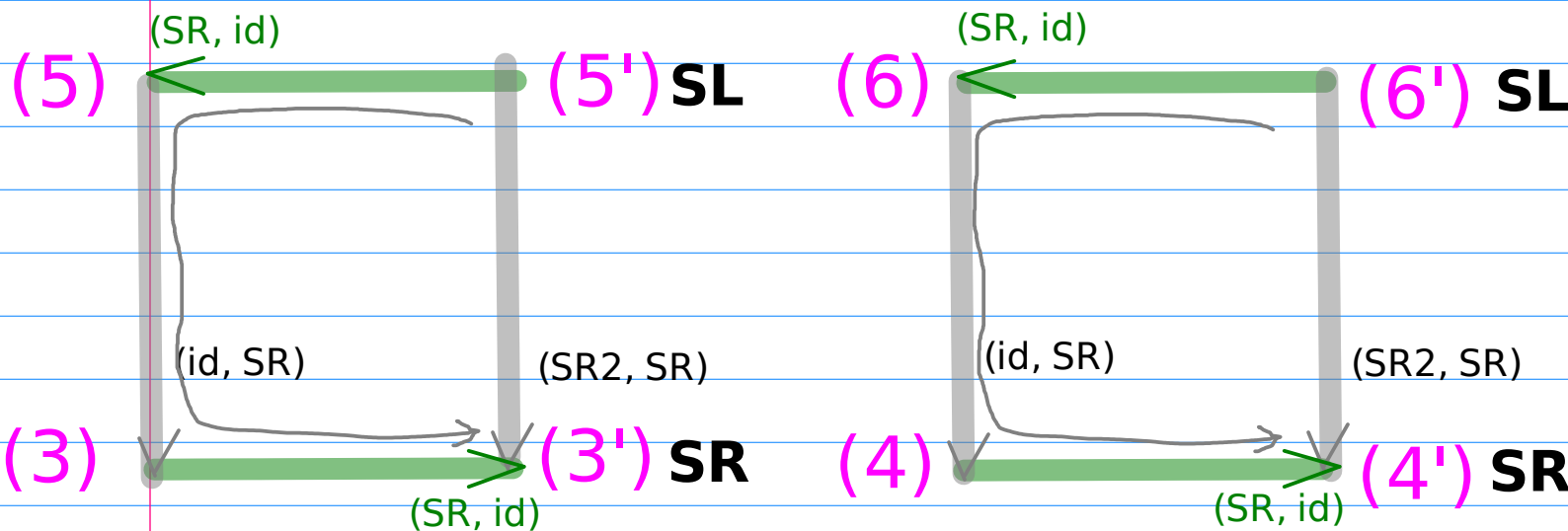
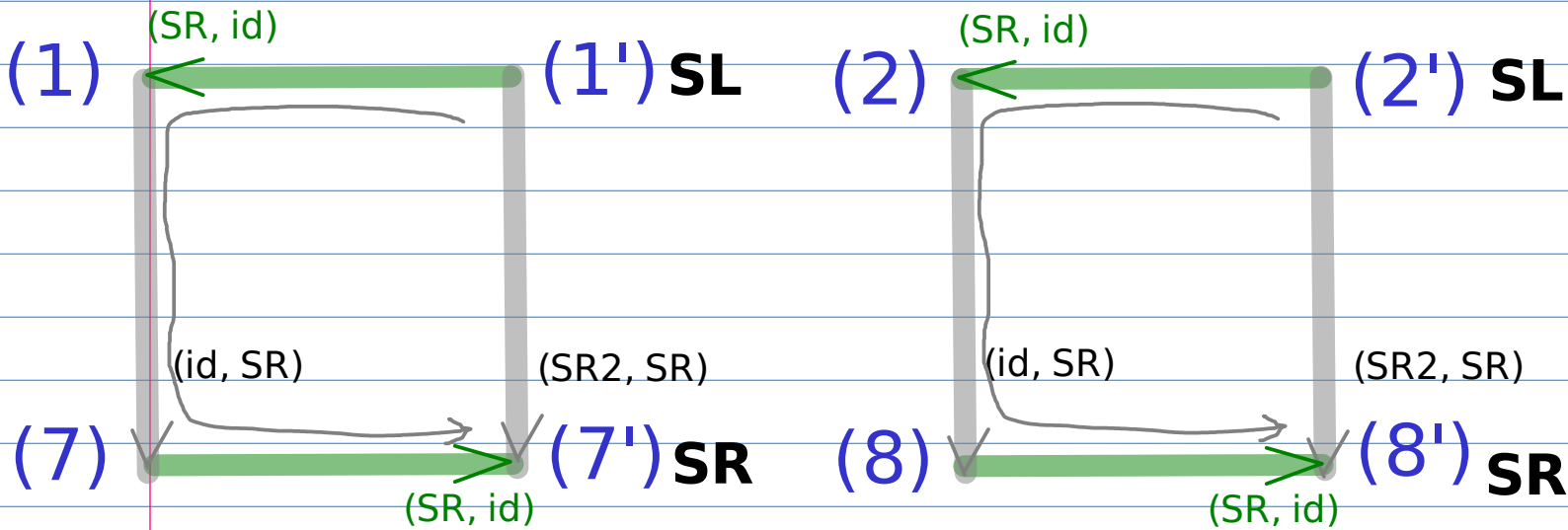
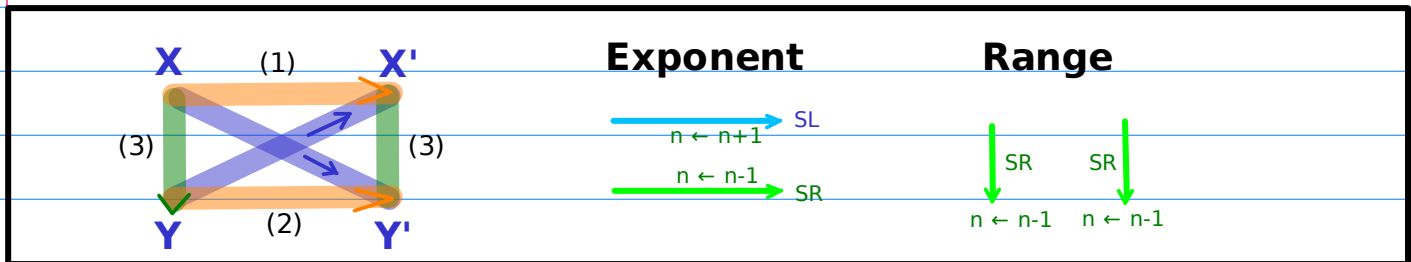
$$(id, SL) + (SL, id) = (SL, SL)$$

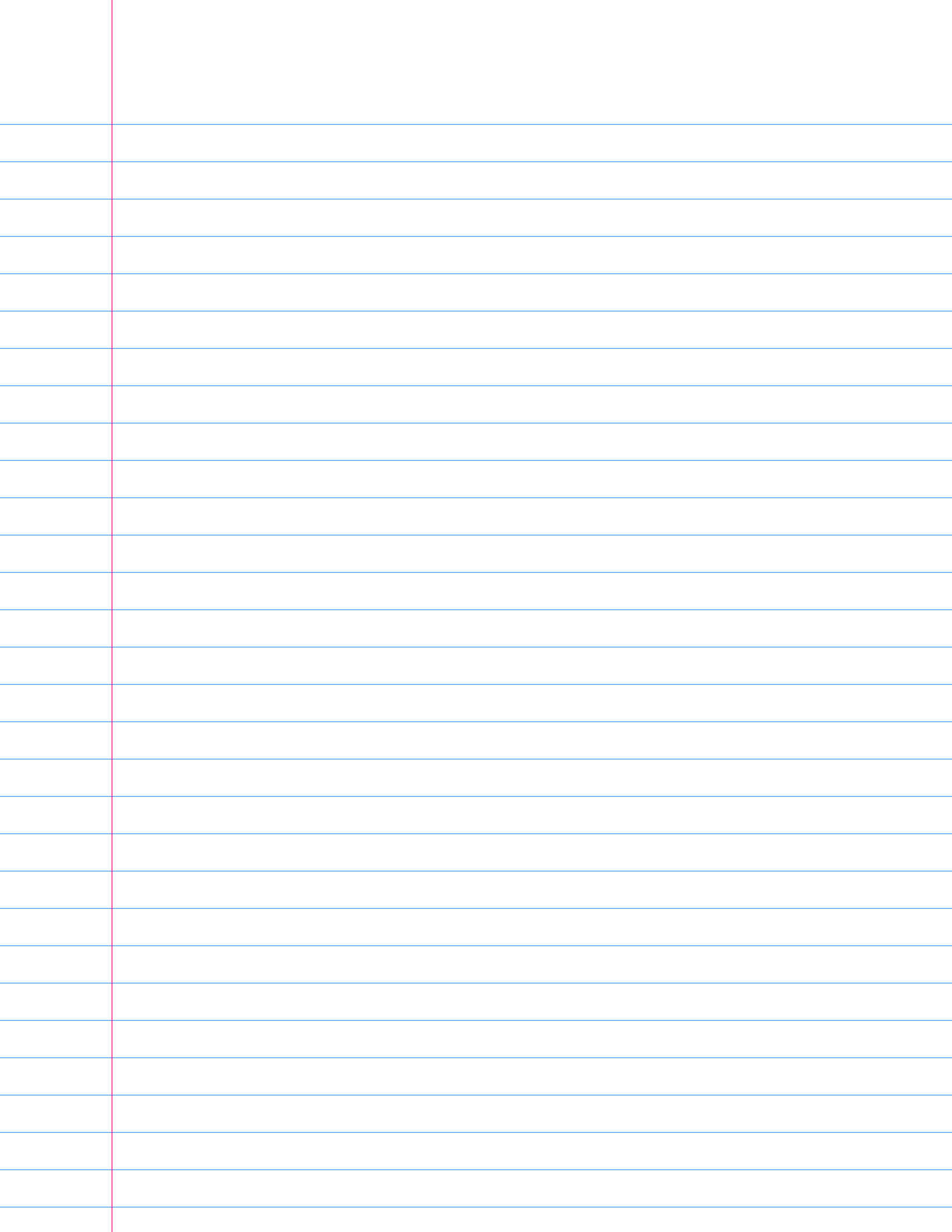


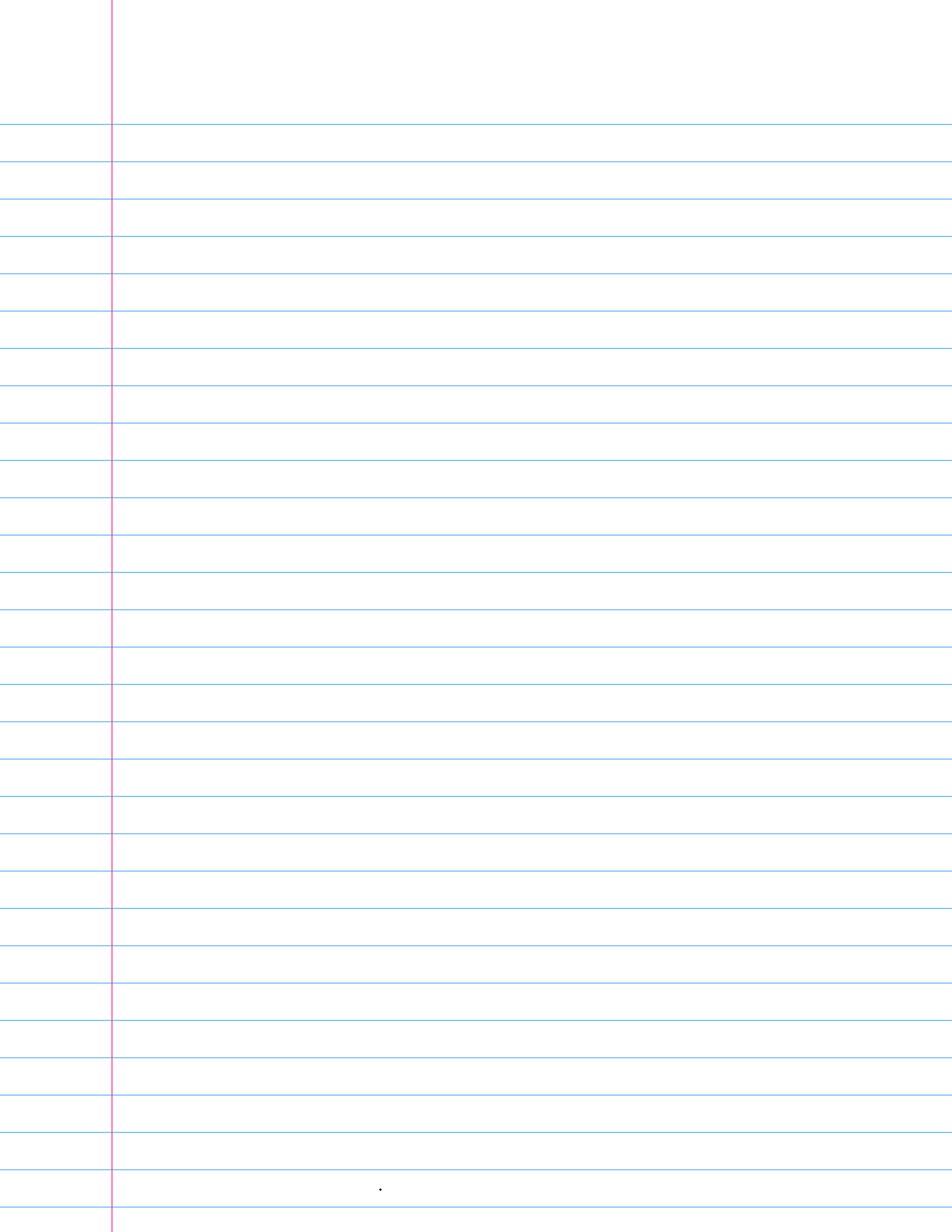
Decomposition of Exp and Rng Shifts (3)

A Shifting Shifted Sequence 1
 Unshifted Sequence $x \rightarrow$ Shifted Sequence 1 x'

$$(SR, id) + (id, SR) + (SR, id) = (SR2, SR)$$





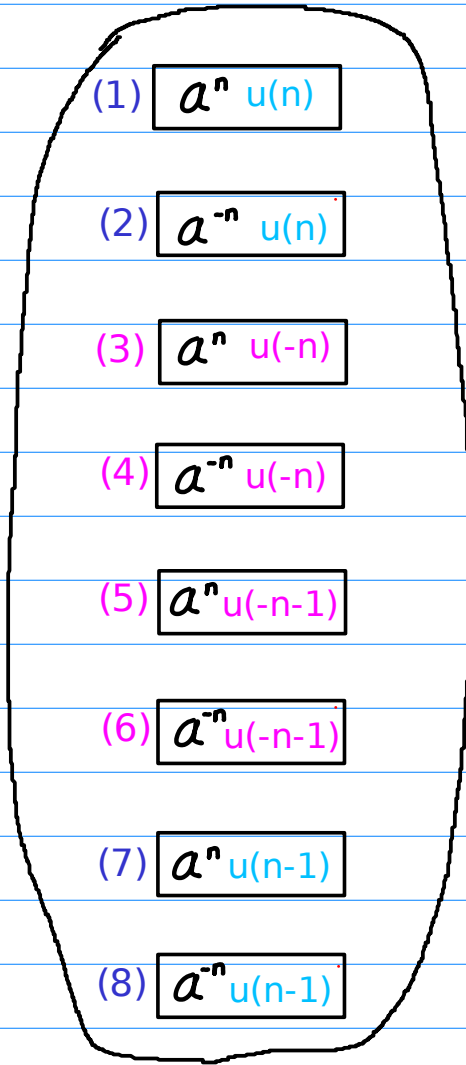
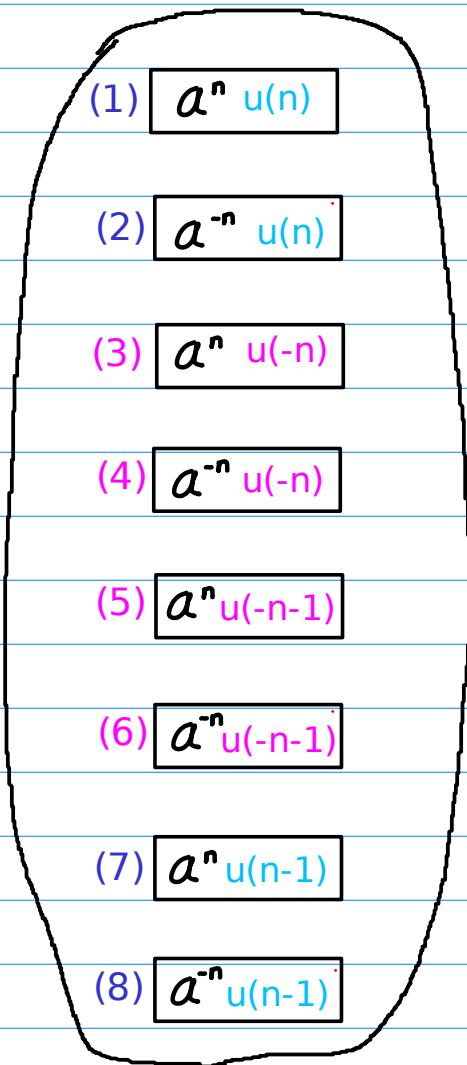


Intra-Permutations over (1) ~ (8)

Unshifted
Sequence $\times$



Unshifted
Sequence $\times$

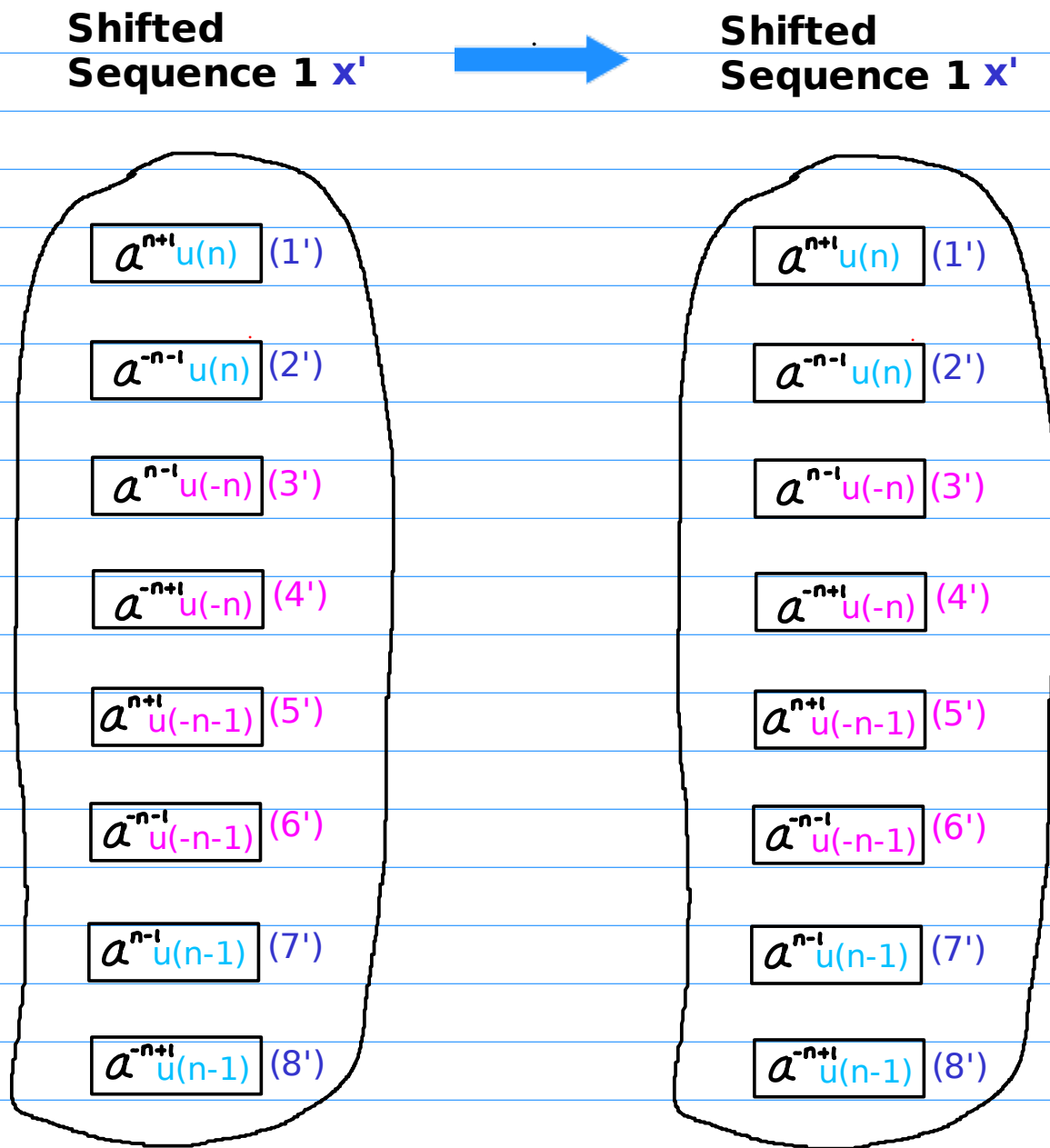


Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Intra-Permutations over (1') ~ (8')



Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

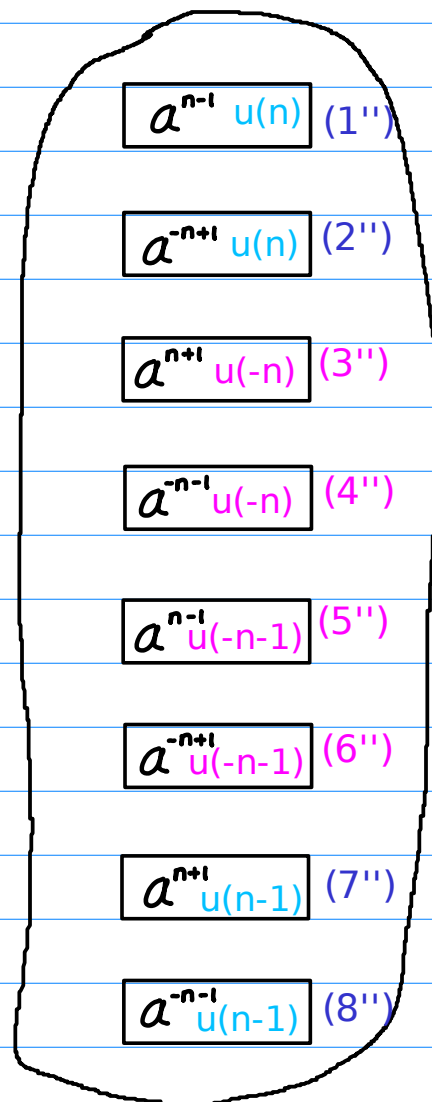
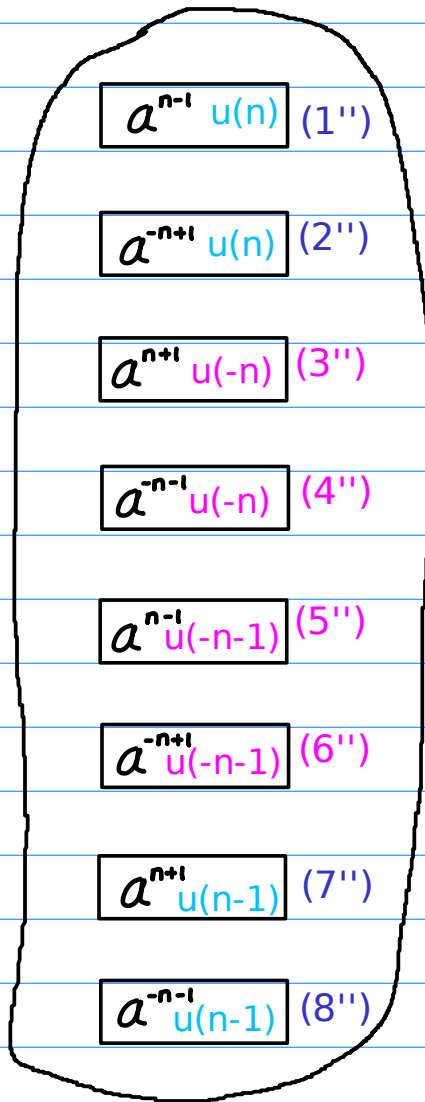
Intra-permutations over shifted sequence

Intra-Permutations over (1') ~ (8')

Shifted
Sequence 1 x''



Shifted
Sequence 1 x''



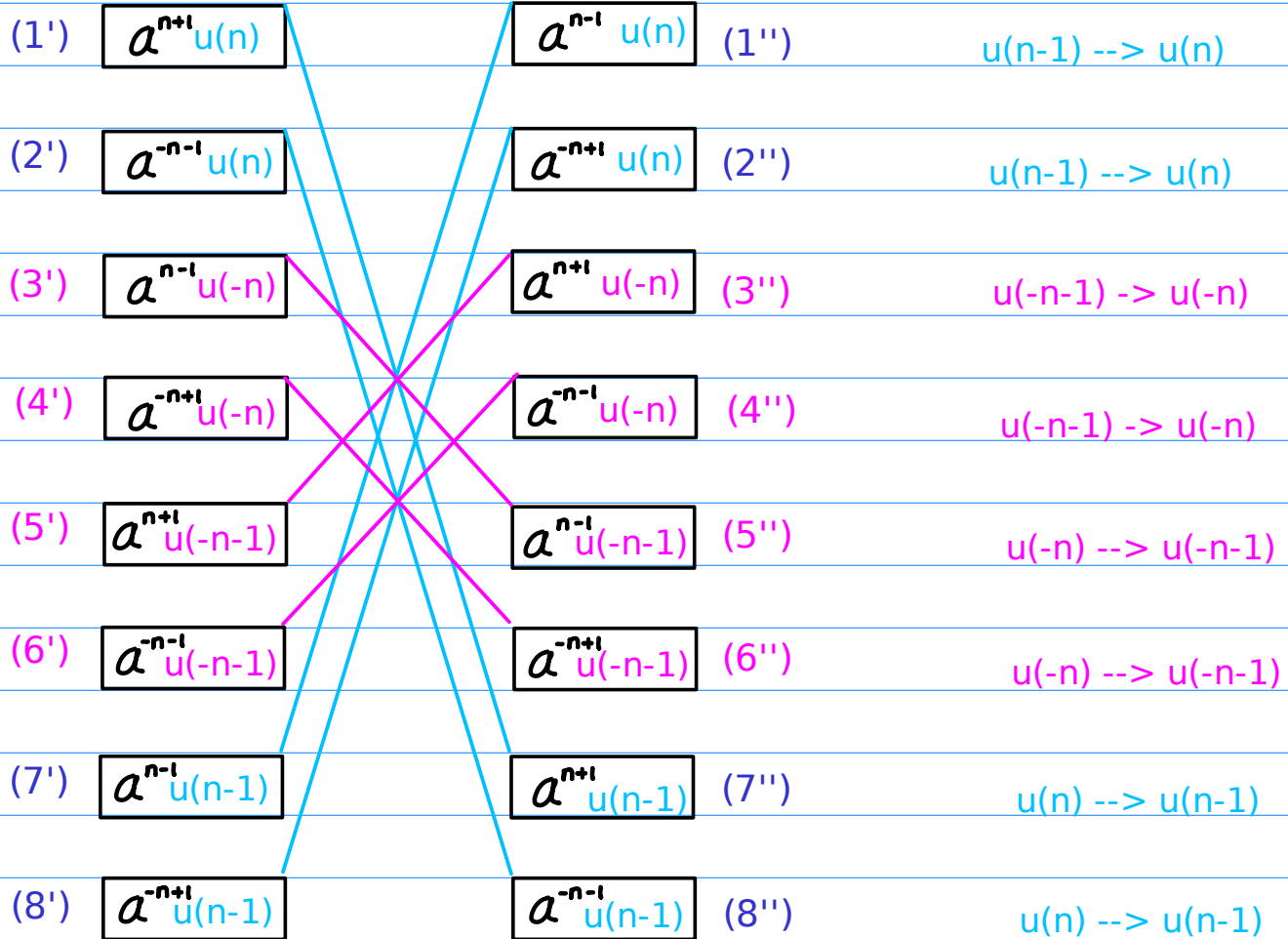
Inter-permutations over unshifted sequence and shifted sequence

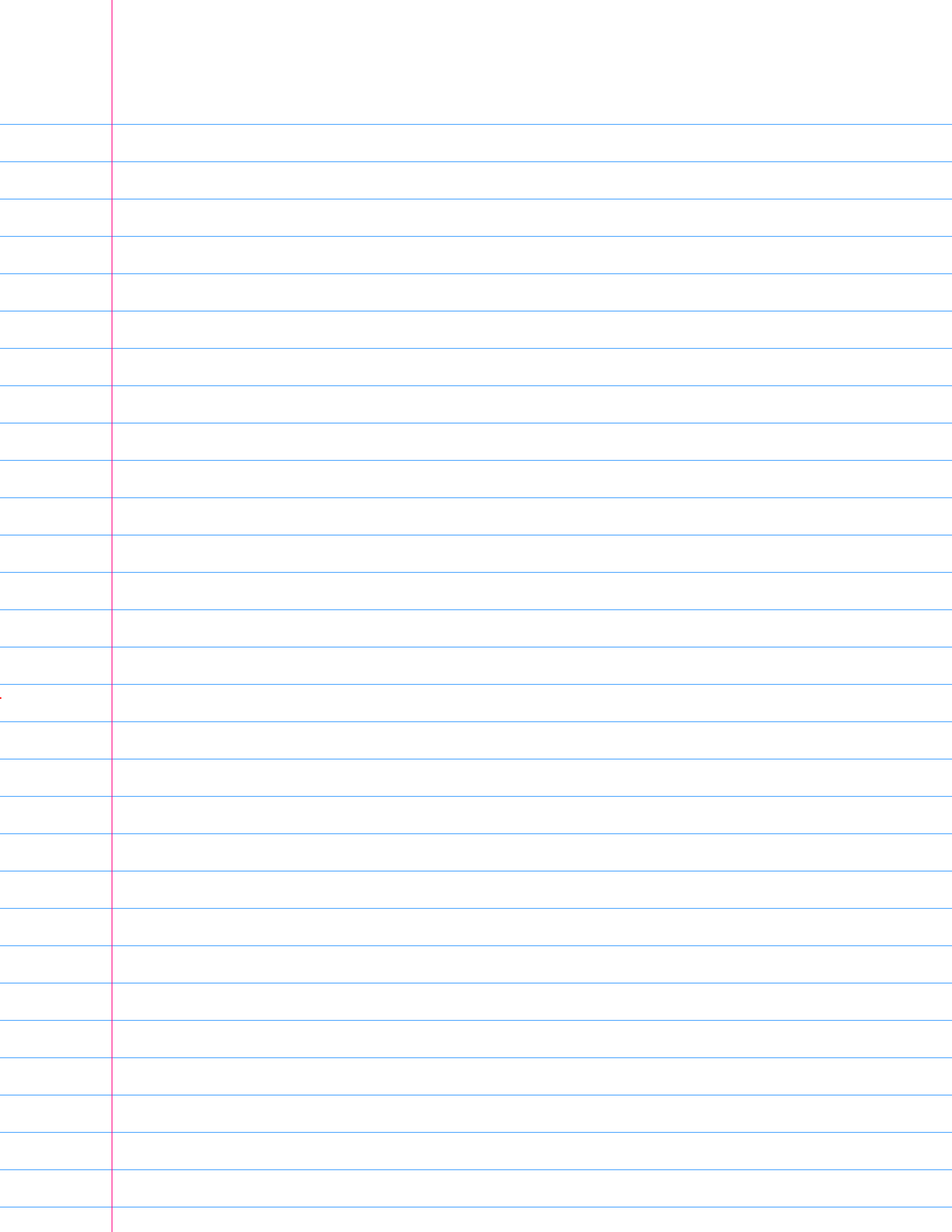
Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Shifted Sequence 1 x'

Shifted Sequence 2 x''





permutation over (1) ~ (8) (x)  (x)

A. Flipping	(1) - (4)	(5) - (8)
Base Inverting	(2) - (3)	(6) - (7)
Range Flipping	(3) - (2)	(7) - (6)
	(4) - (1)	(8) - (5)
B. <u>Range Shifting</u>	(1) - (7)	(5) - (3)
Range Flipping	(2) - (8)	(6) - (4)
Range Complementing	(3) - (5)	(7) - (1)
	(4) - (6)	(8) - (2)
C. Complementary Inverting	(1) - (6)	(5) - (2)
Base Inverting	(6) - (1)	(6) - (1)
Range Complementing	(2) - (5)	(7) - (4)
	(5) - (2)	(8) - (3)

permutation over (1') ~ (8') (x')  (x')

D. Flipping2	(1') - (4')	(5') - (8')
Base Inverting	(2') - (3')	(6') - (7')
<u>Shifted Range Flipping</u>	(3') - (2')	(7') - (6')
	(4') - (1')	(8') - (5')
E. Shifting2	(1') - (7')	(5') - (3')
Shifted Range Flipping	(2') - (8')	(6') - (4')
Range Complementing	(3') - (5')	(7') - (1')
	(4') - (6')	(8') - (2')
F. Complementary Inverting	(1') - (6')	(5') - (2')
Base Inverting	(6') - (1')	(6') - (1')
Range Complementing	(2') - (5')	(7') - (4')
	(5') - (2')	(8') - (3')

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flipping+Complementing)
= Exponent Shifting2 + Range Shifting

permutation over (1'') ~ (8'') (x'')  (x'')

D. Flipping2

Base Inverting

Shifted Range Flipping

(1'') - (4'')	(5'') - (8'')
(2'') - (3'')	(6'') - (7'')
(3'') - (2'')	(7'') - (6'')
(4'') - (1'')	(8'') - (5'')

E. Shifting2

Shifted Range Flipping

Range Complementing

(1'') - (7'')	(5'') - (3'')
(2'') - (8'')	(6'') - (4'')
(3'') - (5'')	(7'') - (1'')
(4'') - (6'')	(8'') - (2'')

F. Complementary Inverting

Base Inverting

Range Complementing

(1'') - (6'')	(5'') - (2'')
(6'') - (1'')	(6'') - (1'')
(2'') - (5'')	(7'') - (4'')
(5'') - (2'')	(8'') - (3'')

Permutation over (1) ~ (8)

Permutations	A	B	C
Base Inverting	X		X
Range Flipping	X	X	
Range Complementing		X	X

Permutation over (1') ~ (8')

Permutations	D	E	F
Base Inverting	X		X
Shifted Range Flipping	X	X	
Range Complementing		X	X

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flipping + Complementing)
= Exponent Shifting2 + Range Shifting

Over (1) ~ (8)

Base Inverting

$$a^{\textcolor{blue}{n}} \longleftrightarrow a^{-\textcolor{blue}{n}}$$

Range Flipping

$$R(\textcolor{blue}{n}) \longleftrightarrow R(-\textcolor{blue}{n})$$

Range Complementing

$$R(\textcolor{blue}{n}) \longleftrightarrow \overline{R(\textcolor{blue}{n})}$$

Over (1') ~ (8')

Base Inverting

$$a^{\textcolor{blue}{n}} \longleftrightarrow a^{-\textcolor{blue}{n}}$$

Shifted Range Flipping

$$a^{\textcolor{blue}{n}} R(\textcolor{blue}{n}) \longleftrightarrow a^{\textcolor{red}{sh}2(\textcolor{blue}{n})} R(-\textcolor{blue}{n})$$

Range Complementing

$$R(\textcolor{blue}{n}) \longleftrightarrow \overline{R(\textcolor{blue}{n})}$$

A.I Flipping

**Base Inverting
Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

D.I Flipping2

**Base Inverting
Shifted Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

B.I Range Shifting

**Range Flipping
Range Complementing**

$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

E.I Shifting2

**Shifted Range Flipping
Range Complementing**

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

C.I Complementary Inverting

**Base Inverting
Range Complementing**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

F.I Complementary Inverting

**Base Inverting
Range Complementing**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

G.I Flipping2

**Base Inverting
Shifted Range Flipping**

$$\begin{aligned}a^n &\longleftrightarrow a^{-n} \\ a^n R(n) &\longleftrightarrow a^{sh2(n)} R(-n) \\ a^n R(n) &\longleftrightarrow a^{-sh2(n)} R(-n)\end{aligned}$$

H.I Shifting2

**Shifted Range Flipping
Range Complementing**

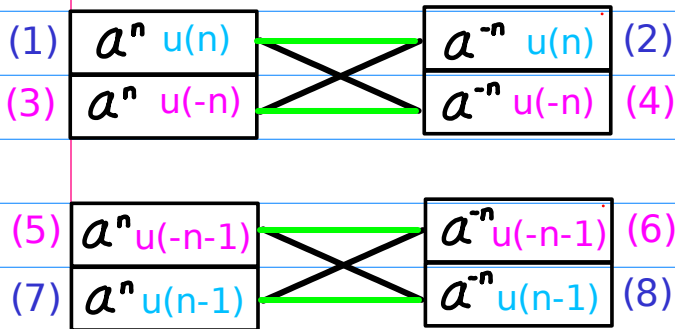
$$\begin{aligned}a^n R(n) &\longleftrightarrow a^{sh2(n)} R(-n) \\ R(n) &\longleftrightarrow \overline{R(n)} \\ a^n R(n) &\longleftrightarrow a^{sh2(n)} \overline{R(-n)}\end{aligned}$$

I.I Complementary Inverting

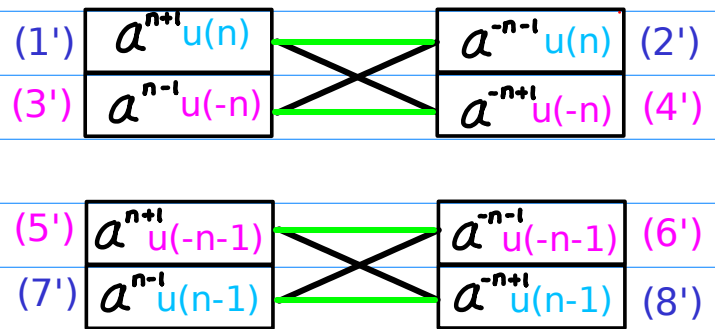
**Base Inverting
Range Complementing**

$$\begin{aligned}a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \\ a^n R(n) &\longleftrightarrow a^{-n} \overline{R(n)}\end{aligned}$$

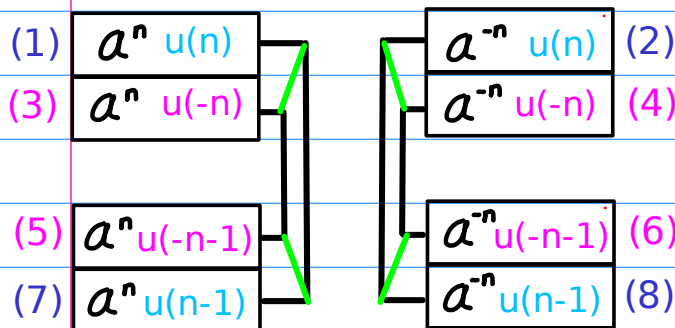
A.I Flipping Base Inverting Range Flipping



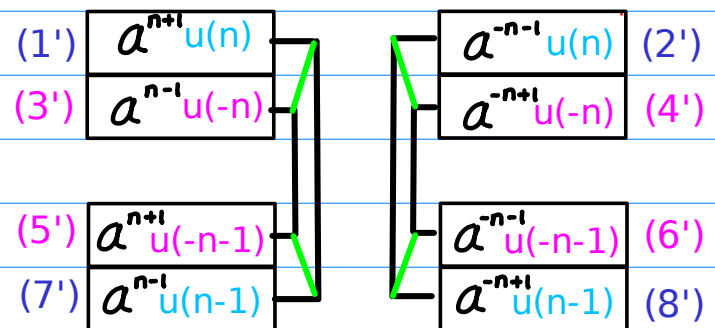
D.I Flipping2 Base Inverting Shifted Range Flipping



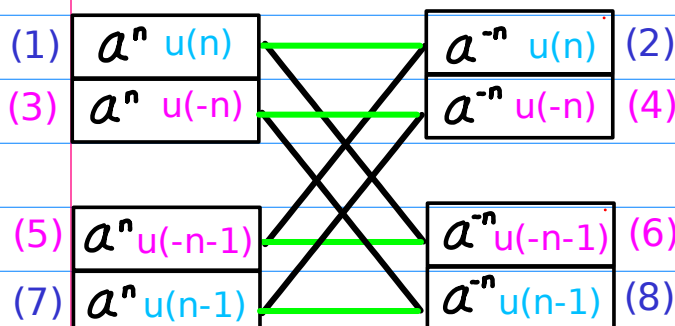
B.I Range Shifting Range Flipping Range Complementing



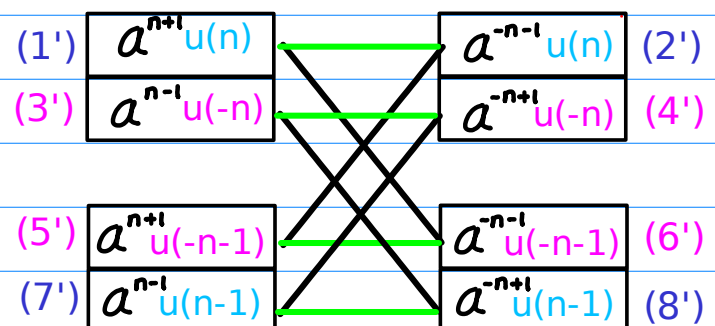
E.I Shifting2 Shifted Range Flipping Range Complementing



C.I Complementary Inverting Base Inverting Range Complementing



F.I Complementary Inverting Base Inverting Range Complementing



$$shl(b^n) \longleftrightarrow shr(b^n)$$

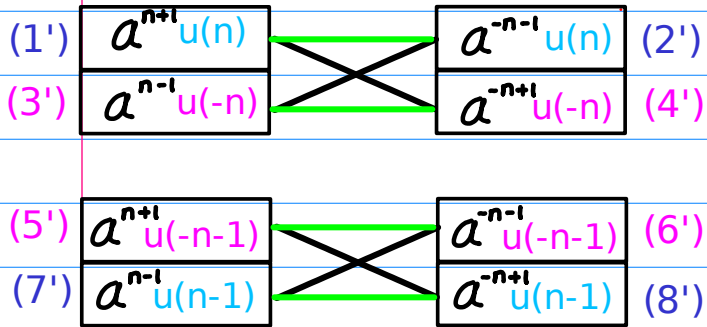
$$shr(b^n) \longleftrightarrow shl(b^n)$$

$$a^{(n+1)} \longleftrightarrow a^{(n-1)}$$

$$a^{-(n+1)} \longleftrightarrow a^{-(n-1)}$$

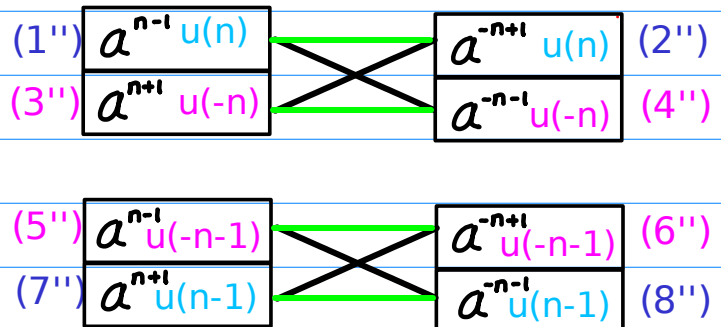
D.I Flipping2

**Base Inverting
Shifted Range Flipping**



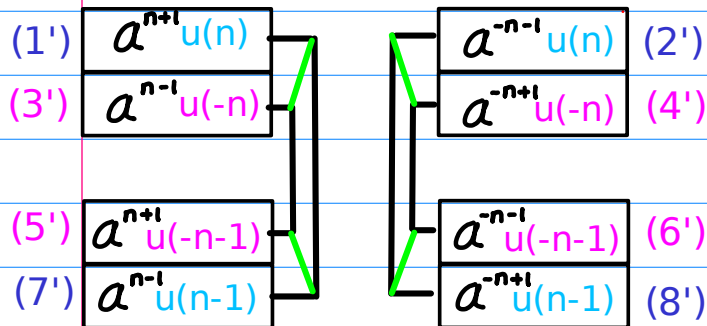
G.I Flipping2

**Base Inverting
Shifted Range Flipping**



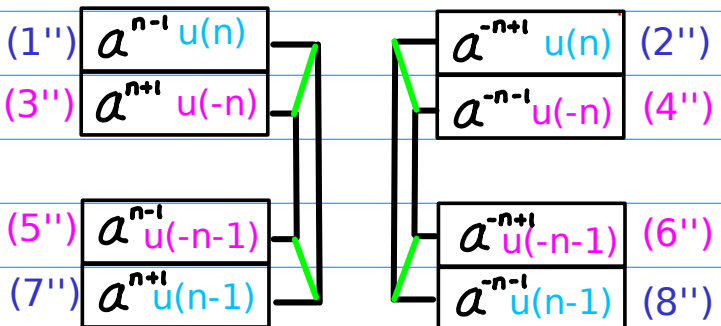
E.I Shifting2

**Shifted Range Flipping
Range Complementing**



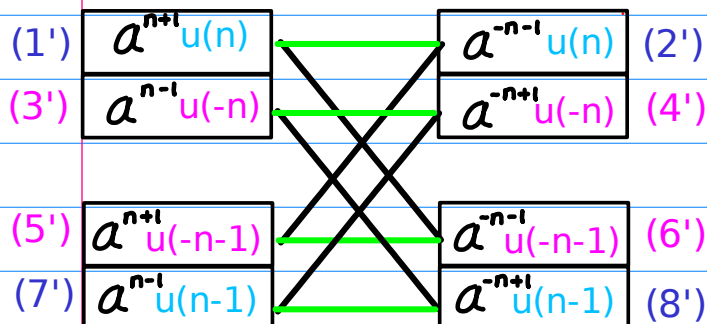
H.I Shifting2

**Shifted Range Flipping
Range Complementing**



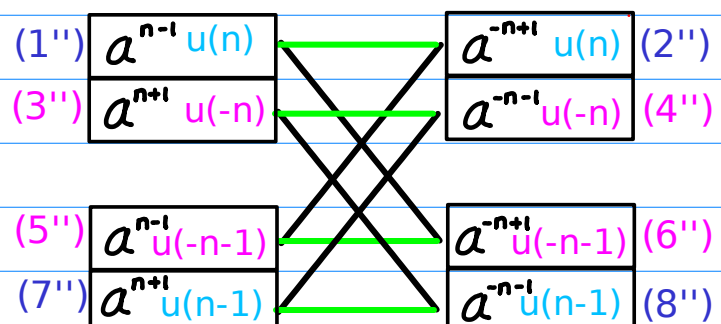
F.I Complementary Inverting

**Base Inverting
Range Complementing**



I.I Complementary Inverting

**Base Inverting
Range Complementing**



D.I Flipping2
Base Inverting
Shifted Range Flipping

G.I Flipping2
Base Inverting
Shifted Range Flipping

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (3') & a^{n-l}u(-n) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-l}u(n) & (2') \\ a^{-n+l}u(-n) & (4') \end{array}$$

$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (3'') & a^{n+l}u(-n) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n+l}u(n) & (2'') \\ a^{-n-l}u(-n) & (4'') \end{array}$$

$$\begin{array}{cc} (5') & a^{n+l}u(-n-1) \\ (7') & a^{n-l}u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n-1) & (6') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (5'') & a^{n-l}u(-n-1) \\ (7'') & a^{n+l}u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n-1) & (6'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

H.I Shifting2
Shifted Range Flipping
Range Complementing

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (5') & a^{n+l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{n-l}u(-n) & (3') \\ a^{n-l}u(n-1) & (7') \end{array}$$

$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (5'') & a^{n-l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{n+l}u(-n) & (3'') \\ a^{n+l}u(n-1) & (7'') \end{array}$$

$$\begin{array}{cc} (2') & a^{-n-l}u(n) \\ (6') & a^{-n-l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n) & (4') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (2'') & a^{-n+l}u(n) \\ (6'') & a^{-n+l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n) & (4'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

F.I Complementary Inverting
Base Inverting
Range Complementing

I.I Complementary Inverting
Base Inverting
Range Complementing

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (5') & a^{n+l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-l}u(n) & (2') \\ a^{-n-l}u(-n-1) & (6') \end{array}$$

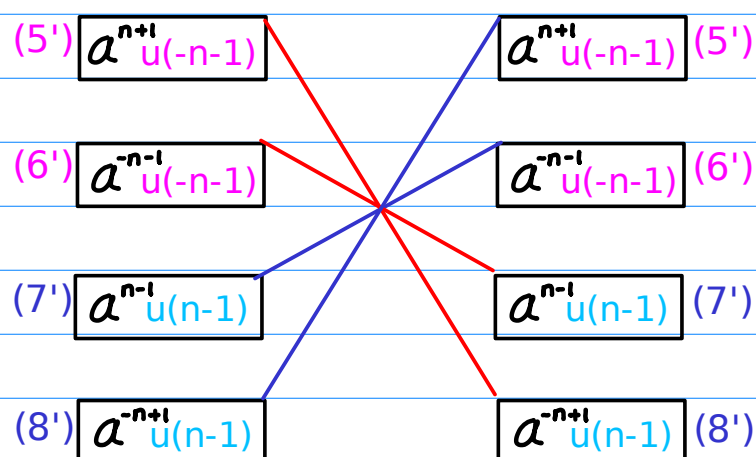
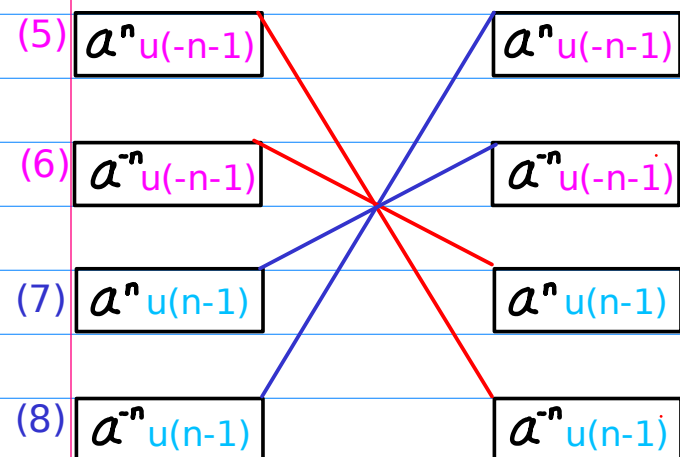
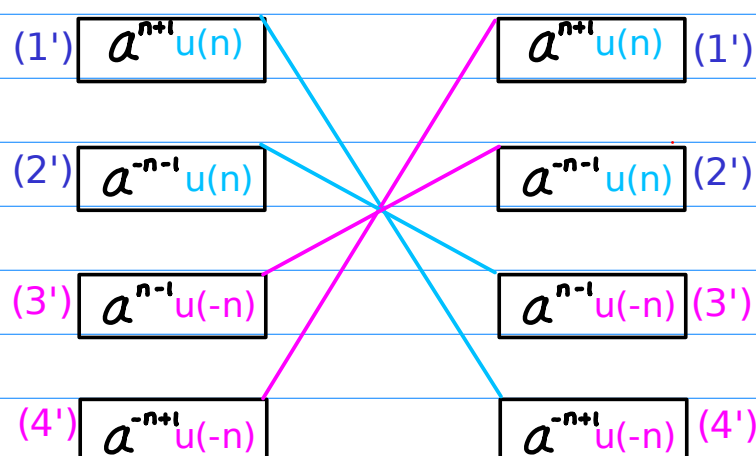
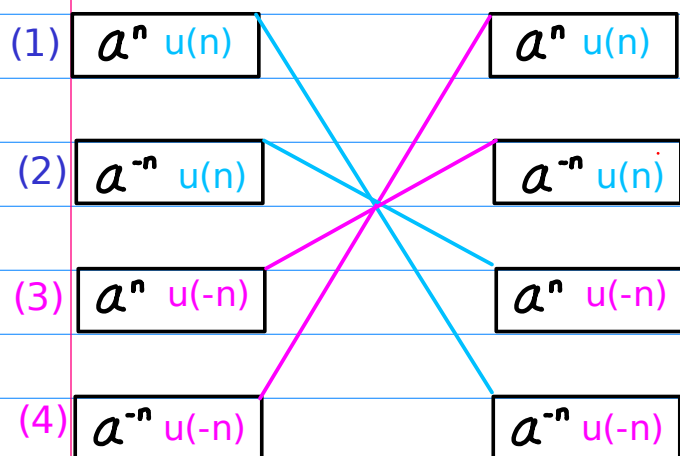
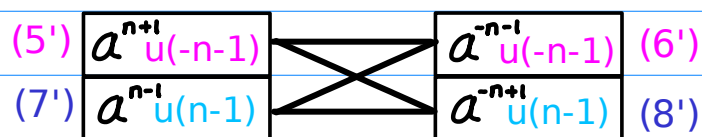
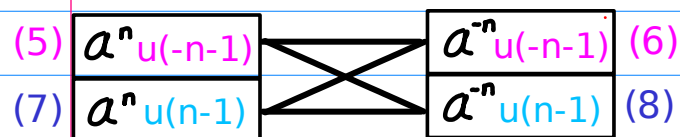
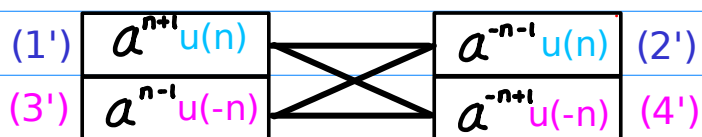
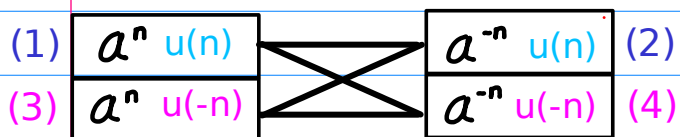
$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (5'') & a^{n-l}u(-n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n+l}u(n) & (2'') \\ a^{-n+l}u(-n-1) & (6'') \end{array}$$

$$\begin{array}{cc} (3') & a^{n-l}u(-n) \\ (7') & a^{n-l}u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n) & (4') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (3'') & a^{n+l}u(-n) \\ (4'') & a^{n+l}u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n) & (4'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

A.I Flipping Base Inverting Range Flipping

D.I Flipping2 Base Inverting Shifted Range Flipping

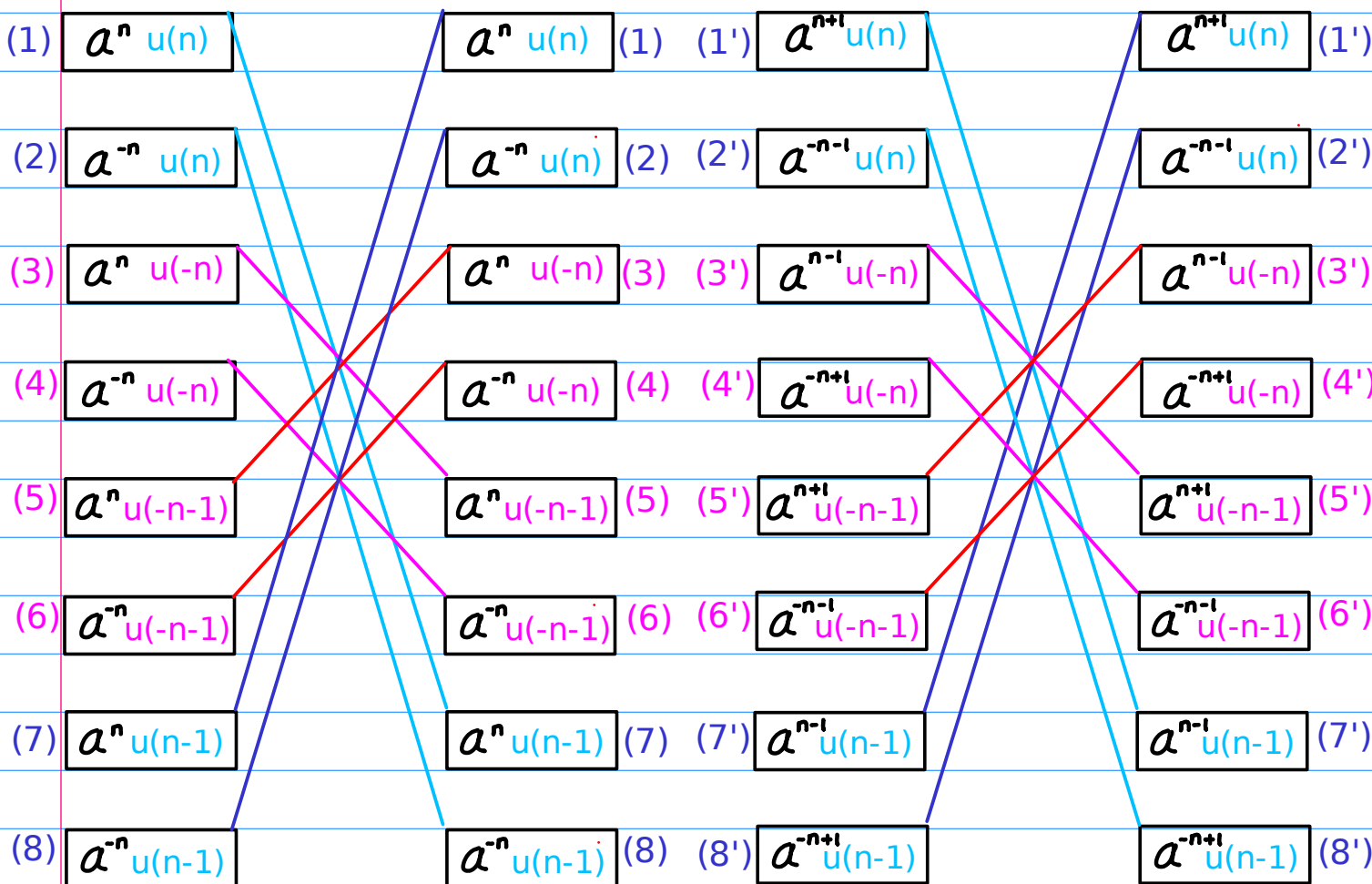
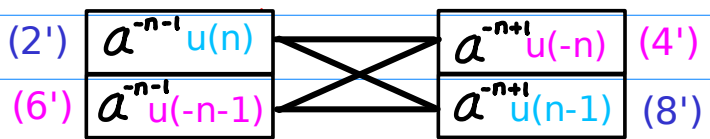
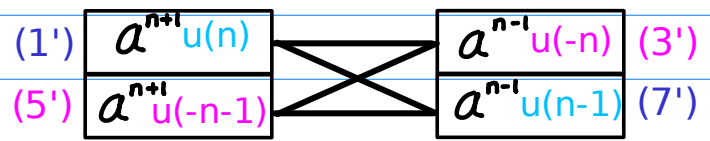
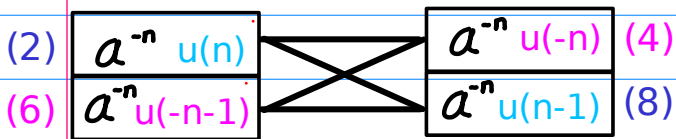
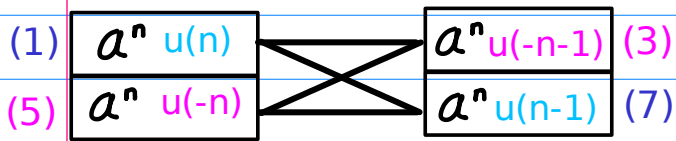


(1) - (4) (5) - (8)
 (2) - (3) (6) - (7)
 (3) - (2) (7) - (6)
 (4) - (1) (8) - (5)

(1') - (4') (5') - (8')
 (2') - (3') (6') - (7')
 (3') - (2') (7') - (6')
 (4') - (1') (8') - (5')

B.I Range Shifting Range Flipping Range Complementing

E.I Shifting2 Shifted Range Flipping Range Complementing



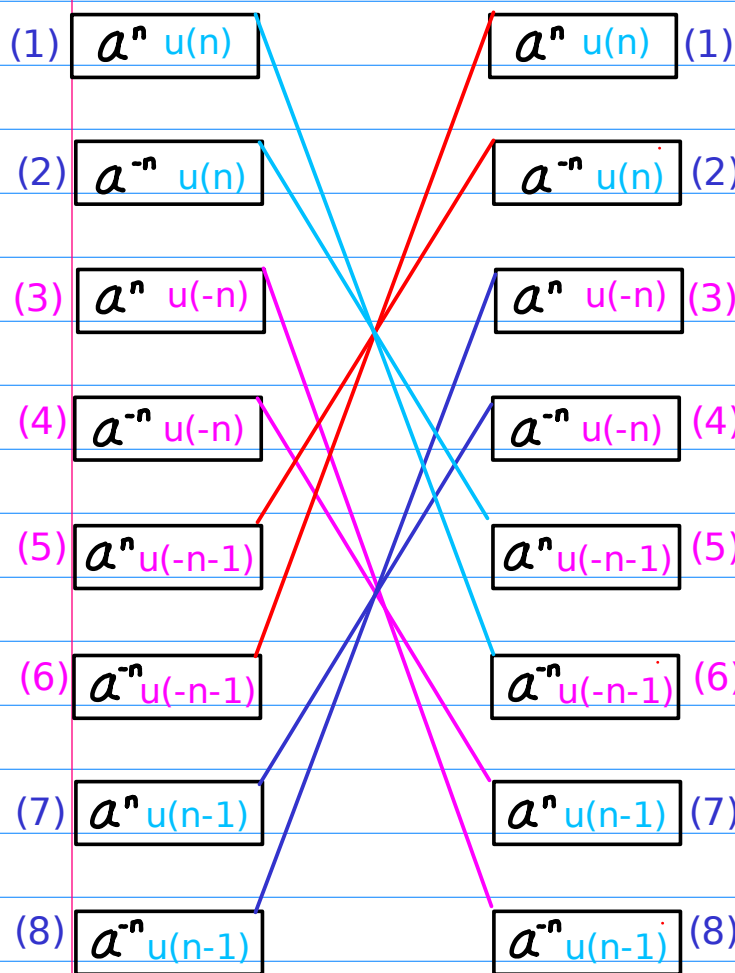
(1) - (7)
(2) - (8)
(3) - (5)
(4) - (6)
(5) - (3)
(6) - (4)
(7) - (1)
(8) - (2)

(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')
(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

C.I Complementary Inverting Base Inverting Range Complementing

$$\begin{array}{cc} (1) & a^n u(n) \\ (5) & a^n u(-n-1) \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} a^{-n} u(n) & (2) \\ a^{-n} u(-n-1) & (6) \end{array}$$

$$\begin{array}{cc} (3) & a^n u(-n) \\ (7) & a^n u(n-1) \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} a^{-n} u(-n) & (4) \\ a^{-n} u(n-1) & (8) \end{array}$$

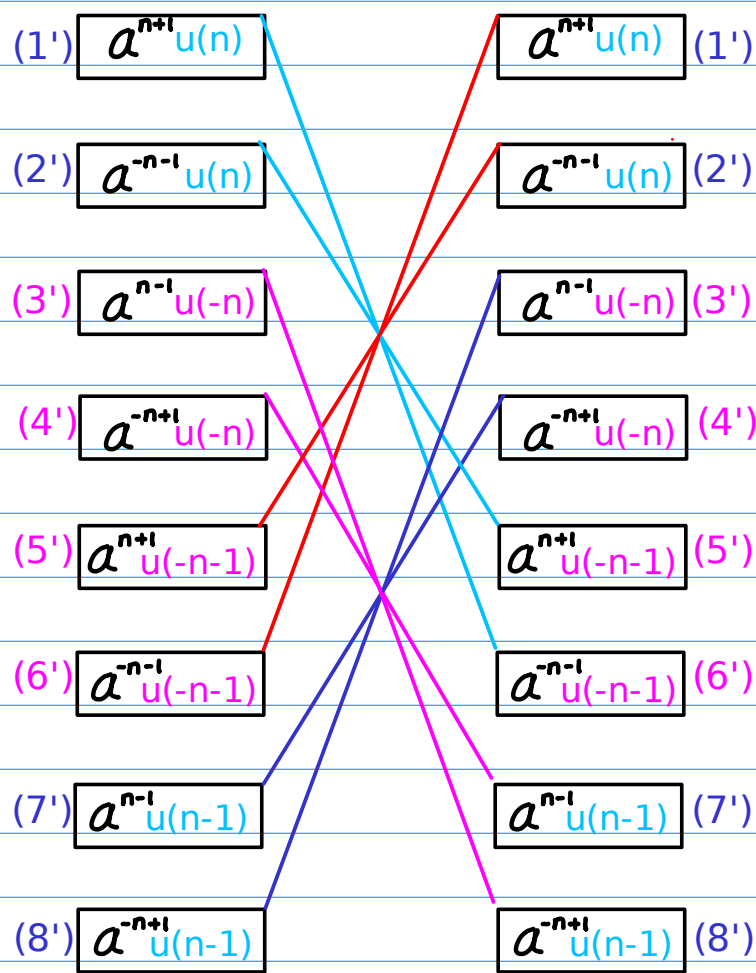


$(1) - (6)$
 $(6) - (1)$
 $(2) - (5)$
 $(5) - (2)$
 $(5) - (2)$
 $(8) - (3)$

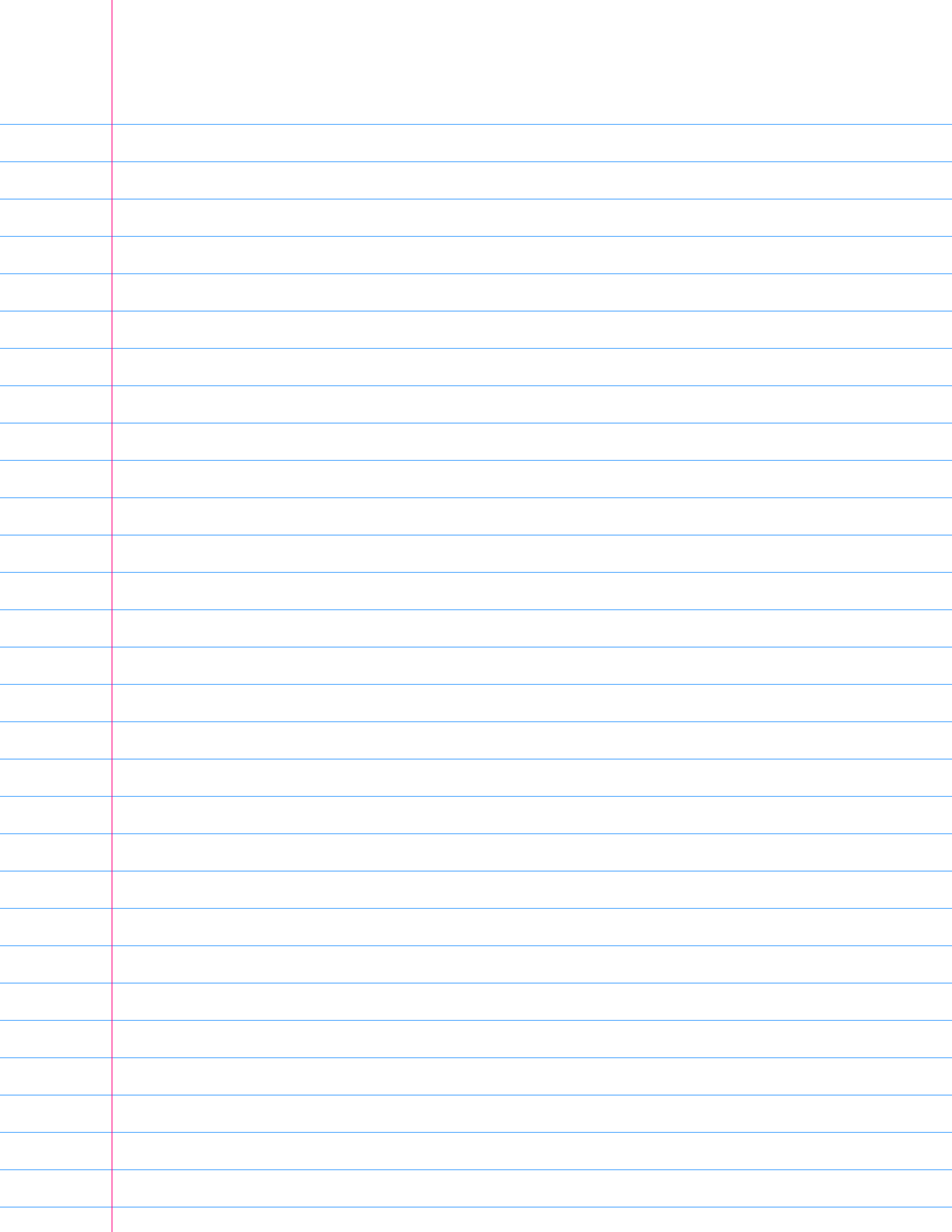
F.I Complementary Inverting Base Inverting Range Complementing

$$\begin{array}{cc} (1') & a^{n+1} u(n) \\ (5') & a^{n+1} u(-n-1) \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} a^{-n-1} u(n) & (2') \\ a^{-n-1} u(-n-1) & (6') \end{array}$$

$$\begin{array}{cc} (3') & a^{n-1} u(-n) \\ (7') & a^{n-1} u(n-1) \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} a^{-n+1} u(-n) & (4') \\ a^{-n+1} u(n-1) & (8') \end{array}$$



$(1') - (6')$
 $(6') - (1')$
 $(2') - (5')$
 $(5') - (2')$
 $(5') - (2')$
 $(8') - (3')$

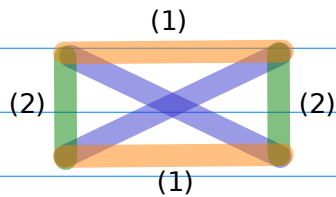


A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{|c|c|} \hline (1) & a^n u(n) \\ \hline (3) & a^n u(-n) \\ \hline \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n} u(n) & (2) \\ \hline a^{-n} u(-n) & (4) \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline (5) & a^n u(-n-1) \\ \hline (7) & a^n u(n-1) \\ \hline \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n} u(-n-1) & (6) \\ \hline a^{-n} u(n-1) & (8) \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-n} R(-n) \end{array}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

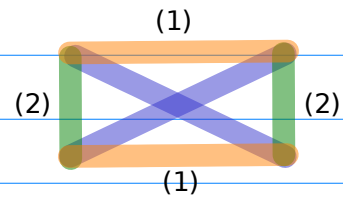
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

$$\begin{array}{|c|c|} \hline (1') & a^{n+1} u(n) \\ \hline (3') & a^{n-1} u(-n) \\ \hline \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n-1} u(n) & (2') \\ \hline a^{-n+1} u(-n) & (4') \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline (5') & a^{n+1} u(-n-1) \\ \hline (7') & a^{n-1} u(n-1) \\ \hline \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n-1} u(-n-1) & (6') \\ \hline a^{-n+1} u(n-1) & (8') \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n) \end{array}$$

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

B.I Range Shifting

(1) Range Complementing

(2) Range Flipping

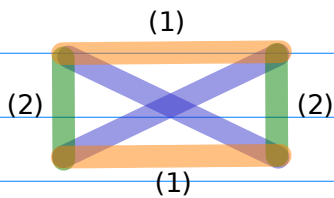
E.I Shifting2

(1) Shifted Range Flipping

(2) Range Complementing

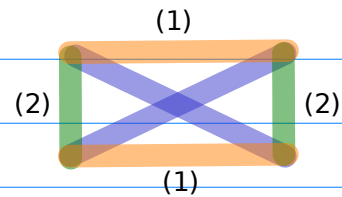
$$\begin{array}{l} (1) \quad a^n u(n) \\ (5) \quad a^n u(-n) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^n u(-n-1) \quad (3) \\ a^n u(n-1) \quad (7) \end{array}$$

$$\begin{array}{l} (2) \quad a^{-n} u(n) \\ (6) \quad a^{-n} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{-n} u(-n) \quad (4) \\ a^{-n} u(n-1) \quad (8) \end{array}$$



$$\begin{array}{l} (1') \quad a^{n+l} u(n) \\ (5') \quad a^{n+l} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{n+l} u(-n) \quad (3') \\ a^{n+l} u(n-1) \quad (7') \end{array}$$

$$\begin{array}{l} (2') \quad a^{-n-l} u(n) \\ (6') \quad a^{-n-l} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{-n-l} u(-n) \quad (4') \\ a^{-n-l} u(n-1) \quad (8') \end{array}$$



$$\begin{array}{l} R(n) \xleftrightarrow{(1)} R(-n) \\ R(n) \xleftrightarrow{(2)} \overline{R(n)} \\ R(n) \longleftrightarrow \overline{R(-n)} \end{array}$$

$$\begin{array}{l} a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} R(-n) \\ R(n) \xleftrightarrow{(2)} \overline{R(n)} \\ a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)} \end{array}$$

b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

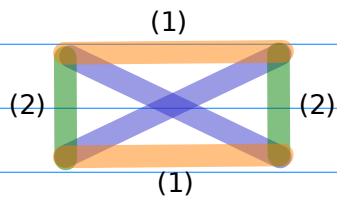
C.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{|c|c|} \hline (1) & a^n u(n) \\ \hline (5) & a^n u(-n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n} u(n) & (2) \\ \hline a^{-n} u(-n-1) & (6) \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline (3) & a^n u(-n) \\ \hline (7) & a^n u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n} u(-n) & (4) \\ \hline a^{-n} u(n-1) & (8) \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

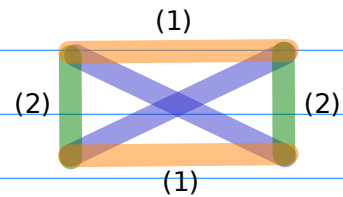
F.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{|c|c|} \hline (1') & a^{n+1} u(n) \\ \hline (5') & a^{n+1} u(-n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n-1} u(n) & (2') \\ \hline a^{-n-1} u(-n-1) & (6') \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline (3') & a^{n+1} u(-n) \\ \hline (7') & a^{n+1} u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{|c|c|} \hline a^{-n-1} u(-n) & (4') \\ \hline a^{-n-1} u(n-1) & (8') \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

b^n	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$$b^n \quad b^{-sh2(n)}$$

$$a^{(n+1)} \quad a^{-(n-1)}$$

$$a^{-(n+1)} \quad a^{(n-1)}$$

$$a^{(n-1)} \quad a^{-(n+1)}$$

$$a^{-(n-1)} \quad a^{(n+1)}$$

$$b^n \quad b^{-sh2(n)}$$

$$a^{(n-1)} \quad a^{-(n+1)}$$

$$a^{-(n-1)} \quad a^{(n+1)}$$

$$a^{(n+1)} \quad a^{-(n-1)}$$

$$a^{-(n+1)} \quad a^{(n-1)}$$

$$b^n \quad b^{-(n)}$$

$$a^{(n+1)} \quad a^{-(n+1)}$$

$$a^{-(n+1)} \quad a^{(n+1)}$$

$$a^{(n-1)} \quad a^{-(n-1)}$$

$$a^{-(n-1)} \quad a^{(n-1)}$$

$$b^n \quad b^{-(n)}$$

$$a^{(n-1)} \quad a^{-(n-1)}$$

$$a^{-(n-1)} \quad a^{(n-1)}$$

$$a^{(n+1)} \quad a^{-(n+1)}$$

$$a^{-(n+1)} \quad a^{(n+1)}$$

$$b^n \quad b^{sh2(n)}$$

$$a^{(n+1)} \quad a^{(n-1)}$$

$$a^{-(n+1)} \quad a^{-(n-1)}$$

$$a^{(n-1)} \quad a^{(n+1)}$$

$$a^{-(n-1)} \quad a^{-(n+1)}$$

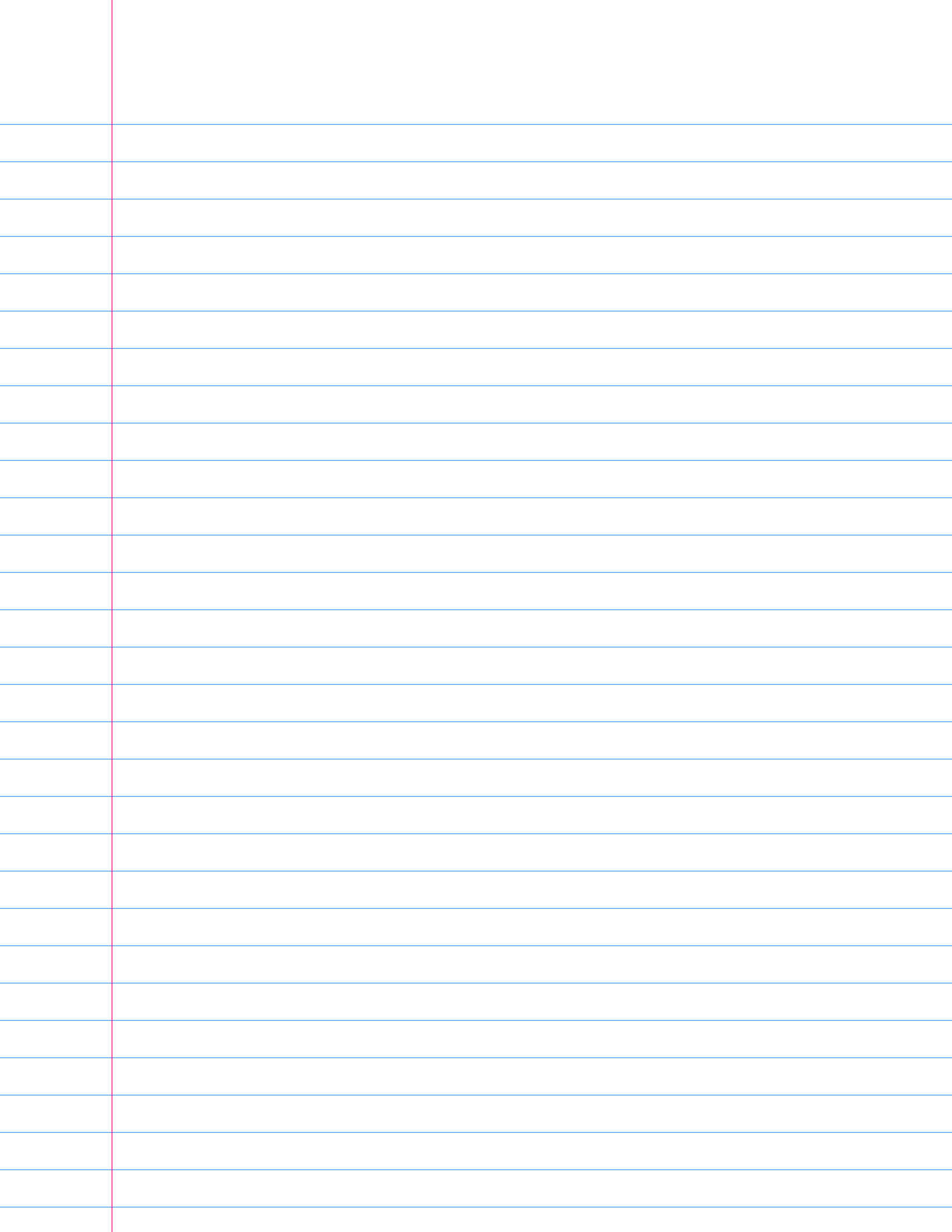
$$b^n \quad b^{sh2(n)}$$

$$a^{(n-1)} \quad a^{(n+1)}$$

$$a^{-(n-1)} \quad a^{-(n+1)}$$

$$a^{(n+1)} \quad a^{(n-1)}$$

$$a^{-(n+1)} \quad a^{-(n-1)}$$



A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-n} R(-n)
 \end{array}$$

D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

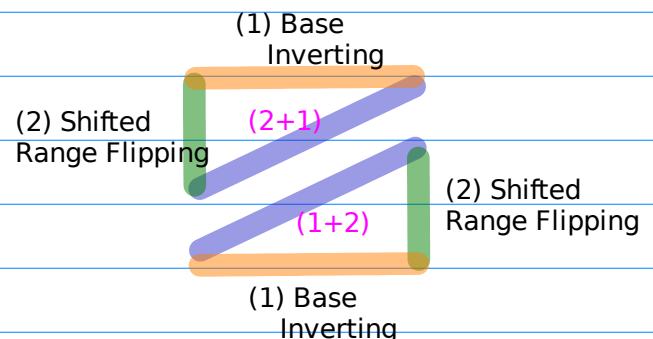
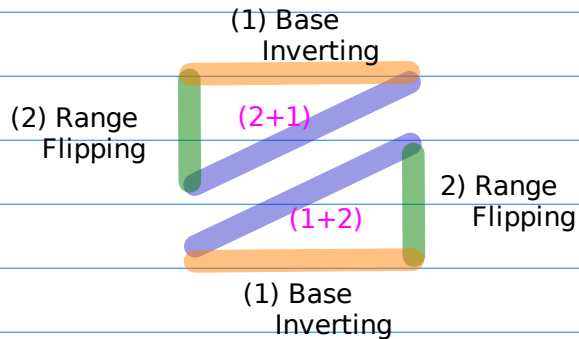
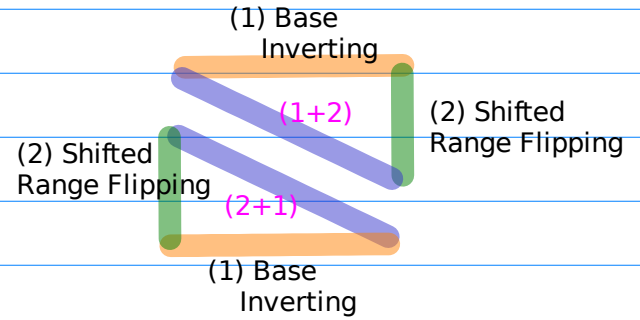
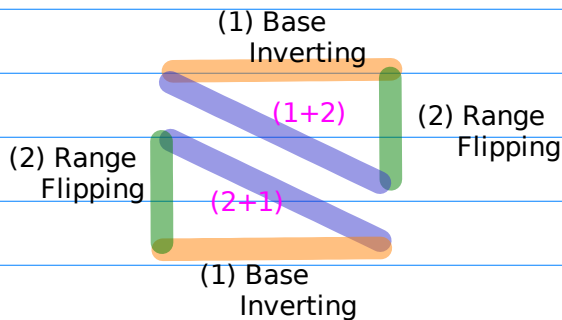
$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n R(-n) \\
 & \xrightarrow{(1)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 & \xrightarrow{(1)} & a^{-sh2(n)} R(-n)
 \end{array}$$



B.I Range Shifting

(1) Range Complementing

(2) Range Flipping

$$R(n) \xleftrightarrow{(1)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

E.I Shifting2

(1) Shifted Range Flipping

(2) Range Complementing

$$a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

$$\xrightarrow{(2)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

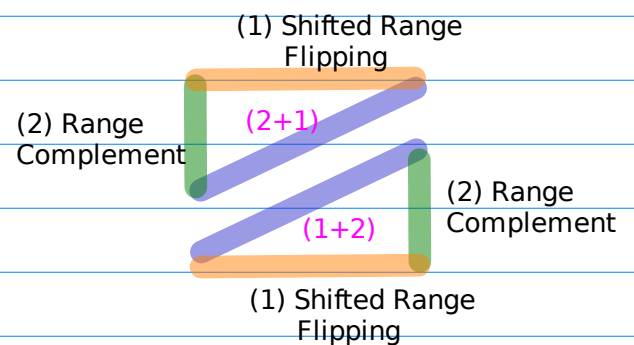
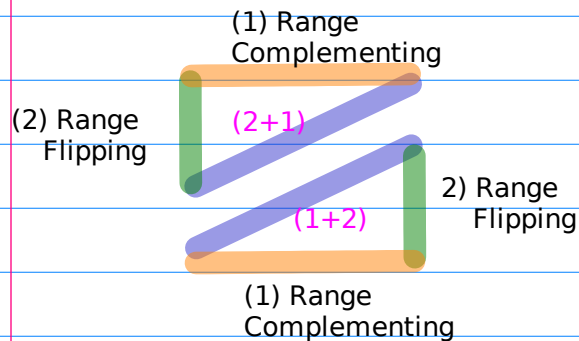
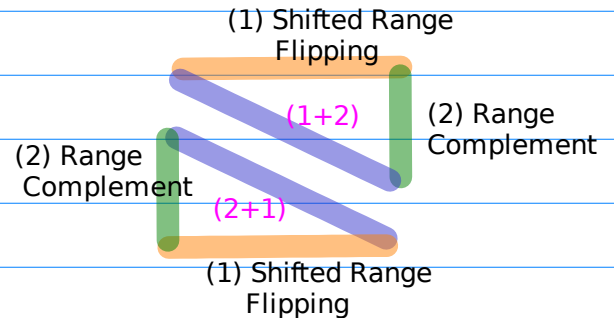
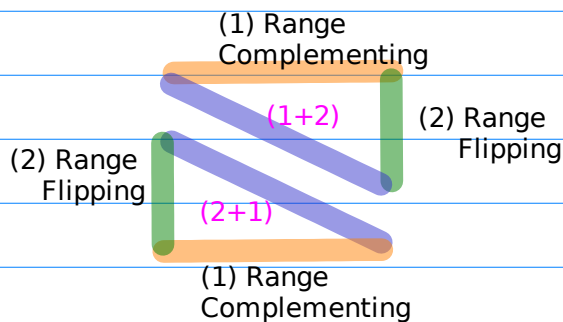
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n rng(n) \xrightarrow{(2)} a^n \overline{R(n)}$$

$$\xrightarrow{(1)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$

$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$



C.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$

F.I Complementary Inverting

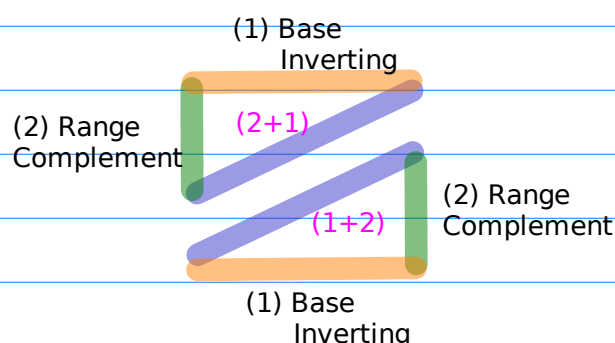
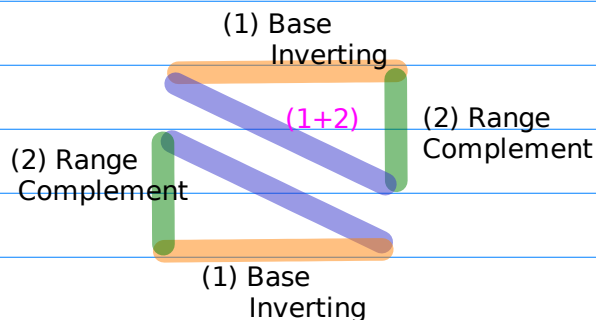
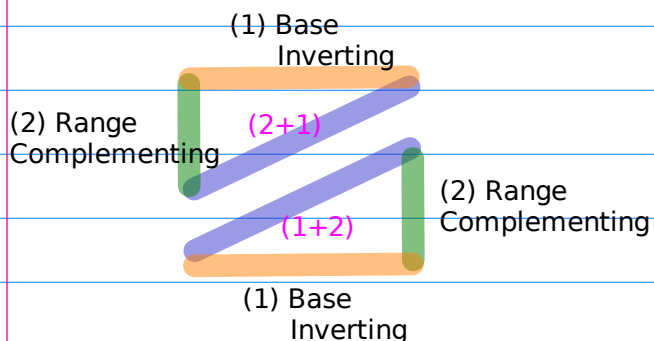
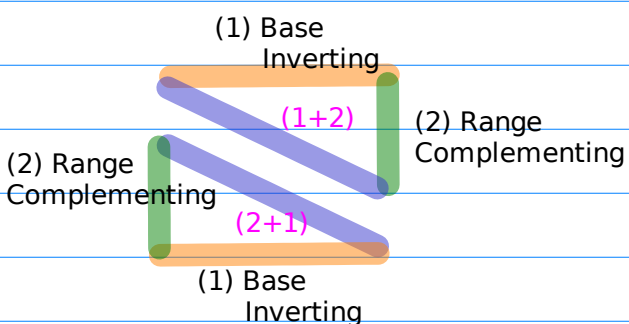
(1) Base Inverting

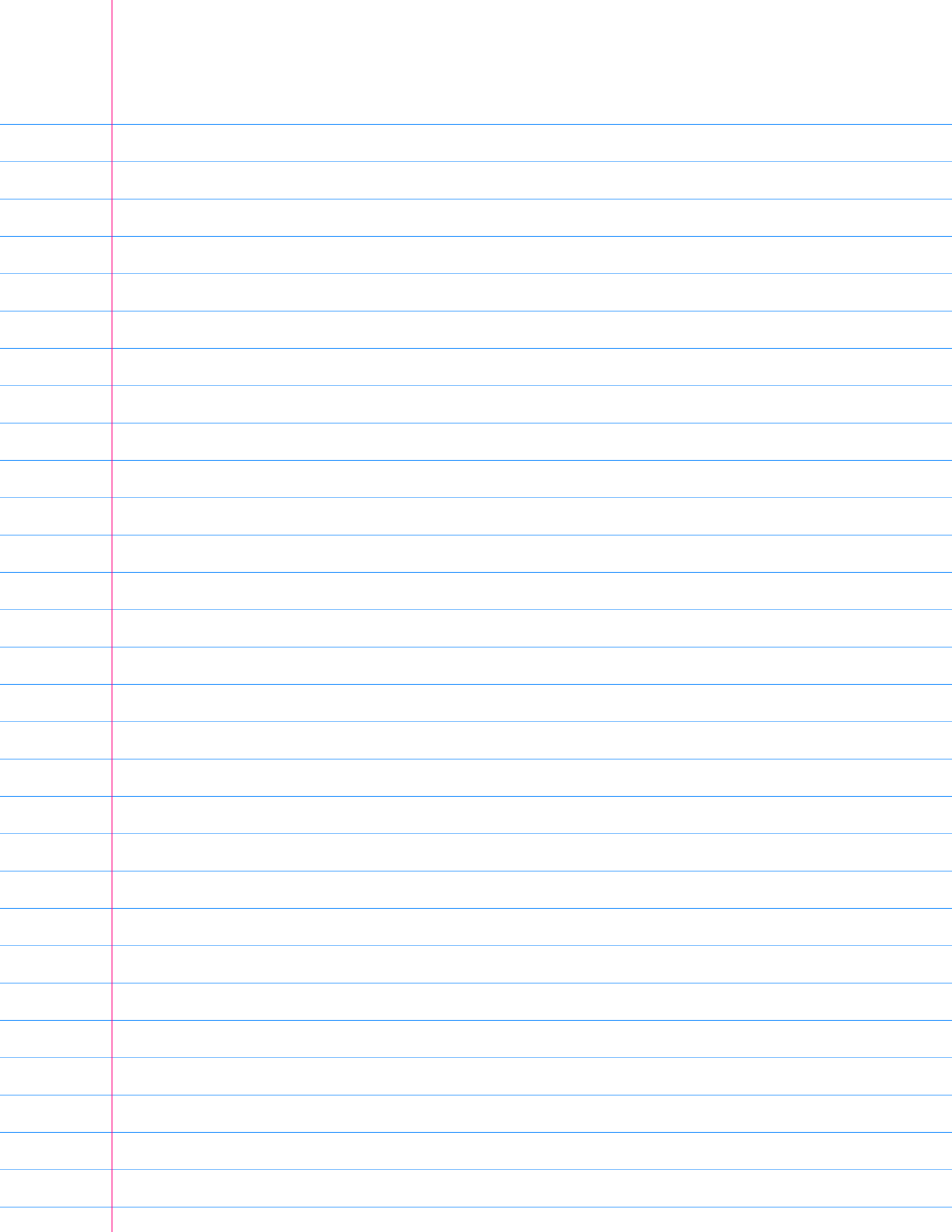
(2) Range Complementing

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

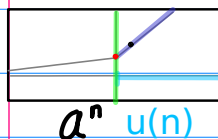
$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$



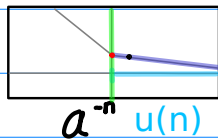


A.II Flipping Base Inverting Range Flipping

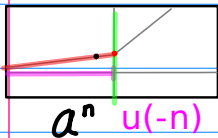
(1) 0000



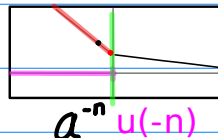
(2) 0001



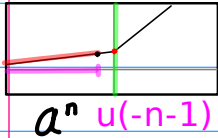
(3) 0010



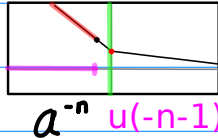
(4) 0011



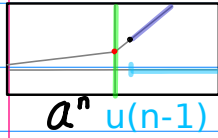
(5) 0100



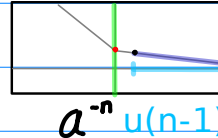
(6) 0101



(7) 0110

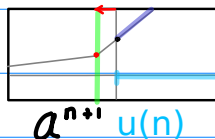


(8) 0111

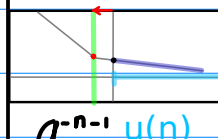


D.II Flipping2 Base Inverting Shifted Range Flipping

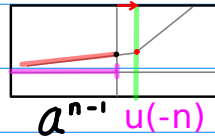
(1') 1000



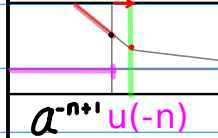
(2') 1001



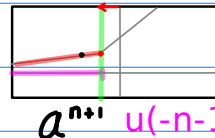
(3') 1010



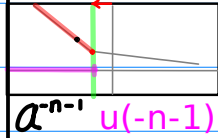
(4') 1011



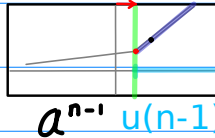
(5') 1100



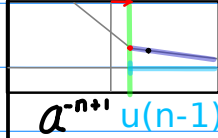
(6') 1101



(7') 1110



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n \longleftrightarrow a^{-n}$$

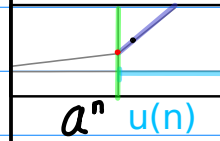
$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

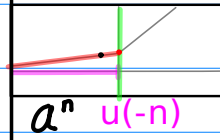
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

B.II Range Shifting Range Flipping Range Complementing

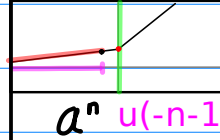
(1) 0000



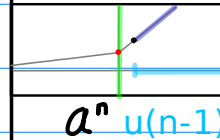
(3) 0010



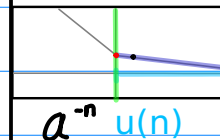
(5) 0100



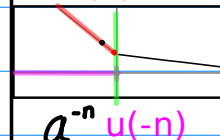
(7) 0110



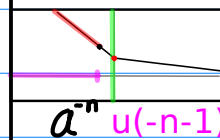
(2) 0001



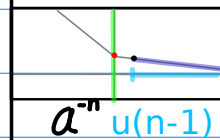
(4) 0011



(6) 0101



(8) 0111



Range Shifting
= Range Flipping
+ Range Complementing

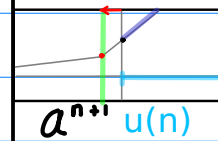
$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow R(-n)$$

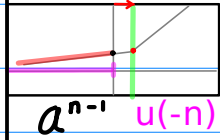
$$R(n) \longleftrightarrow \overline{R(n)}$$

E.II Shifting2 Shifted Range Flipping Range Complementing

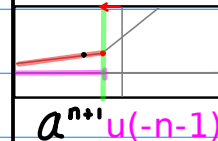
(1') 1000



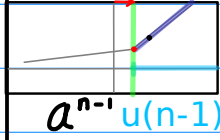
(3') 1010



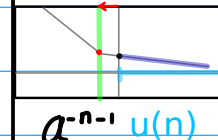
(5') 1100



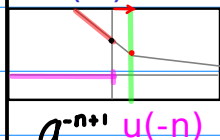
(7') 1110



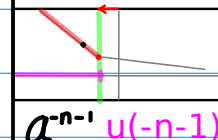
(2') 1001



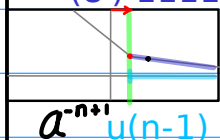
(4') 1011



(6') 1101



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

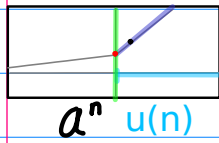
$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

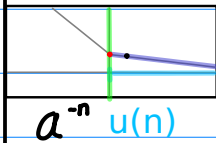
$$R(n) \longleftrightarrow \overline{R(n)}$$

C.II Complementary Inverting Base Inverting Range Complementing

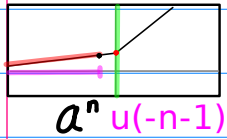
(1) 0000



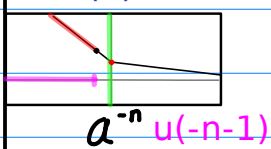
(2) 0001



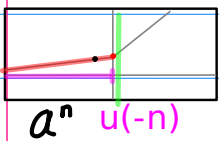
(5) 0100



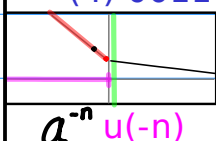
(6) 0101



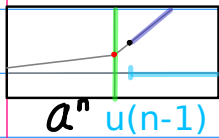
(3) 0010



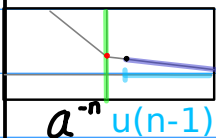
(4) 0011



(7) 0110



(8) 0111

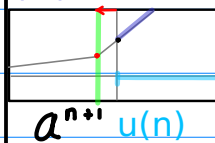


$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

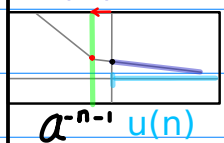
$$\begin{aligned} a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \end{aligned}$$

F.II Complementary Inverting Base Inverting Range Complementing

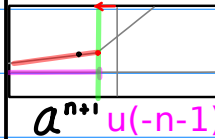
(1') 1000



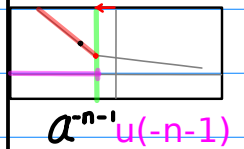
(2') 1001



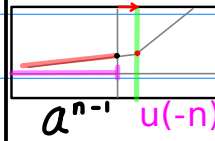
(5') 1100



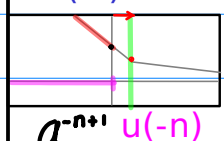
(6') 1101



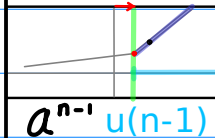
(3') 1010



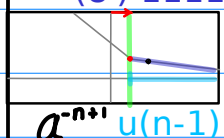
(4') 1011



(7') 1110

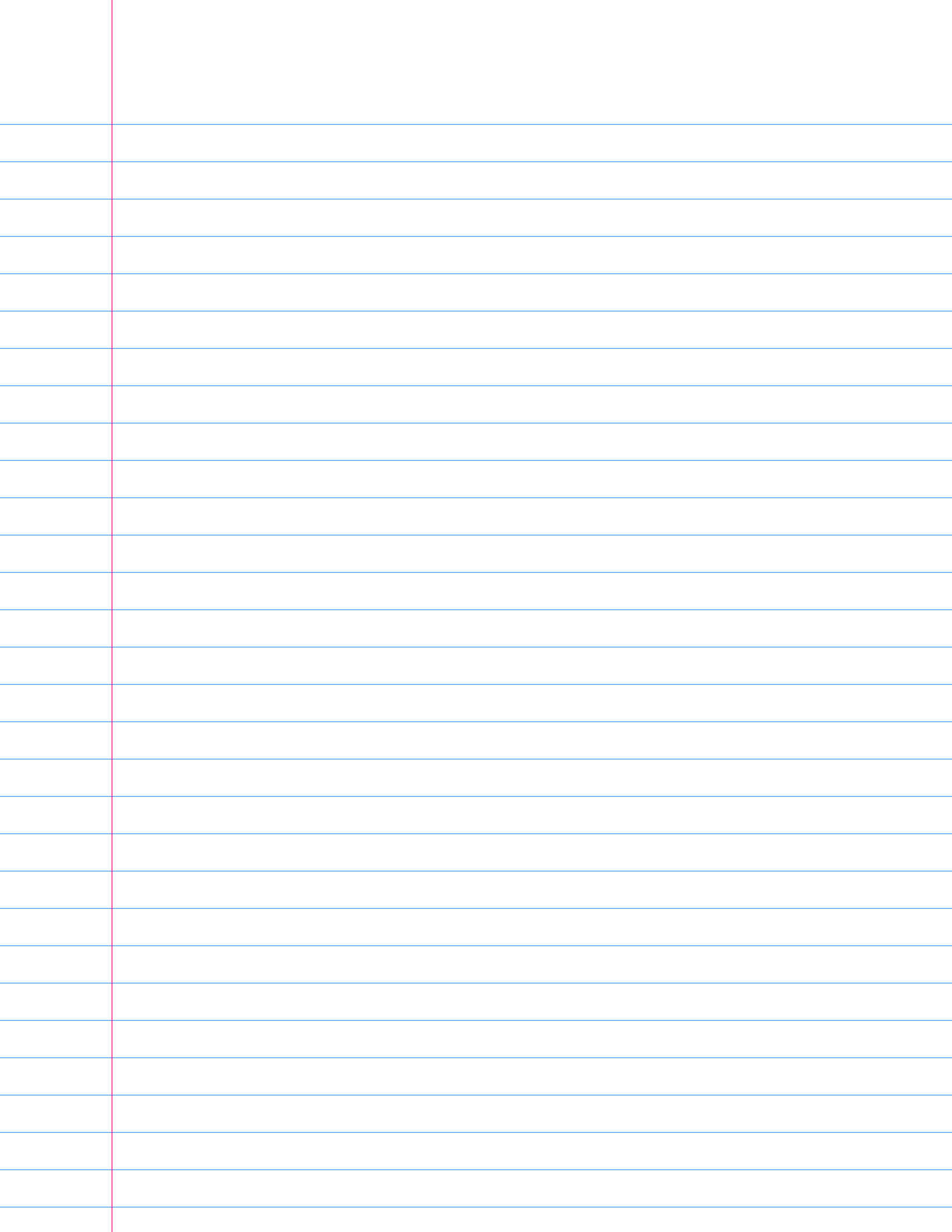


(8') 1111



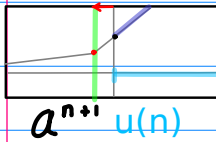
$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

$$\begin{aligned} a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \end{aligned}$$

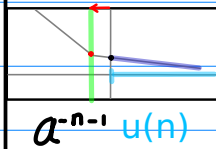


D.II Flipping2 Base Inverting Shifted Range Flipping

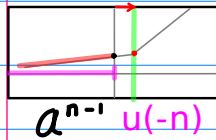
(1') 1000



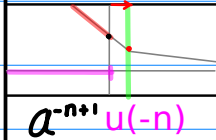
(2') 1001



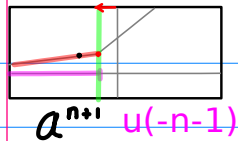
(3') 1010



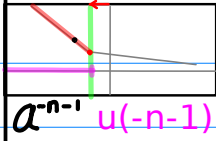
(4') 1011



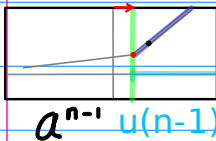
(5') 1100



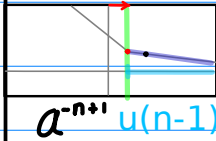
(6') 1101



(7') 1110



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

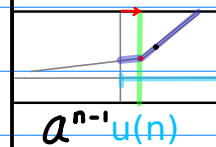
$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

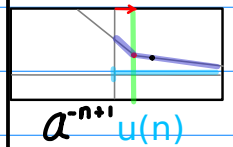
$$a^n R(n) \longleftrightarrow a^{-\text{sh } 2(n)} R(-n)$$

G.II Flipping2 Base Inverting Shifted Range Flipping

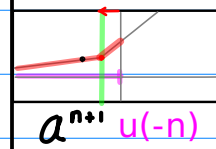
(1'') 1000



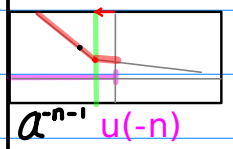
(2'') 1001



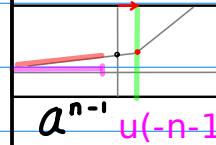
(3'') 1010



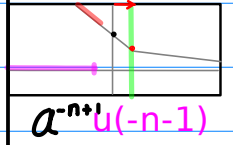
(4'') 1011



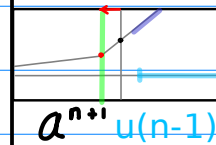
(5'') 1100



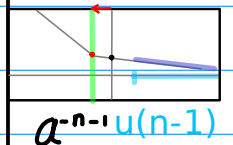
(6'') 1101



(7'') 1110



(8'') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

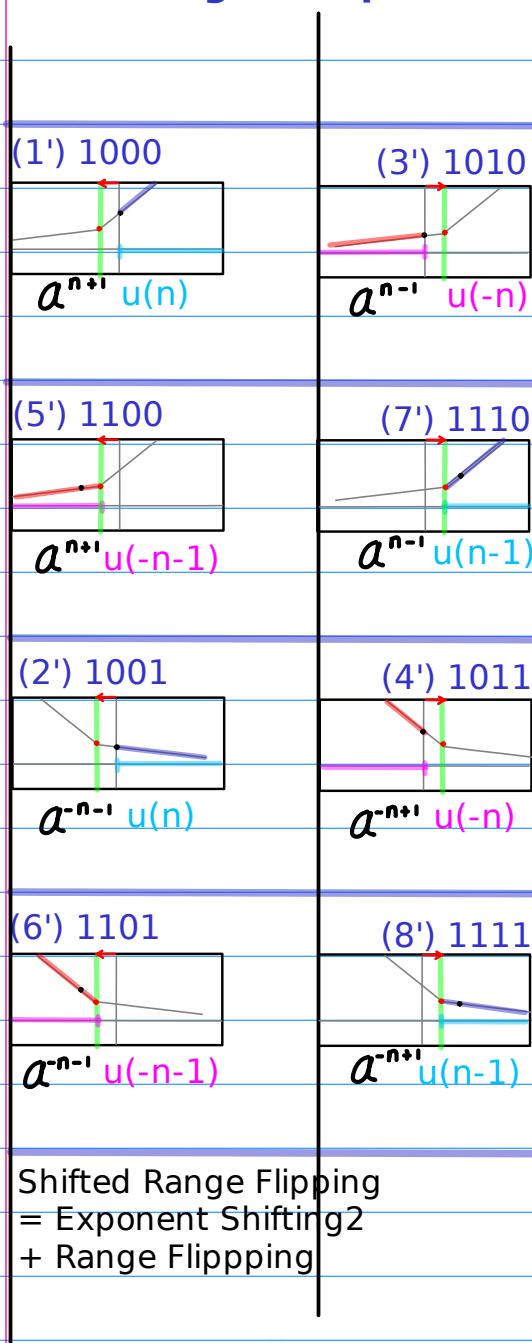
$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-\text{sh } 2(n)} R(-n)$$

E.II Shifting2 Shifted Range Flipping Range Complementing



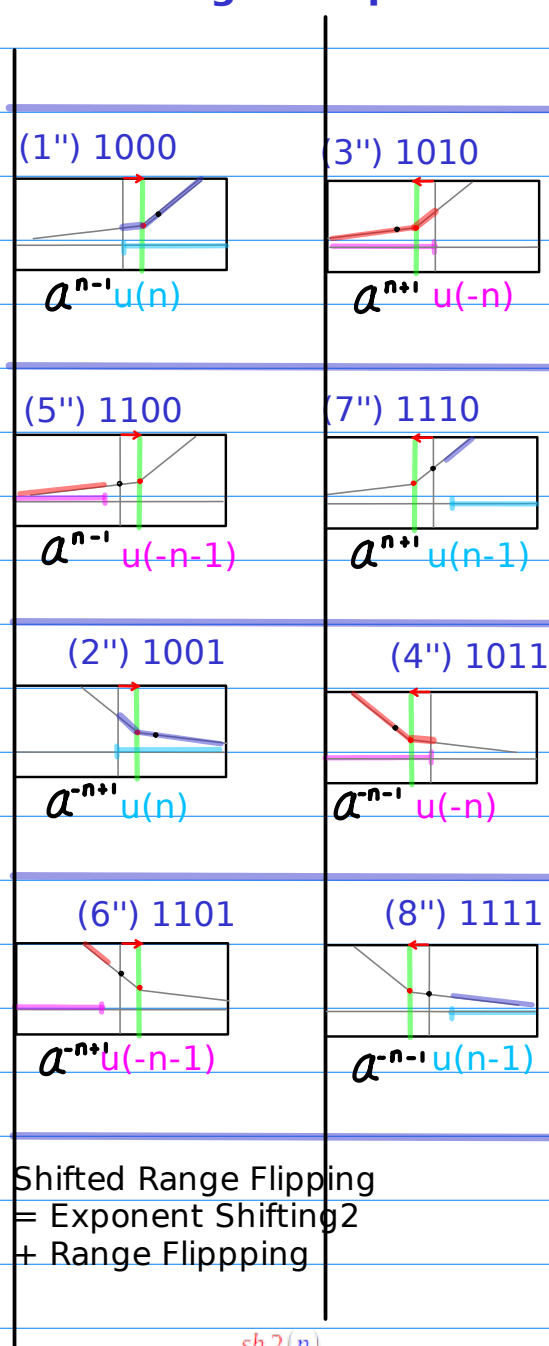
$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n) \quad \boxed{\times} \quad a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

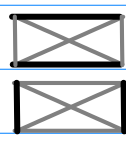
$$R(n) \longleftrightarrow \overline{R(n)} \quad \boxed{\times} \quad R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)} \quad \boxed{\times} \quad a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)}$$

H.II Shifting2 Shifted Range Flipping Range Complementing



$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$



$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

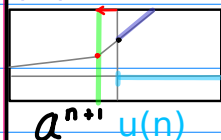
$$R(n) \longleftrightarrow \overline{R(n)}$$



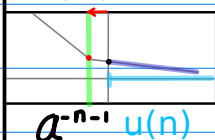
$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)}$$

F.II Complementary Inverting Base Inverting Range Complementing

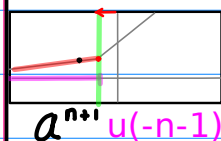
(1') 1000



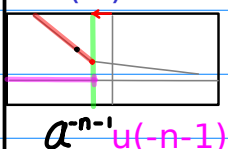
(2') 1001



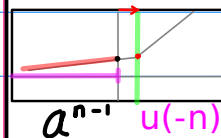
(5') 1100



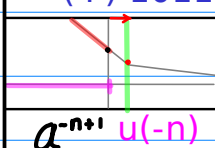
(6') 1101



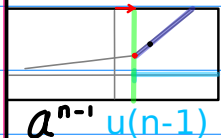
(3') 1010



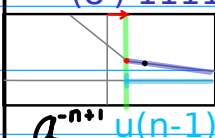
(4') 1011



(7') 1110

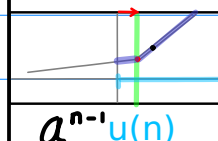


(8') 1111

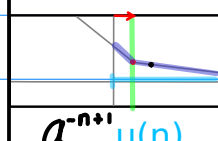


I.II Complementary Inverting Base Inverting Range Complementing

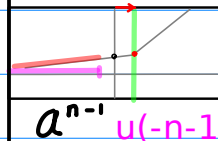
(1'') 1000



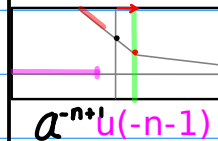
(2'') 1001



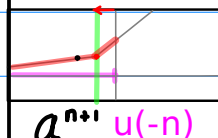
(5'') 1100



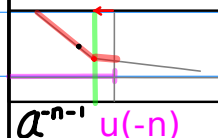
(6'') 1101



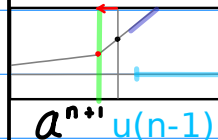
(3'') 1010



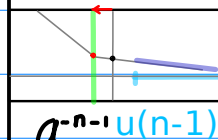
(4'') 1011



(7'') 1110

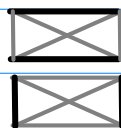


(8'') 1111



Shifted Range Flipping
= Exponent Shifting²
+ Range Flipping

$$\begin{array}{ccc} a^n & \longleftrightarrow & a^{-n} \\ R(n) & \longleftrightarrow & \overline{R(n)} \end{array}$$



$$\begin{array}{ccc} a^n & \longleftrightarrow & a^{-n} \\ R(n) & \longleftrightarrow & \overline{R(n)} \end{array}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$



$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

Shifted Sequence 1 x'

