

# Laurent Series and z-Transform - Case Examples 0.B

20170205

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|                     |        |   |   |
|---------------------|--------|---|---|
|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$ |
| $ z  < \frac{1}{2}$ | $f(z)$ | Case ③                                  | Case ④                                    |
|                     | $X(z)$ | Case ③                                  | Case ④                                    |
| $ z  > 2$           | $f(z)$ | Case ③                                  | Case ④                                    |
|                     | $X(z)$ | Case ③                                  | Case ④                                    |

$$\text{Case ①} \quad f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \quad X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

$$\text{Case ②} \quad f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \quad X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

$$\text{Case ③} \quad f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \quad X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

$$\text{Case ④} \quad f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \quad X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

|                     |        |   |   |
|---------------------|--------|---|---|
|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$ |
| $ z  < \frac{1}{2}$ | $f(z)$ | case ① $(n > 0)$                        | case ② $(n > 0)$                          |
|                     | $X(z)$ | case ② $(n \leq 0)$                     | case ① $(n < 0)$                          |
| $ z  > 2$           | $f(z)$ | case ② $(n < 0)$                        | case ② $(n \leq 0)$                       |
|                     | $X(z)$ | case ② $(n > 0)$                        | case ① $(n > 0)$                          |

$$\text{case ①} \quad f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

$$X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

$$\text{case ②} \quad f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

$$X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

$$\text{case ③} \quad f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

$$X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

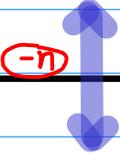
$$\text{case ④} \quad f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

$$X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$ |
|---------------------|--------|---|---|
| $ z  < \frac{1}{2}$ | $f(z)$ | case ① $(n > 0)$                        | $(-n)$                                    |
|                     | $X(z)$ |   | case ① $(n < 0)$ $(-n)$                   |
| $ z  > 2$           | $f(z)$ | case ① $(n < 0)$                        | $(-n)$                                    |
|                     | $X(z)$ |   | case ① $(n > 0)$ $(-n)$                   |

|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$ |
|---------------------|--------|---|---|
| $ z  < \frac{1}{2}$ | $f(z)$ |   | case ② $(n > 0)$                          |
|                     | $X(z)$ | case ② $(n \leq 0)$ $(-n)$              |   |
| $ z  > 2$           | $f(z)$ |   | case ② $(n \leq 0)$ $(-n)$                |
|                     | $X(z)$ | case ② $(n > 0)$ $(-n)$                 |   |

|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$   |
|---------------------|--------|---|---|
| $ z  < \frac{1}{2}$ | $f(z)$ | Case III                                |  |
|                     | $X(z)$ | Case III                                |   |
| $ z  > 2$           | $f(z)$ | Case III                                |  |
|                     | $X(z)$ | Case III                                |   |

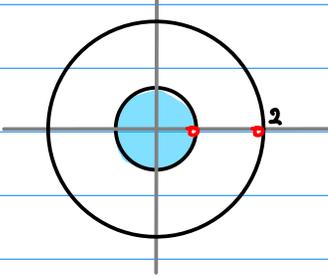
|                     |        | ① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$   | ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$ |
|---------------------|--------|---|---|
| $ z  < \frac{1}{2}$ | $f(z)$ |  | Case IV                                   |
|                     | $X(z)$ |   | Case IV                                   |
| $ z  > 2$           | $f(z)$ |  | Case IV                                   |
|                     | $X(z)$ |   | Case IV                                   |

1.8

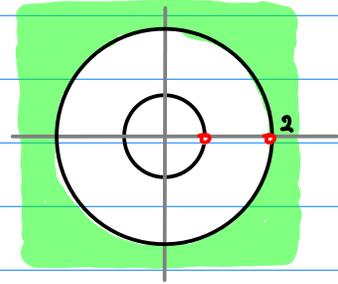
$$f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

$$= \frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

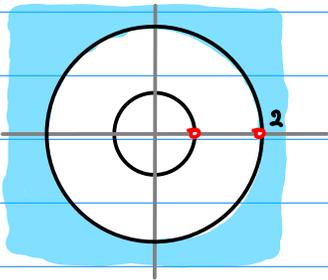
$$= \frac{z}{1-2z} + \frac{z}{1-0.5z}$$



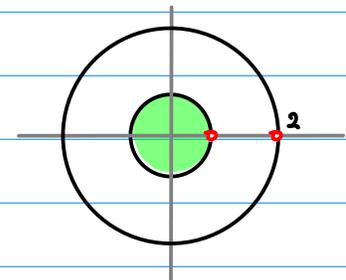
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n \equiv \equiv$$



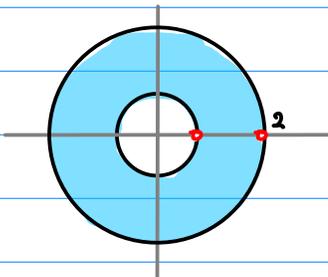
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$



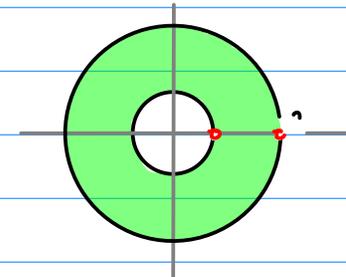
$$\sum_{n=-1}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^n \equiv \equiv$$



$$\sum_{n=-1}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$



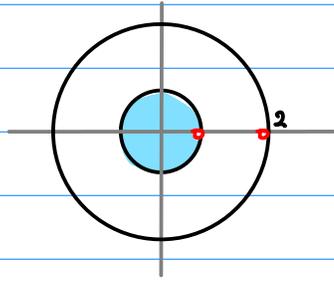
$$\sum_{n=-1}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \equiv \equiv$$



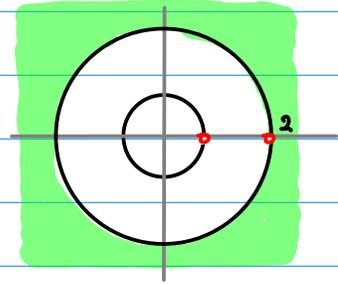
$$\sum_{n=-1}^{\infty} 2^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

2. B

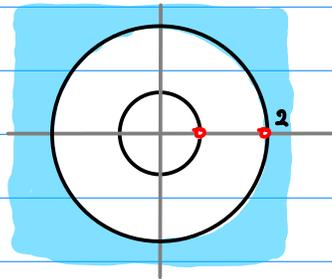
$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$



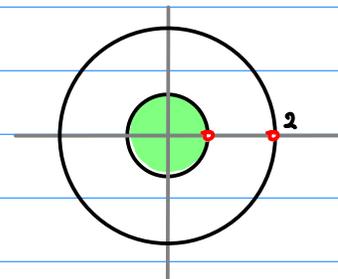
$$\sum_{n=-1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$



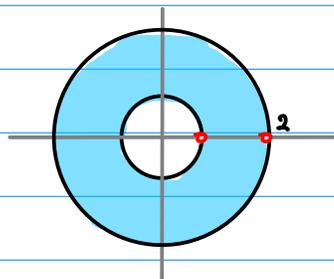
$$\sum_{n=-1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$



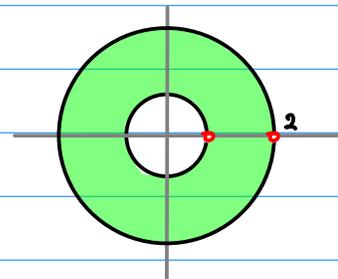
$$\sum_{n=0}^{-\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$



$$\sum_{n=0}^{-\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$

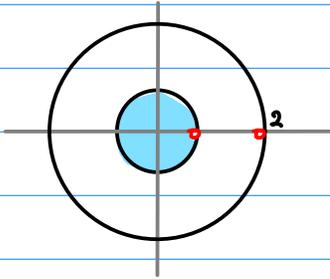


$$\sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{-\infty} 2^{n+1} z^{-n}$$

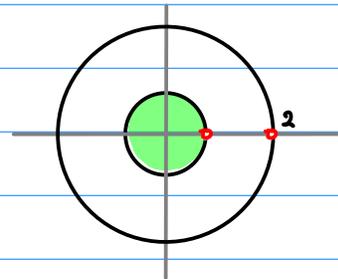


$$\sum_{n=0}^{-\infty} 2^{n+1} z^n + \sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

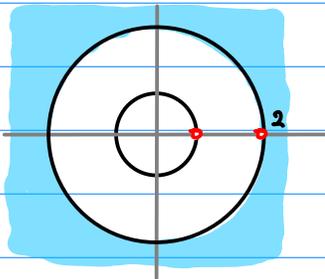
3.B  $f(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)} = X(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)}$



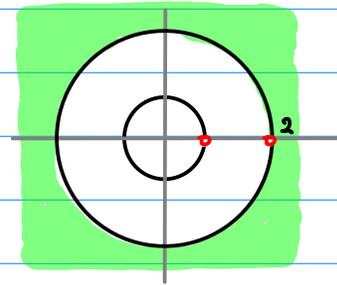
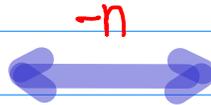
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$



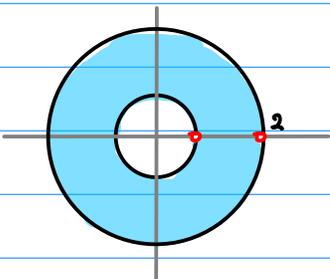
$$\sum_{n=0}^{\infty} \left[ 2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^{-n}$$



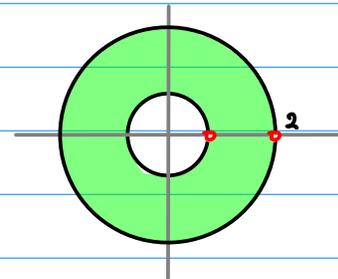
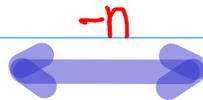
$$\sum_{n=-1}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$\sum_{n=-1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^{-n}$$



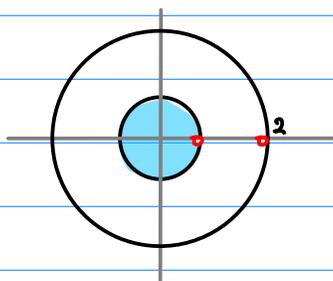
$$\sum_{n=-1}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$



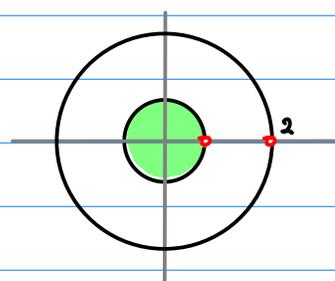
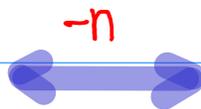
$$\sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.B

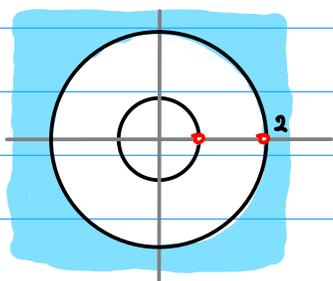
$$f(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



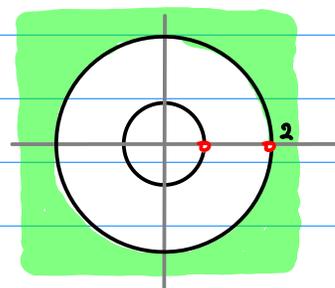
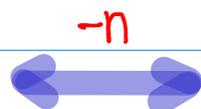
$$\sum_{n=-1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$



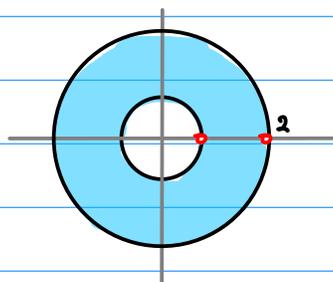
$$\sum_{n=-1}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$



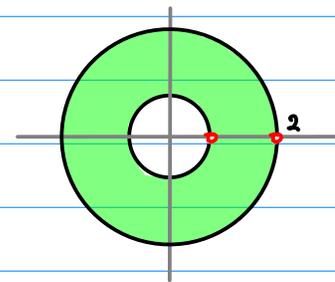
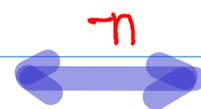
$$\sum_{n=0}^{\infty} \left[ 2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^n$$



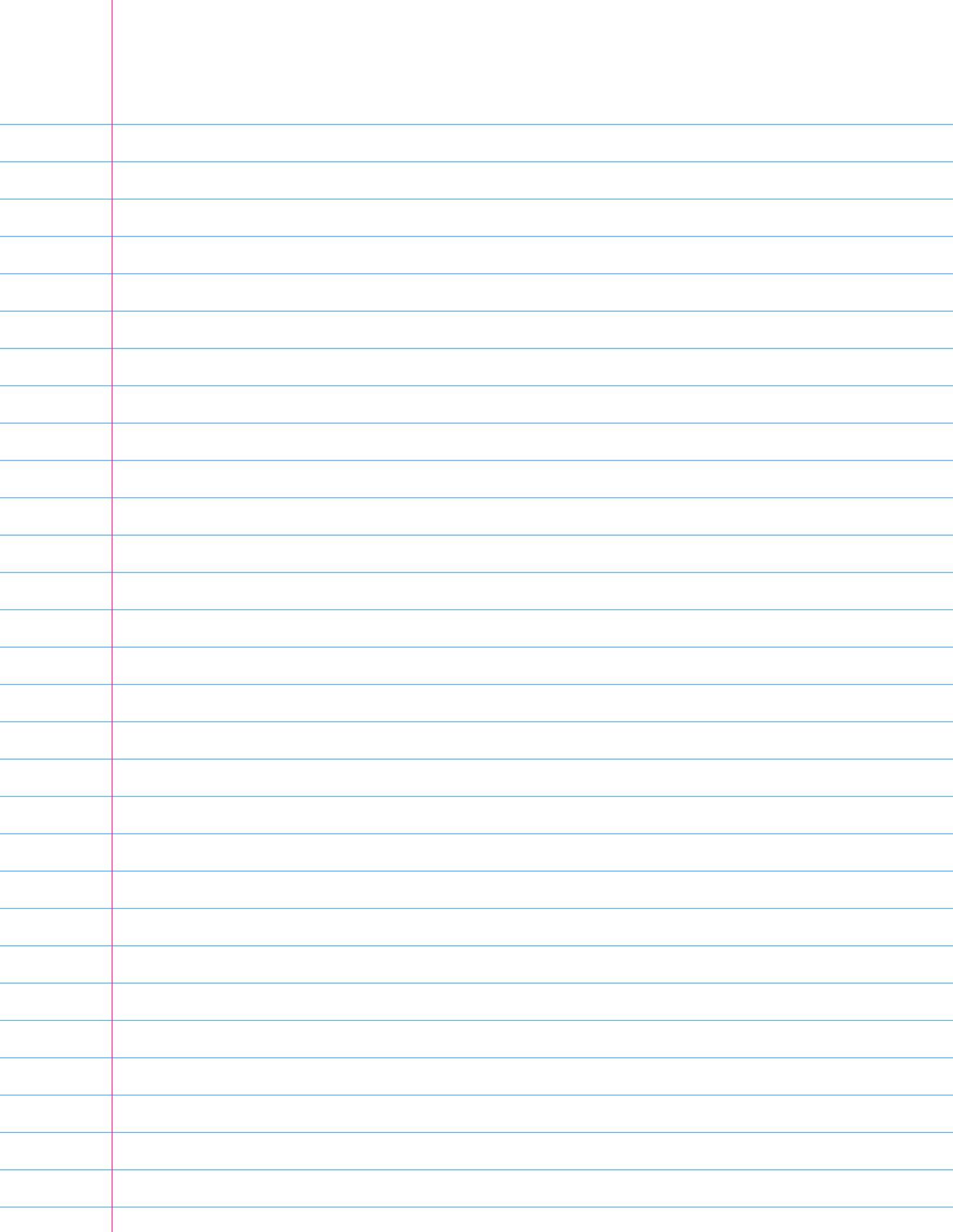
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$



$$\sum_{n=0}^{\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$



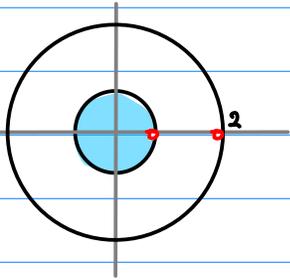
$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} 2^{n+1} z^{-n}$$



1.8

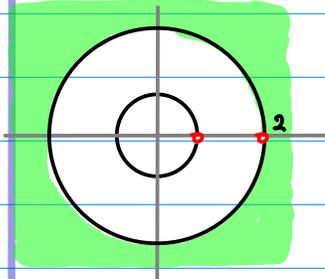
$$f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

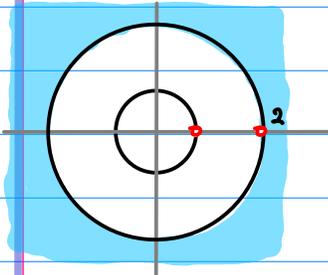
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

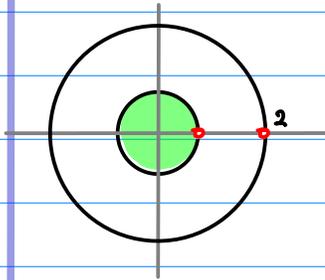
$$X(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

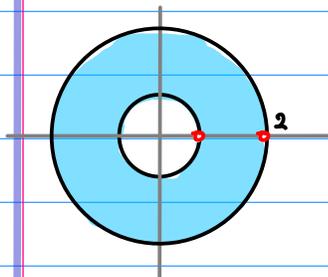
$$f(z) = \sum_{n=-1}^{-\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

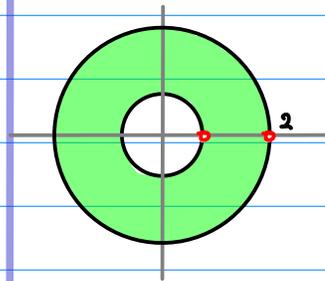
$$X(z) = \sum_{n=-1}^{-\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



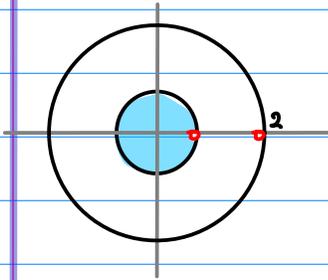
$$x_n = \begin{cases} 2^{n+1} & (n \geq 0) \\ (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$X(z) = \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

2. B

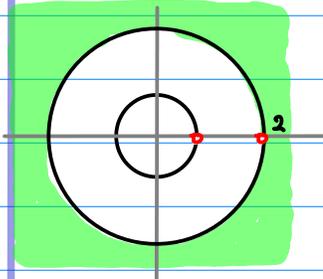
$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

I



$$a_n = \begin{cases} [(\frac{1}{2})^{n+1} - 2^{n+1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

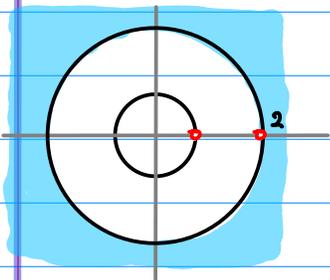
$$f(z) = \sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$



$$x_n = \begin{cases} [(\frac{1}{2})^{n-1} - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

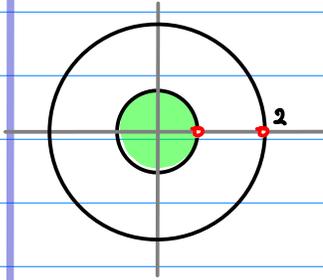
$$X(z) = \sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^n$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ [2^{n+1} - (\frac{1}{2})^{n+1}] & (n \leq 0) \end{cases}$$

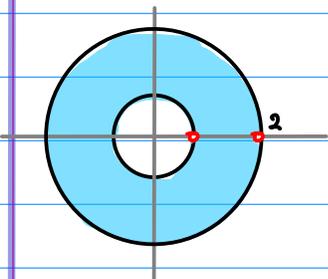
$$f(z) = \sum_{n=-\infty}^{\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$



$$x_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - (\frac{1}{2})^{n-1}] & (n \leq 0) \end{cases}$$

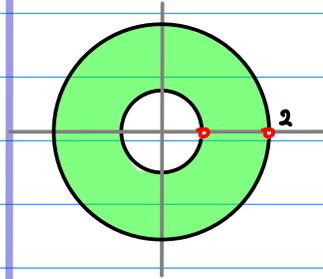
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^n$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{\infty} 2^{n-1} z^{-n} + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n}$$



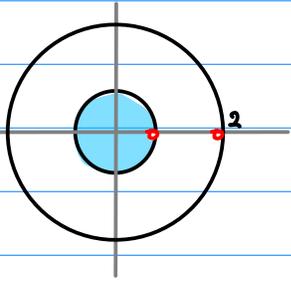
$$x_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$

3.B

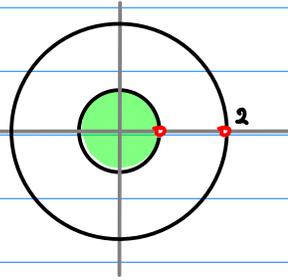
$$f(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)} = X(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

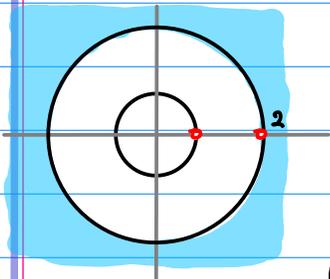
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

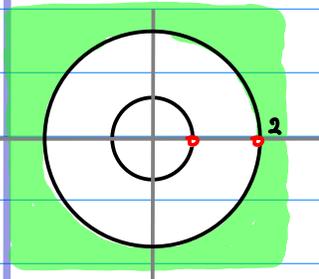
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

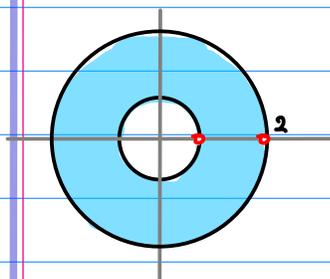
$$f(z) = \sum_{n=-1}^{\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

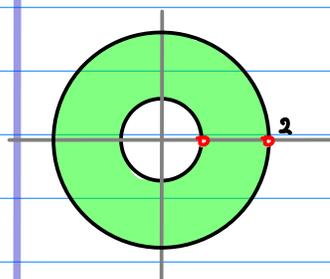
$$X(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



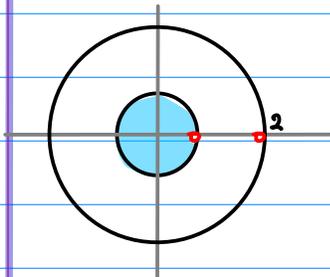
$$x_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.B

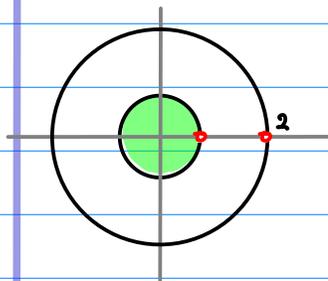
$$f(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

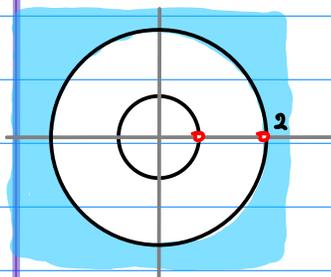
$$f(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

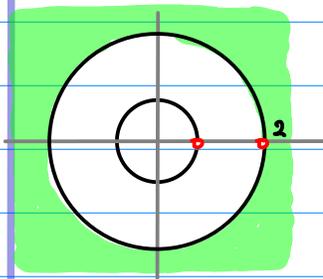
$$X(z) = \sum_{n=-1}^{-\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

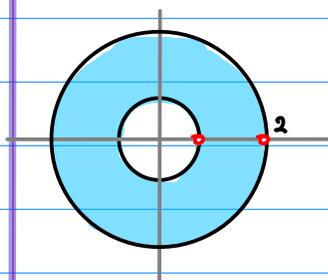
$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

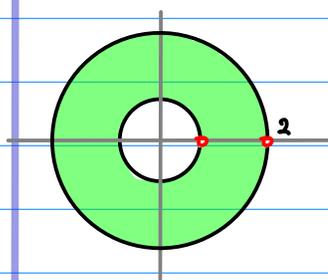
$$X(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$

III



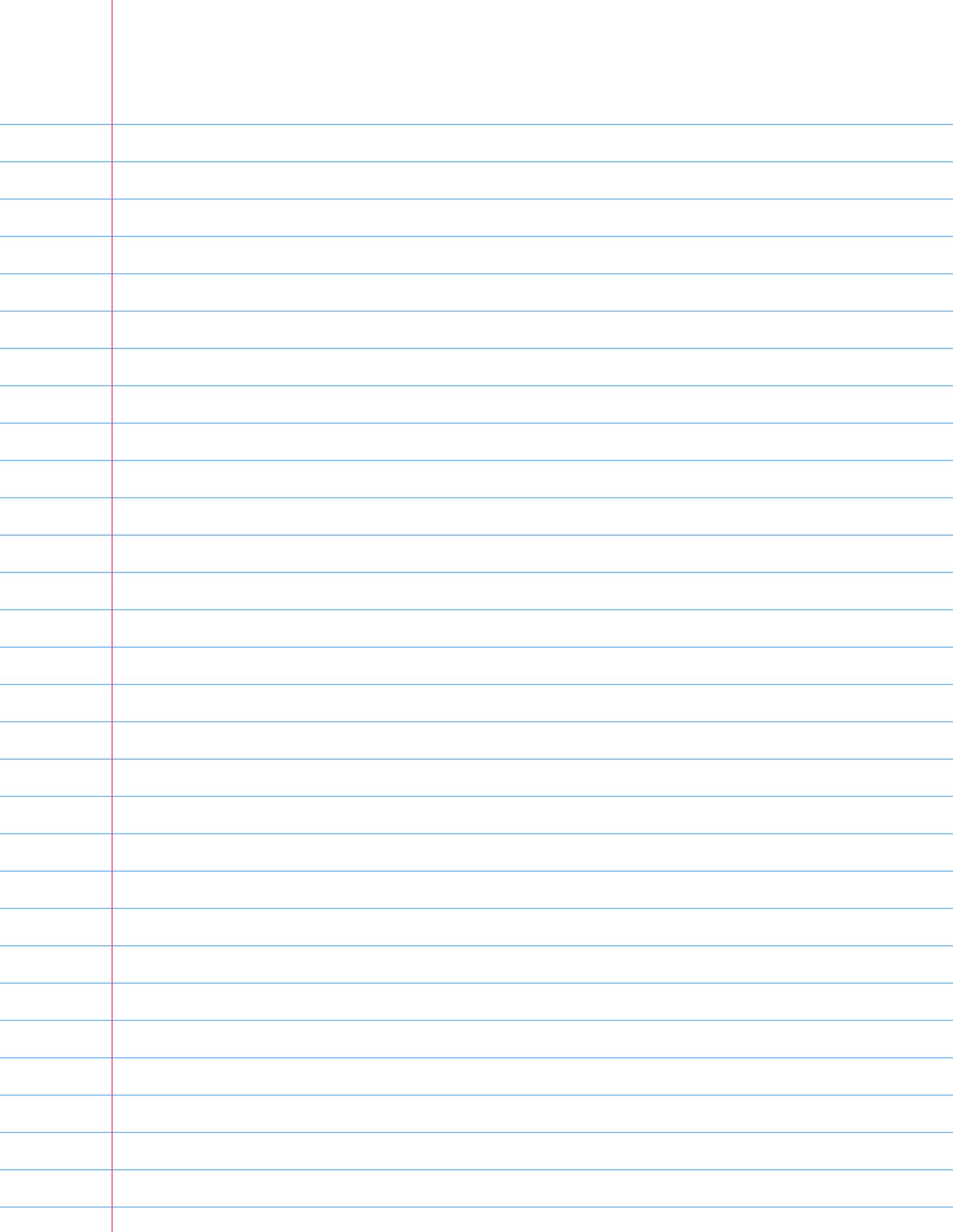
$$a_n = \begin{cases} 2^{n-1} & (n > 0) \\ (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

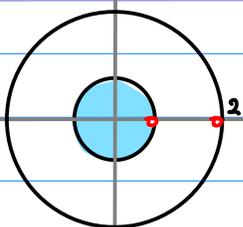
$$X(z) = \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n}$$



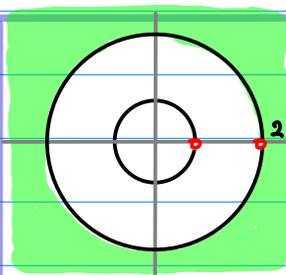
$f(z) \xleftrightarrow{z^{-1}} X(z)$

$a_n = x_n$

Ⓘ

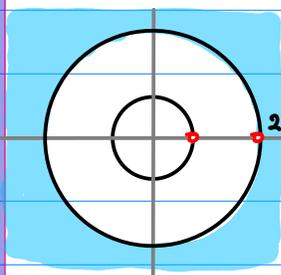


$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

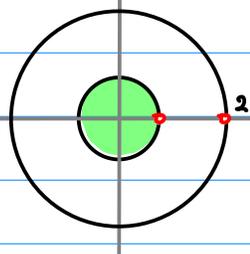


$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⓙ

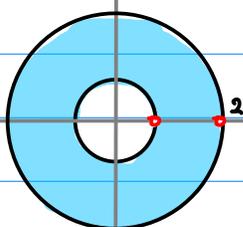


$$\begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

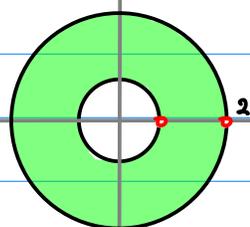


$$\begin{cases} 0 & (n > 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n \leq 0) \end{cases}$$

Ⓚ

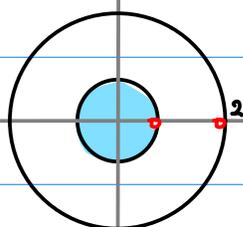


$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

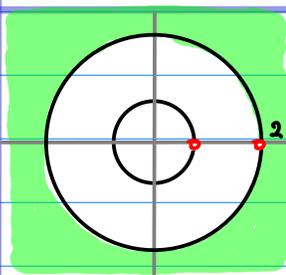


$$\begin{cases} (\frac{1}{2})^{n+1} & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

Ⓘ

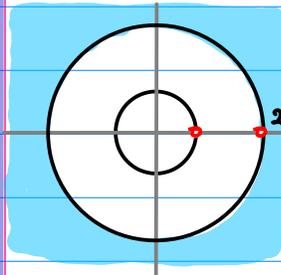


$$\begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

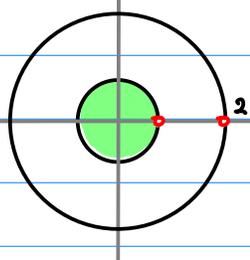


$$\begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⓙ

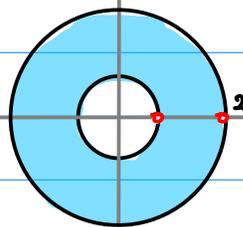


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

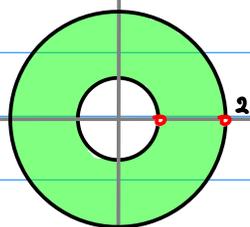


$$\begin{cases} 0 & (n \geq 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n < 0) \end{cases}$$

Ⓚ



$$\begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

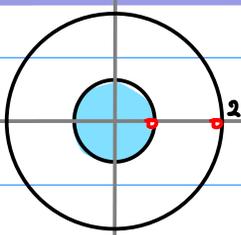


$$\begin{cases} (\frac{1}{2})^{n-1} & (n \geq 0) \\ 2^{n-1} & (n < 0) \end{cases}$$

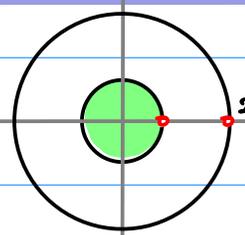
$$f(z) = X(z)$$

$$a_n \overset{-n}{\longleftrightarrow} x_n$$

Ⓘ

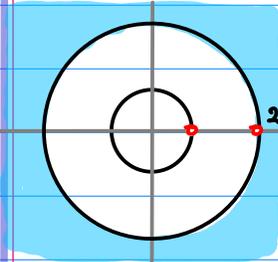


$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

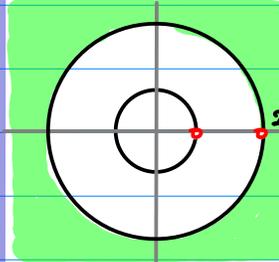


$$\begin{cases} 0 & (n > 0) \\ (\frac{1}{2})^{n-1} - 2^{n-1} & (n \leq 0) \end{cases}$$

Ⓙ

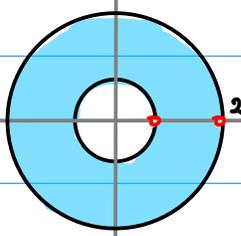


$$\begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

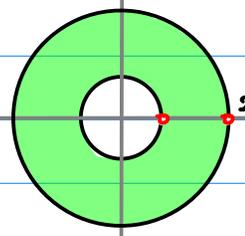


$$\begin{cases} 2^{n-1} - (\frac{1}{2})^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⓚ

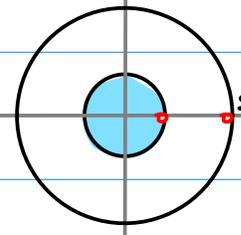


$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

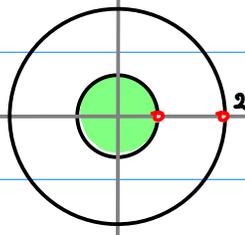


$$\begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⓛ

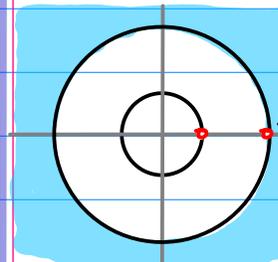


$$\begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

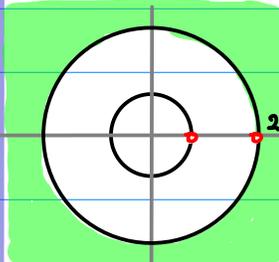


$$\begin{cases} 0 & (n \geq 0) \\ (\frac{1}{2})^{n+1} - 2^{n+1} & (n < 0) \end{cases}$$

Ⓜ

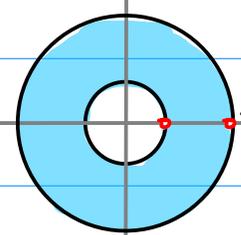


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

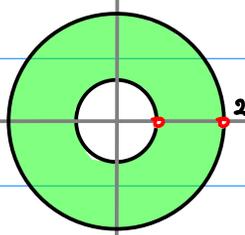


$$\begin{cases} 2^{n+1} - (\frac{1}{2})^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⓨ



$$\begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$f(z) \xleftrightarrow{z^{-1}} X(z)$$

$$a_n = x_n$$

I

$$P_1 = 0.5 \\ P_2 = 2$$

$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$P_1 = 0.5 \\ P_2 = 2$$

II

$$P_1 = 0.5 \\ P_2 = 2$$

$$\begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$P_1 = 0.5 \\ P_2 = 2$$

III

$$P_1 = 0.5 \\ P_2 = 2$$

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$P_1 = 0.5 \\ P_2 = 2$$

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$$P_1 = 0.5 \\ P_2 = 2$$

$$\begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

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$$\begin{cases} 0 & (n > 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n \leq 0) \end{cases}$$

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